

ON THE KINETIC ENERGY SPECTRUM NEAR THE GROUND

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ABSTRACT

For six stations in the northeastern United States, the spectrum of horizontal wind speed was analyzed using 10 yr of 1-min averaged, hourly surface reports. The fast Fourier transform technique was employed to estimate the spectrum between 1 cycle/2 hr and 1 cycle/2 yr.

The kinetic energy spectra show two major spikes at periods of 24 hr and 1 yr. However, most of the energy is contained in the traveling cyclones and anticyclones with periods between 2 and 7 days. The apparent discrepancy between Van der Hoven's results and our results concerning the existence of an important diurnal cycle in the kinetic energy can be explained by Blackadar's theory of the diurnal wind variation with height. Van der Hoven's spectrum represents conditions near the top of the surface layer, while our data were taken well within the surface layer. A line-by-line investigation of the diurnal peak reveals a very sharp line at 2400 hr with two side lobes 3.9 min away from the main line. These side lobes are probably caused by an annual modulation of the diurnal cycle.

The spectra tentatively corrected for aliasing give some indication of the existence of a spectral gap between small-scale turbulence and mesoscale phenomena.

1. INTRODUCTION

Van der Hoven (1957) made a detailed analysis of the power spectrum of the horizontal wind speed in which he analyzed measurements taken at Brookhaven National Laboratory, Long Island, at a height of about 100 m. By piecing together various sets of observations, he was able to present a composite picture of the contribution to the total variance of the wind speed from different frequency ranges. The kinetic energy spectrum thus determined covered periods from 4 sec to about 2 mo. He convincingly showed that most of the variance of the wind speed can be explained by the passage of large, synoptic scale pressure systems with periods of about 4 days. Turbulence of the order of minutes also gave some contribution, although it was much smaller. However, between these two regions he found a broad section of the spectrum centered near the period of 1 hr with very little energy connected with it; this last portion of the spectrum was therefore called the "spectral gap" region. His analysis further showed a small rise in the spectrum for periods of about 12 hr, but surprisingly there was not much energy near the 24-hr period.

More recent investigations by Bysova et al. (1967) using 40 hr of wind speed data from a 300-m tower at

Obninsk (U.S.S.R.) also show a pronounced minimum in the spectrum between the turbulent and mesometeorological wind fluctuations.

Recently, 10 yr of hourly wind records have become available on magnetic tape for several weather stations in the United States. This made it feasible to repeat and extend Van der Hoven's analysis to include the spectrum from periods of a few hours up to a few years. Another contributing factor was the advance in data analysis made possible by the introduction of the "fast Fourier transforms" (FFT), recently developed by Cooley and Tukey (1965) for calculating the Fourier components directly from a long time series in very little computation time. This practically eliminates the difficulties connected with the piecing together of various portions of the spectrum.

In contrast to Van der Hoven's work, our analysis shows a major spike in the wind spectrum at a period of 24 hr. Therefore, an important part of this paper will be concerned with an investigation of the diurnal variability in the kinetic energy.

2. DATA AND DATA ANALYSIS

In 1965, records of hourly surface data for several U.S. stations covering the period Jan. 1, 1949, through

Dec. 31, 1958, were stored on magnetic tape at The Travelers Research Center, Inc., Conn., under contract for the U.S. Air Force.

DATA

A group of three stations (Caribou, Old Town, and Portland) in Maine, representing the northeastern part of the United States, and another group in the Great Lakes Region (Detroit and Sault Sainte Marie, both in Michigan, and Duluth, Minn.) have been investigated. The climate at all of these stations is influenced to a high degree by the proximity of water, with the exception possibly of Caribou. As we shall see later, there appear to be no major differences in the shape of the spectrum at the stations considered. Therefore, in this paper we have arbitrarily selected Caribou for a more detailed study. In a future study we intend to include other groups of stations having a more continental climate as well as stations at a lower latitude.

The wind observations were taken 1 hr apart and represent 1-min averages. The height of the wind sensor is not the same for all stations but varies from 30 to 80 ft above the ground. Other factors that make the results for the different stations not strictly compatible are 1) differences in station elevation above sea level and 2) differences in exposure of the wind sensor due to, e.g., neighboring buildings. In some cases the location of the wind sensor was appreciably changed during the 10 yr of record. However, we have included all 10 yr in our analyses.

Although Caribou is located only 150 mi from the Atlantic coast, its climate can be classified as a typical continental type. Old Town and Portland have an "east coast" maritime climate; the winds are generally light.

The climate of the group of stations in the Great Lakes Region has some maritime characteristics because of the location of the stations close to the Great Lakes. Rather frequent changes in the weather pattern occur, since nearly all atmospheric disturbances that move eastward across the country pass close enough to affect the weather.

Further climatological information is given in table 1.

DATA ANALYSIS

In the present study we are interested in the contribution from different frequency bands to the variance of the horizontal wind speed (i.e., in the kinetic energy spectrum). The contribution from each frequency range is estimated by calculating the sum of the squares of the coefficients in the cosine and sine transforms (the Fourier coefficients) at the particular frequency. Recently a method for efficiently computing these coefficients, called the fast Fourier transform (FFT), has been reported by Cooley and Tukey (1965). This method produces savings of up to 99 percent of computer time over conventional methods of finding the Fourier coefficients. The FFT apparently not only reduces the computation time but also slightly reduces round off errors associated with these compu-

TABLE 1.—Climatological information from "Local Climatological Data—With Comparative Data" published by the U.S. Weather Bureau (1958)

	Station identification	Lat. (° N.)	Long. (° W.)	Station elevation above sea level (ft)	Wind instruments above ground (ft)	Reported changes in exposure of wind instruments
Caribou, Maine.....	CAR	46.9	68.0	620	33	none
Old Town, Maine.....	OLD	44.9	68.7	124	27	(¹)
Portland, Maine.....	PWM	43.7	70.3	61	55	none
Detroit, Mich.....	DET	42.4	83.0	619	81	none
Sault Ste. Marie, Mich.	SSM	46.5	84.4	721	33	(²)
Duluth, Minn.....	DLH	46.8	92.2	1409	55	(³)

¹ We could find no reports on the height of the wind instruments before 1954. It is, however, more likely that the height was not changed during the 10 yr of record.

² On June 15, 1949, the wind instruments were relocated on another building. The elevation above ground was changed from 43 ft to 33 ft on the same date.

³ On July 1, 1950, the wind-recording equipment was moved from the city to the Duluth Airport.

tations. The computation time is reduced by a factor of $(\log_2 N)/N$ where N is the number of data points in the time series (Group on Audio and Electroacoustics (G-AE) Subcommittee on Measurement Concepts, 1967). For further details the reader is referred to the papers in a special issue of the *IEEE Transactions on Audio and Electroacoustics* (June 1967), entitled "On Fast Fourier Transform and Its Application to Digital Filtering and Special Analysis."

A fast Fourier transform subroutine was coded for the Univac 1108. This subroutine replaces a time series of length 2^m (m an integer) with the Fourier coefficients for the time series. Since the maximum value of m allowed by the program equals 14, the time series may contain a maximum of 16,384 data points. In the case of hourly data, one can analyze a series of at least 1 yr in one pass (8,760 points). In order to analyze a series of 10 yr, we shall replace the values in the original time series by the averages over 6 hr. The number of data points is then reduced from 87,600 to 14,600.

Because of the finite length record, it is impossible to resolve Fourier coefficients corresponding to frequencies separated by less than a certain amount. This limit of resolution measured by ΔF has the value of 1 over the period of the entire data record, i.e., 1 cycle/1 yr or 1 cycle/10 yr, according to whether a 1-yr or a 10-yr period is being analyzed.

In order to clarify these statements, let us follow the reasoning given by Bingham et al. (1967, pp. 57-58). The finite Fourier transforms resolve exactly any combination of sine terms and cosine terms of frequencies $F_0, F_1, F_2, \dots, F_j, \dots$ ($=0, \Delta F, 2\Delta F, \dots, j\Delta F \dots$). Thus, if a component $c_j \cos(2\pi F_j t + \phi_j)$ were added to the time series, the transform would be affected in the coefficients a_j and b_j (the coefficient a_j would be replaced by $a_j + c_j \cos \phi_j$, and b_j by $b_j + c_j \sin \phi_j$), but the other coefficients a_k and b_k , where $k \neq j$, would not be affected. However, if a component $c \cos(2\pi F t + \phi)$ were added to

the time series, with F not equal to one of the F_j , all Fourier coefficients would be affected by an amount proportional to $(|F-F_j|/\Delta F)^{-1}$ for $|F-F_j| > 2\Delta F$. Thus the influence of a sinusoidal term at frequency F is largest for frequencies F_j nearest F , although its influence extends to coefficients at frequencies F_j many times ΔF away. This "spill-over" of the influence may be decreased by decreasing ΔF (which means considering longer records).

Another means of sharpening the resolution lies in a filtering procedure known as "hanning" the Fourier coefficients (see Blackman and Tukey, 1958, pp. 14-15). This may be done directly by applying the hanning weights $-\frac{1}{4}$, $\frac{1}{2}$, $-\frac{1}{4}$ to the coefficients at frequencies $F-\Delta F$, F , $F+\Delta F$, respectively, or indirectly by multiplying the original time series by a *data window* function before processing the data. If the time t runs from 0 to T , a data window can be obtained by multiplying the data series by a cosine bell:

$$(1 - \cos 2\pi t/T)/2.$$

This curve has a maximum value of 1 in the middle of the series and falls off smoothly to 0 at the beginning and at the end of the series. After applying the window, the influence of a sinusoid of frequency F on the other coefficients tends to $(|F-F_j|/\Delta F)^{-3}$ for $|F-F_j| > 4\Delta F$. That is, the influence of a line at frequency F is now restricted to a smaller neighborhood of that frequency.

Prior to applying the data window, the mean of the series and a least-squares linear trend were computed and subtracted from the series. The presence of such long-term variations would tend to bias the spectrum by introducing extra variance in the lower frequencies.

After the data were conditioned by the mean and trend removal and by the data window, the time series was extended by adding sufficient zeros to attain the number of 2^{14} data points required by the subroutine. As a result, the number of Fourier components computed increased from half of 8,760 (or half of 14,600) to 8,193 at frequencies that divide the frequency interval $0 \leq F \leq 1/(2\Delta t)$ into 8,192 equal parts (in our case Δt equals 1 hr or 6 hr). Thus, we have an apparent increase in resolution over this interval. The increase in resolution is not real, however, since the Fourier coefficients are no longer independent, but must be related to each other in such a manner as to produce the extra zeros. This introduces an interaction between components similar to the spill-over mentioned above and emphasizes the need of hanning the data with the cosine bell.

A plot of the individual power estimates versus frequency will in general be very "rough" showing many individual small peaks. Often, the location and magnitude of these peaks is climatologically insignificant, being due to *sampling fluctuations* rather than any systematic physical interaction. It is thus desirable to average out these peaks to obtain a more useful presentation. Of course, we then give up much detail in resolution. How-

ever, with a long time series covering nearly three decades of frequency, the resolution is generally one or two orders of magnitude greater than required in any case.

There are two possible means of performing the averaging to account for sampling fluctuations. One method is to break up the entire record into several parts of equal length, next to compute spectral estimates for each part, and finally to average these estimates at the corresponding frequencies. In effect, this corresponds to taking several samples. An alternative method, which we actually use here, is to calculate the Fourier coefficients and then to average estimates in several frequency bands (Hinich and Clay, 1968). Incidentally, the loss of resolution in the frequency domain produced by this averaging tends to compensate for the fictitious increased resolution produced by the subtended zeros.

In our case, we have split up the frequency scale into about 80 bands. We distributed the band limits according to a logarithmic scale between the lowest and highest frequencies attainable. The lowest frequency is evidently $1/(N\Delta t)$, and the highest (the Nyquist or folding) frequency $1/(2\Delta t)$, where N is the total number of observations and Δt is the time interval between observations. For example, for a 10-yr record the resolution in the vicinity of 1 yr is about one-tenth of a year or 36 days; however, near periods of 1 day the resolution is approximately 1/3650 day or 24 sec. In our logarithmic scale with 80 bands, the power in the band near 1 day represents the average over several hundred individual estimates (see table 3 in the Appendix), while in the bands near 1 yr only one or two estimates are included.

In order to have some assurance that significant information is not being averaged out along with variations due to sampling fluctuations, some form of *confidence statistics* would be useful. If the individual estimates within each band are distributed very tightly around the mean for the band, more confidence would be attached to that mean than if the individual estimates were widely scattered. Again, more confidence is attached to a mean if many estimates are used to compose it.

A statistic with these properties is given by

$$DF = n M^2 / (M^2 + V)$$

where n is the number of estimates in the band, M is the mean, and V the variance. The quantity DF is referred to as the number of *equivalent degrees of freedom* and, according to Blackman and Tukey (1958, pp. 21-25) the chi-square distribution with DF degrees of freedom has the same mean-variance relation as the estimates in that band.

It should be emphasized that a low number of degrees of freedom does not mean that the data in this band is unreliable, but merely that the computed mean does not represent the estimates in the band very well. Thus, if resonance, for example, produces a very large, narrow peak inside the band, say at the 24-hr period, the degrees

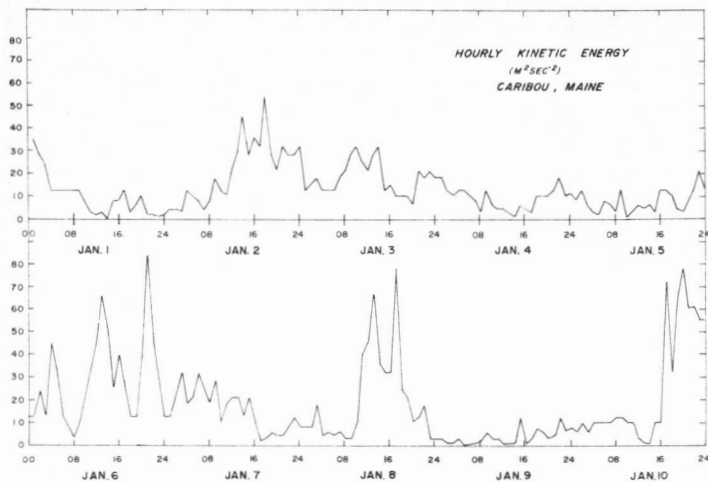


FIGURE 1.—Hourly kinetic energy at Caribou, Maine, for the first 10 days of January 1949.

of freedom estimate for that band is sharply reduced (see, e.g., table 3 in the Appendix of this paper). In such cases a close, line-by-line examination of this band may be indicated.

3. THE ALIASING PROBLEM

As mentioned earlier, the reported wind speeds represent 1-min averages and are spaced 1 hr apart. Figure 1 shows a plot of the reported hourly kinetic energy for Caribou, Maine, for the first 10 days of January 1949. The graph gives evidence of high-frequency fluctuations in the data.

This intuitive conclusion is backed up by Van der Hoven's results at Brookhaven (fig. 2) which show that the energy in wind fluctuations below 2 hr cannot be neglected. The power in the frequency range between 1 cycle/2 hr and 1 cycle/1 min causes an aliasing problem. However, the aliasing is not as bad as it might appear from figure 2. The observations in the high-frequency part of the spectrum were taken during the passage of a hurricane near Brookhaven. A more normal situation would certainly give lower values for the maximum in the minute range; a maximum value between 0.5 and 1.0 m² sec⁻² might be expected (table 2) instead of the value of 3.0 m² sec⁻² shown in figure 2.

In the present data sample the power in the non-resolvable frequencies between 1 cycle/2 hr and 1 cycle/1 min is added to and cannot be distinguished from the real power in the resolvable range of frequencies between 1 cycle/10 yr and 1 cycle/2 hr. The higher frequencies that are aliased into a resolvable frequency F are:

$$2F_N - F, 2F_N + F, 4F_N - F, 4F_N + F, 6F_N - F, 6F_N + F, \text{ etc.},$$

where F_N = Nyquist or folding frequency = 1 cycle/2 hr. For example, the power in periods of approximately 72, 51, 33, 28, 21, 19, 15.6, 14.4, 12.4, 11.6, 10.3, 9.7, 8.8 min, etc. will be added to the power in a period of 6 hr.

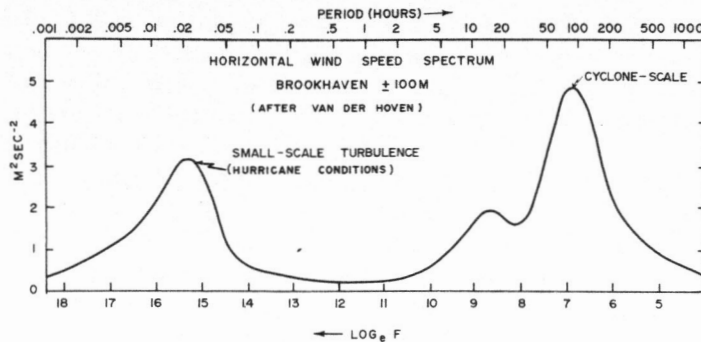


FIGURE 2.—Spectrum wind speed at Brookhaven National Laboratory, Long Island, at about 100-m height (after Van der Hoven, 1957). Frequency F in cycles/4096 days.

TABLE 2.—Spectral intensity in small-scale turbulence maximum as a function of roughness length

	Caribou	Old Town	Portland	Detroit	Sault Ste. Marie	Duluth
z (ft)	33	27	55	81	33	55
V (m sec ⁻¹)	5.75	3.85	4.59	4.95	4.61	6.35
$(P \times F)_{max}$ (m ² sec ⁻²) ($z_0=20$ cm)	.36	.18	.18	.17	.23	.34
$(P \times F)_{max}$ (m ² sec ⁻²) ($z_0=50$ cm)	.60	.31	.28	.26	.39	.53
$(P \times F)_{max}$ (m ² sec ⁻²) ($z_0=100$ cm)	1.00	.54	.42	.38	.64	.82

z = observation height above ground.

V = mean horizontal wind speed.

$(P \times F)_{max}$ = product of power density and frequency in small-scale turbulence maximum.

z_0 = roughness length.

Before going into more detail, let us first discuss the form in which the spectra in this paper will be presented. For each frequency band, the value of the product of the mean power density (P) and the mean frequency (F) will be plotted as the ordinate and the natural logarithm of the mean frequency ($\log_e F$) as abscissa. This commonly used scale gives perhaps a better illustration of the contribution of the various ranges of meteorological interest than a curve of simply the mean power density versus the frequency. In both cases, the area under the curve between the two frequencies F_1 and F_2 gives that portion of the total variance (kinetic energy) that is explained by phenomena in this frequency range

$$\int_{\log_e F_1}^{\log_e F_2} (P \times F) d \log_e F = \int_{F_1}^{F_2} P dF.$$

In our graphs we chose to let the frequency decrease from left to right, in order to have the time scale increase in that direction.

Let us assume that the aliased part of the spectrum between 1 cycle/1 min and the folding frequency of 1 cycle/2 hr resembles *white noise*, i.e., the power is not a function of frequency. The effect of aliasing will then be to add to the true power between the folding frequency

and 1 cycle/2 yr a constant amount, independent of frequency. Aliasing as described above is, of course, independent of the way in which the spectrum is represented. However, different representations of the spectrum might give different impressions of the effect of aliasing. In the graphs in this paper, the product of the power and the frequency (not the power itself) is plotted along the y -axis. Each time that one decreases the frequency by e (in other words, the value of the x -coordinate in the graphs is decreased by 1), the contribution to the y -coordinate (power \times frequency) due to aliasing will decrease by e , simply because the frequency decreases by e . Thus, the effects of aliasing tend to show up mostly near the folding frequency of 1 cycle/2 hr. If the true spectrum were to show a decrease of power with increasing frequency near the folding frequency instead of being independent of frequency, the effects of aliasing would be still more concentrated near this frequency.

For a better understanding of the aliasing effect, we have performed two experiments for Caribou, Maine. In the first experiment we used all hourly surface reports and analyzed the spectrum in the range from the Nyquist frequency of 1 cycle/2 hr up to 1 cycle/2 yr. In the second experiment we only used one observation every 6 hr and could, therefore, determine only the spectrum between the new Nyquist frequency of 1 cycle/12 hr and 1 cycle/2 yr. In this last experiment, the power between 2 hr and 12 hr is aliased further into the rest of the spectrum. The results shown in figure 3 indicate that most of the distortion is confined to the spectrum in the vicinity of the Nyquist frequency, i.e., to the range from 12 hr up to about 2 days. In the next section we shall make use of this result in order to make a rough correction to the spectrum of Caribou for the effects of aliasing from the range of periods of 1 min to 2 hr.

4. THE KINETIC ENERGY SPECTRUM

In this section, power spectra of the surface wind speed will be presented which cover cycles from 1 cycle/2 hr to 1 cycle/2 yr. A split up of the spectral analysis in two parts was necessary because of the limitation in the total number of data points in the computer program for the fast Fourier transform.

METHOD OF ANALYSIS OF THE SPECTRUM

The spectrum for periods of 24 hr and up was obtained by an analysis of the 10 yr of record. The hourly data were first averaged over nonoverlapping 6-hr intervals. Next, these 6-hr averages were used as input data for estimating the spectrum from 12 hr and up. The averaging process reduces the amplitude of each wave in the spectrum by

$$R(F) = \sin(\pi FT) / (\pi FT)$$

where $R(F)$ = the response function for a wave with frequency F , F = frequency in cycle/hr, and T = the filtering interval = 6 hr (Holloway, 1958). The original

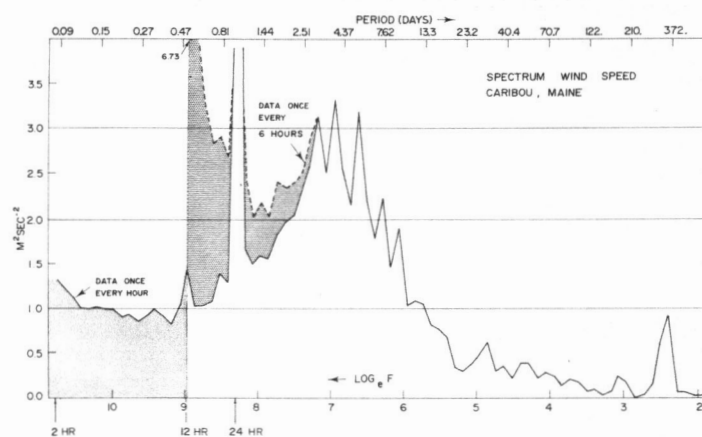


FIGURE 3.—Example of the effects of aliasing on the wind speed spectrum. Dashed line gives the spectrum at Caribou, Maine, if power between 2 and 12 hr is aliased. Frequency F in cycles/4096 days.

spectral power was restored by multiplying the computed power at each frequency by $1/R^2(F)$.

The spectrum for periods between 2 hr and 1 day was obtained by analyzing each year of the 10 yr of record separately and then averaging the spectral estimates thus obtained. In general, the effects of aliasing are most severely felt in this part of the spectrum, i.e., close to the Nyquist frequency.

THE HIGH FREQUENCY PART OF THE SPECTRUM

Our concern at this point is to estimate what the most probable shape of the spectrum will be between periods of 2 hr and 1 min. Since Van der Hoven made his study, further evidence has been accumulated that there exists a broad gap in the spectrum. For example, Bysova et al. (1967) analyzed a continuous record of 40 hr of wind velocity fluctuations measured at Obninsk (U.S.S.R.). Their analysis shows at all levels (25, 75, 150, and 300 m) a pronounced spectral gap between periods of about 15 min and 7 hr. This gap, which separates the mesoscale phenomena (periods of the order of a few hours) from the high-frequency turbulence (periods of the order of minutes), appears to be centered at a period of about 1 hr.

The value of the high-frequency maximum of $P \times F$ can be estimated using a general relationship found by Busch and Panofsky (1968) for the maximum

$$(P \times F) / u^{*2} \approx 1$$

where u^* = friction velocity = $kV / (\log_e z/z_0)$ in neutral air, k = von Kármán constant = 0.4, z_0 = roughness length, z = height observation above ground, and V = mean wind speed. For simplicity, effects of stability have been neglected in the formula for the friction velocity. Table 2 gives the calculated values of $P \times F$ in this maximum, assuming several different values of the roughness length.

In the case of Caribou, we rather arbitrarily selected a value of $0.8 \text{ m}^2 \text{ sec}^{-2}$ in the high-frequency maximum

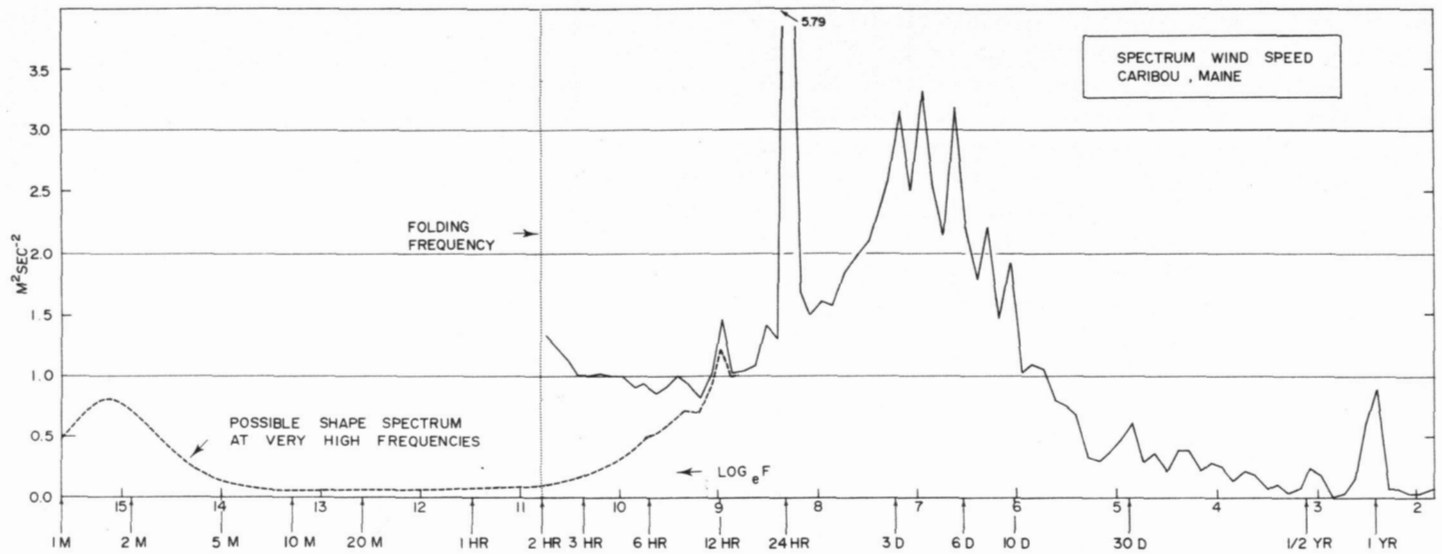


FIGURE 4.—Results of an attempt to correct the spectrum at Caribou, Maine, for the effects of aliasing from periods between 1 min (the basic averaging period of the wind reports) and 2 hr (the Nyquist frequency). Frequency F in cycles/4096 days.

corresponding with a roughness length between 50 cm and 100 cm. The location of the maximum does not seem to be very well fixed. For higher wind speeds it generally shifts to higher frequencies. Again arbitrarily, but perhaps not unreasonably, we selected a location of about 1 cycle/2 min. Having fixed the height and the location of the high-frequency maximum, we drew freehand a hypothetical line for the "true" spectrum (see dashed line in fig. 4). This line gives our estimate of how the spectrum would look if measurements were available for 10 yr with a 1-min instead of a 1-hr separation. In drawing the dashed curve, we used the following guidelines:

1) The conservation of variance of the wind speed. In other words, the area under the dashed curve between the basic averaging frequency of the data (1 cycle/1 min) and the folding frequency F_N (1 cycle/2 hr) should be approximately equal to the area between the solid (computed) and dashed curves to the right of F_N .

2) The assumption of an approximately white power distribution in the aliased portion of the spectrum to the left of F_N . The contribution to the spectrum at the right of F_N due to aliasing will then decrease by e , if F decreases by e (see discussion in section 3). This consideration rather limits the range of acceptable possibilities in drawing the true spectrum curve to the right of F_N . If one would assume, e.g., significantly more power to the left of F_N than is done in figure 4, the dashed curve would become negative near the folding frequency—an impossible situation. After having drawn the dashed curve in the way described, the total amount of aliased power is directly determined. However, considerable freedom is still left in constructing the shape of the spectrum between the folding frequency and 1 cycle/1 min; more information is needed.

3) The assumption of the existence (which seems to be well proved, Busch and Panofsky, 1968) and next of the magnitude and location of the high-frequency maximum as tentatively derived above in table 2. Finally, one can infer that very little energy is left to be distributed between the high-frequency maximum and the folding frequency.

Although our method of estimating the true spectrum is of course very approximate, the final curve in figure 4 shows that our results using a very long time series are certainly compatible with the existence of a spectral gap in the vicinity of 1 cycle/2 hr.

DETAILED DISCUSSION OF THE SPECTRUM

Starting on the left side of figure 4 for Caribou, Maine, one notices first the rise in the spectrum in the vicinity of a period of 2 min due to small-scale turbulence with a maximum value of $0.8 \text{ m}^2 \text{ sec}^{-2}$. Then there follows the spectral gap region with intensities of the order of $0.1 \text{ m}^2 \text{ sec}^{-2}$ or less between roughly 10 min and 2 hr. Next, in the range from 2 hr up to 2 days, the level of activity starts to rise from the low values in the spectral gap up to a very high level in the "cyclone rise."

Superposed on this part of the spectrum is a minor peak at 12 hr and a major peak at 24 hr. A high, broad plateau of activity of about $3 \text{ m}^2 \text{ sec}^{-2}$ connected with the traveling cyclones and anticyclones is found at periods between approximately 2 and 8 days. After this, the level of activity drops quite rapidly and shows only minor (probably not significant) bumps near periods of 1 and 2 mo. Finally, we reach periods of one-half and 1 yr where again important peaks are found.

A striking feature of the "cyclone rise" is the appearance of spikes. By making the resolution coarser (or by a little

smoothing on the spectrum) it is relatively easy to get rid of the spikes. However, we think the picture as it is may give the reader more insight into the inaccuracies (or rather the effects of sample fluctuations on the spectrum). These fluctuations play a role even when one analyzes a very long sample such as 10 yr of data. It is clear that one should not interpret the peaks (at 3.1, 3.9, and 5.5 days) as true periodicities comparable to the diurnal and annual periods or to their higher harmonics. If one looks at the hundreds of individual power estimates that make up each peak, one soon realizes that each band consists of a large number of peaks and valleys. The high average value in a band means only that the general level of activity is high in that portion of the spectrum. If one studies the spectra for individual years, spikes seem to occur each year but not necessarily at the same frequencies. Averaging of the 10 individual years would largely smooth out these spikes.

For more detailed information on the spectrum for Caribou, e.g., the number of equivalent degrees of freedom for each band, the reader is referred to table 3 in the Appendix.

In figure 5, the measured kinetic energy spectra for the six stations considered in the present study are plotted, both with (full line) and without (broken line) applying a correction for aliasing. The area under each solid curve was normalized; and, because the total variance is not the same for the different stations, the scale along the y -axis is also different. The general shape of these spectra is quite similar, in spite of apparent differences in detail. For example, in the case of Caribou and Duluth the intensity in the cyclone rise appears to be much larger than in any of the other stations. On the other hand, Detroit shows an exceptionally pronounced annual cycle.

The *intensity in the cyclone rise* for Caribou and Duluth has a value between 2 and 3 $\text{m}^2 \text{sec}^{-2}$, while for the other stations the intensity is only 1 to 1.5 $\text{m}^2 \text{sec}^{-2}$. According to the description in the *Local Climatological Data—With Comparative Data* (U.S. Weather Bureau, 1958), Caribou Airport is located on the top of high land which is about on the same level as most of the surrounding, gently rolling hills. The exposed location of this airport probably causes the greater activity in the cyclone and anticyclone scale, while in the case of Duluth Airport the large value must be related to its high elevation above sea level (see table 1). The differences in intensity can thus be explained by assuming that the observations at Caribou and Duluth are more representative of the conditions at higher levels in the atmosphere.

One can expect that, in general, the *annual period* in the kinetic energy will show up most prominently at continental stations and at stations located at high latitudes. Since the six stations considered in this study are located in a relatively narrow latitude belt between 42° N. and 47° N., only the effect of continentality might be noticeable. However, there is no clear indication of this effect in the graphs given in figure 5, possibly because

the stations are all located close to either the Atlantic Ocean or to the Great Lakes.

One other important point is that—if one allows for the effects of aliasing (see dashed curves in fig. 5)—all spectra tend to show low values near the Nyquist frequency of 1 cycle/2 hr.

COMPARISON WITH VAN DER HOVEN'S RESULTS

If we compare figures 2 and 4, good qualitative agreement is generally found between the spectrum at Brookhaven, Long Island, determined by Van der Hoven, and our spectrum for Caribou, Maine. As we have pointed out before, the small-scale turbulence maximum in the minute range has an abnormally large value in the case of Brookhaven, due in part to hurricane conditions present in its determination. The difference in magnitude of the cyclone peak, i.e., 5 $\text{m}^2 \text{sec}^{-2}$ for Brookhaven and less than 3 $\text{m}^2 \text{sec}^{-2}$ for Caribou, could be related to the difference in location. However, it appears more likely that it is mainly due to the difference in elevation above ground level at which the observations were taken (respectively, 100 m and 10 m). Another contributing factor may be that Van der Hoven's observations were made during the winter half year, while ours are representative of the entire year.

One important qualitative difference, however, is the surprising lack of a diurnal peak in the Brookhaven data even though there is a semidiurnal peak of comparable magnitude. The most probable reason for this discrepancy lies in the fact that the Brookhaven observations were taken at a height of 100 m, which is near the top of the surface layer (see fig. 6, taken from a report by Singer and Raynor, 1957), while our data represent conditions within the surface layer. Both Van der Hoven's results at the top of the surface layer and our results within the surface layer are in very close agreement with Blackadar's description (1959) of the typical diurnal wind variation with height.

5. THE DIURNAL CYCLE IN THE KINETIC ENERGY

In table 4 (see Appendix) the hourly kinetic energy averaged for each of the 10 yr is given as a function of local time (see also fig. 7). For each station and for each year there is a large and systematic diurnal variation. The maximum value is found between 2 and 3 p.m., and the minimum in the early morning. The phase of this diurnal variation changes rather rapidly with height. Singer's measurements (fig. 6) show that around 120 m the phase of the diurnal cycle is shifted by 180° compared to the phase at the surface. At this height the maximum wind speed is observed at night, and a minimum around midday.

The change of wind speed with elevation and time of the day was clearly explained by Blackadar (1959) as being related to the *coupling* and *decoupling* of the surface and upper layers. Because of the frictional drag exerted

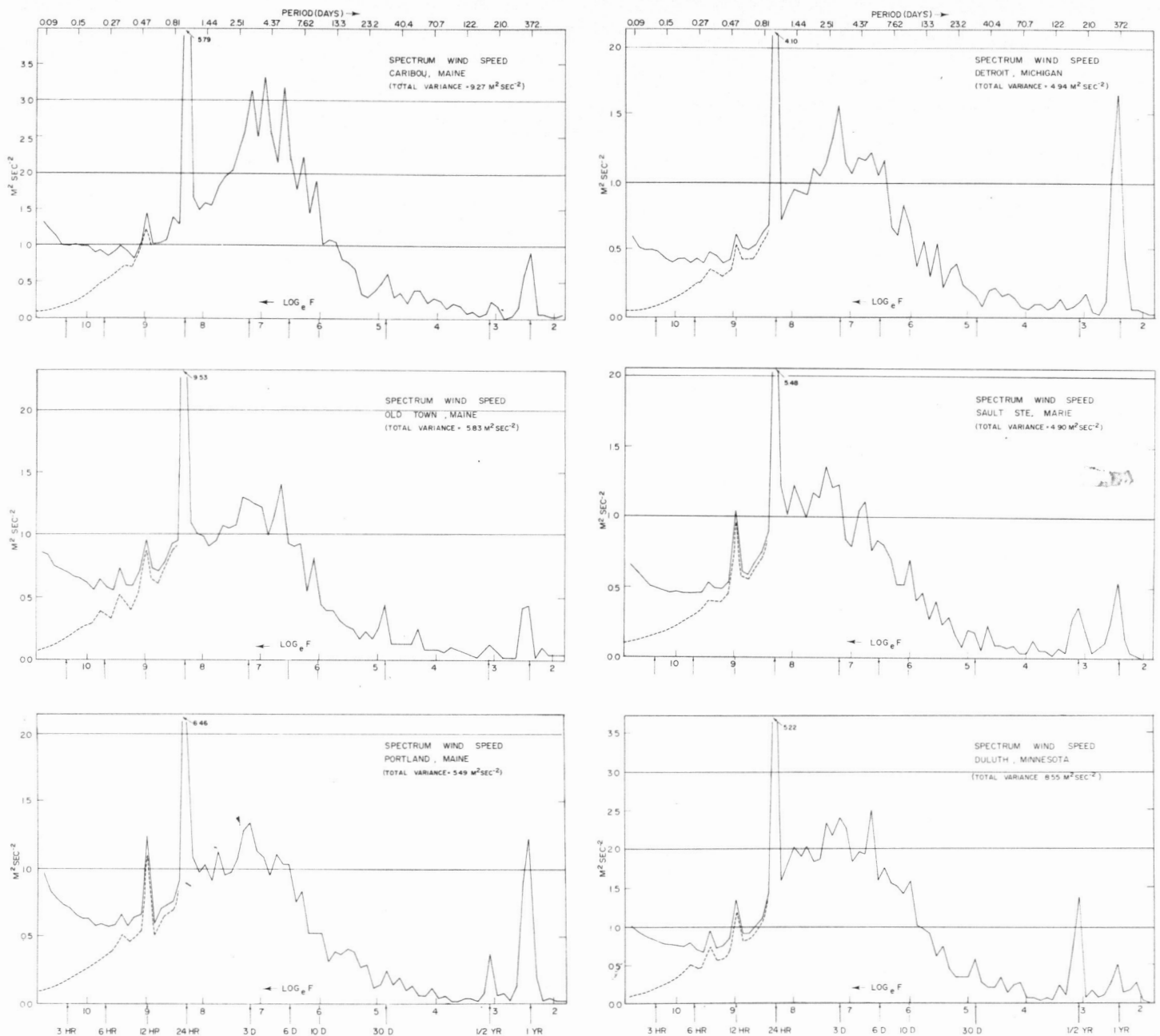


FIGURE 5.—Spectra wind speed determined from 10 yr of hourly wind reports. The dashed curves represent a rough estimate of the spectrum corrected for the effects of aliasing. Frequency F in cycles/4096 days.

on the air by the earth's surface, the wind speed generally increases with height above the ground. During the day, convective mixing will transfer momentum from higher levels to the surface layer. The wind speed in this layer will thus increase until an equilibrium is reached between the supply of momentum from above and the loss due to friction with the earth's surface; but this same process will slow down the upper layers during the daytime. However, at night convective mixing stops; and the surface and upper layers become effectively decoupled by the formation of a temperature inversion. The air near

the surface then slows down, while the air in the upper layers speeds up.

The spectra presented by Bysova et al. (1967) for heights of 25, 75, 150, and 300 m seem also to show an important contribution at frequencies near 1 cycle/24 hr, the highest contributions being found at 300 m.

The diurnal variation presented in table 4 for the different stations shows a remarkable variability from year to year. We have the impression that all this variability cannot be explained away by changes in exposure of the wind instruments and that there may well be a

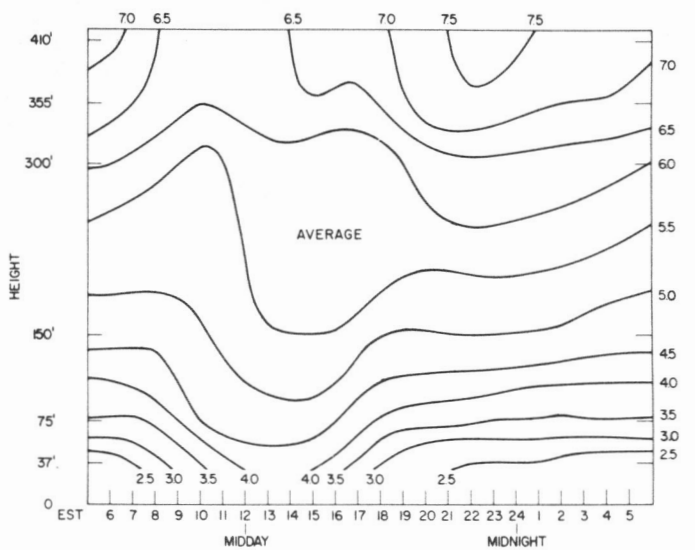


FIGURE 6.—Diurnal variation of wind speed ($m\ sec^{-1}$) with height at Brookhaven National Laboratory, Long Island (after Singer and Raynor, 1957).

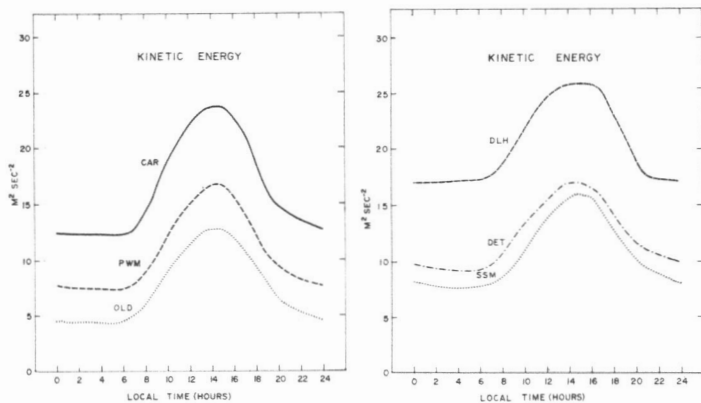


FIGURE 7.—Ten-year average of the kinetic energy as a function of local time at Caribou, Old Town, and Portland, Maine; Detroit and Sault Sainte Marie, Mich.; and Duluth, Minn.

long-term cycle superposed. However, we do not want to pursue this topic further since this is beyond the aim of our present investigation. Our purpose in showing the results for the different years is to make clear that the diurnal cycle shows up in the results for *each* year.

THE LACK OF A DIURNAL CYCLE IN THE SPECTRUM OF THE ZONAL AND MERIDIONAL WIND COMPONENTS

It may be of interest to compare the present results with the spectrum obtained from separate analyses of the west-to-east (u) and the south-to-north (v) components of the wind (fig. 8). In the second type of analysis there is a significant increase (by a factor of 3) in total variance because the wind direction adds a new degree of freedom. However, the most interesting difference (compare figs. 4

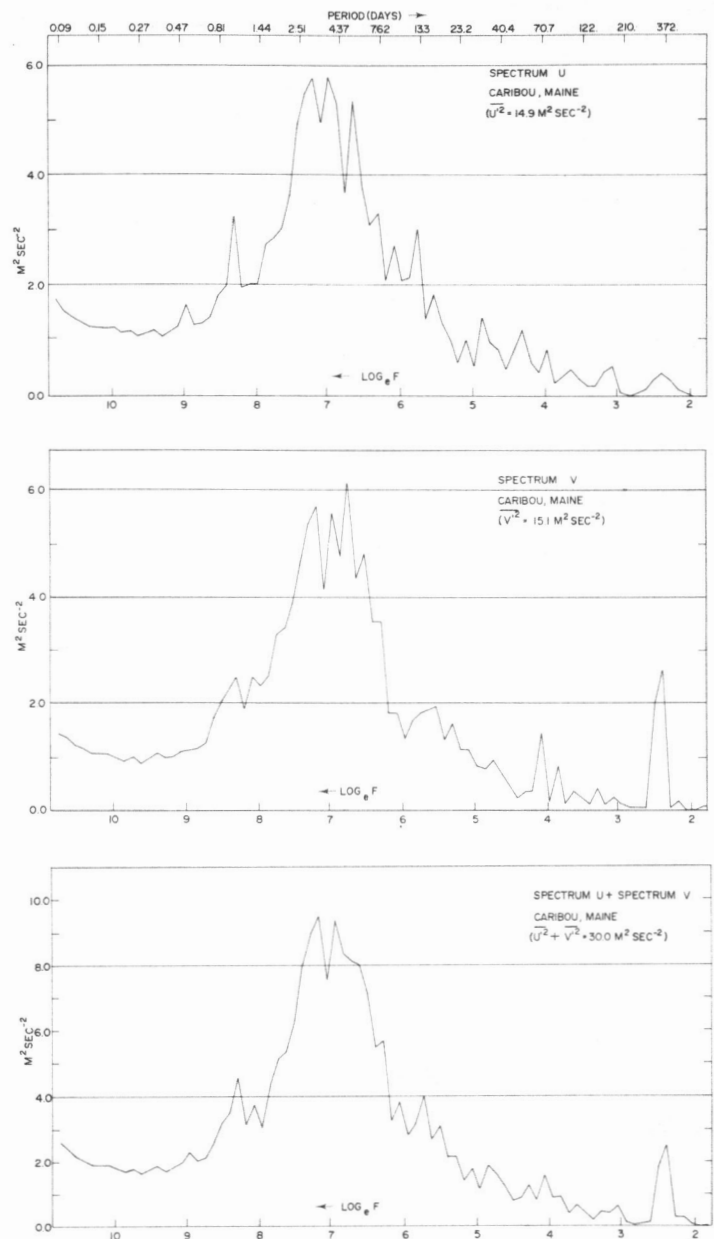


FIGURE 8.—Spectra of the west-to-east (u) and the south-to-north (v) components of the wind for Caribou, Maine, determined from 10 yr of hourly wind reports. Frequency F in cycles/4096 days.

and 8) lies in the fact that the diurnal peak does not show more prominently in the u - and v -spectra.

The lack of an important diurnal cycle in the zonal and meridional wind components is indeed very surprising. The interpretation is probably that the wind direction at the stations considered is—at least near the surface—highly variable and that it does not change systematically during the course of a day. The 10-yr mean u - and v -components and the total wind speed as a function of the time of the day are shown in figure 9. If stations with strong land- and sea-breeze effects had been selected

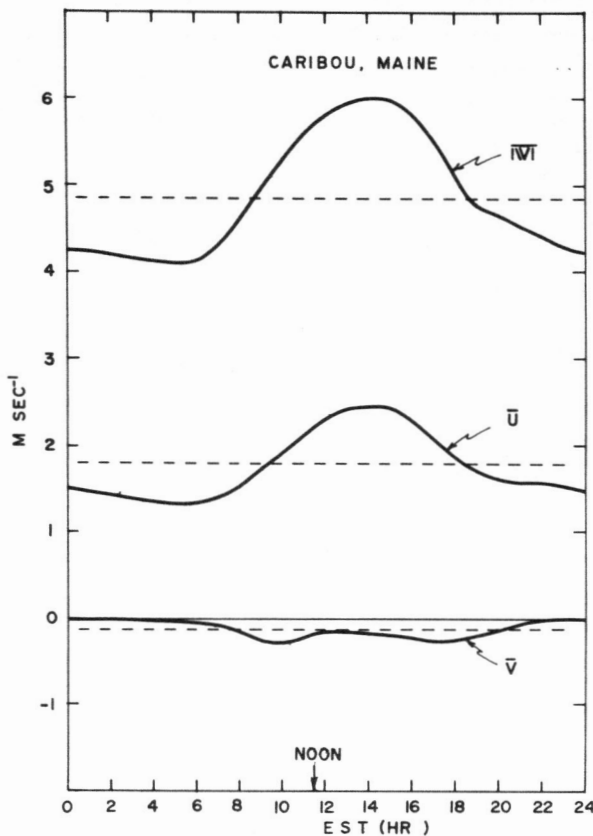


FIGURE 9.—Ten-year average of the zonal (u), the meridional (v) wind component and the total wind speed ($|v|$) as a function of local time.

in this study, the diurnal cycle would certainly have been more prominent in their u - and v -spectra.

We might also mention the strong annual periodicity in the v -component of the wind, which does not show up in the u -component. The same is true for the other stations in Maine (not shown here). However, in the cases of Detroit and Sault Sainte Marie the annual period is most dominant in the u -component, while for Duluth both the u - and v -components show an important annual cycle. It is not obvious what causes these differences in the u - and v -spectra.

THE DIURNAL SPIKE IN THE KINETIC ENERGY SPECTRUM

From the kinetic energy spectra as shown in figure 5, the diurnal band was selected for further study. The logarithm of the *individual* power estimates (not multiplied by the frequency) is graphed on a linear frequency scale in figure 10.

A sharp spike at exactly 24 hr dominates the picture. However, a little distance away from this spike the intensity drops off rapidly. In addition to the main peak, there are on both sides secondary maxima of about equal intensity. A plausible explanation for the occurrence of these side lobes, which are located at a distance of about 4 min from the peak, seems to lie in an analogy with the

“beating” phenomenon in acoustics. Our basic assumption is that the amplitude A of the diurnal cycle in the kinetic energy is not constant throughout the year, but has an annual variation

$$A = A_1 + A_2 \cos(2\pi t/T),$$

where t is measured in days and $T = 365.25$ days. A spectral analysis would show at 24 hr the intensity of that part (A_1) of the diurnal cycle which is constant. However, the remaining signal would be interpreted as being the sum effect of two cycles at $(1 + 1/365.25)$ and $(1 - 1/365.25)$ cycles per day, corresponding with periods of respectively 24.066 hr and 23.934 hr. These frequencies are indicated in figure 10 by arrows.

$$\begin{aligned} A \cos(2\pi t) &= (A_1 + A_2 \cos(2\pi t/T)) \cos(2\pi t) \\ &= A_1 \cos(2\pi t) + (1/2)A_2 \cos 2\pi t(1 + 1/T) \\ &\quad + (1/2)A_2 \cos 2\pi t(1 - 1/T). \end{aligned}$$

With some imagination one can also detect a weaker semiannual modulation of the diurnal cycle at periods of 24.13 hr and 23.87 hr. The meaning of this effect is that the modulation during the year is not a pure sine wave but that also higher harmonics are involved.

In order to test our hypothesis for this beating phenomenon, we investigated the diurnal cycle in January and in July (fig. 11). Indeed, the results show that there is a strong annual modulation of the diurnal cycle. The largest amplitude is found during July, when there is the strongest convective exchange between the surface and higher levels.

Finally, we would like to comment on one other interesting feature shown in figure 10, i.e., the close correspondence in magnitude and shape of the diurnal spike and to a lesser degree of the side lobes between the different stations. One would expect that the intensity in the diurnal cycle would be strongly dependent on the latitude and longitude of the station, in the sense that the intensity would increase with decreasing latitude and also with increasing continentality. However, the stations used in this study are not particularly suitable for investigating these questions.

6. SUMMARY AND CONCLUSIONS

The spectrum of horizontal wind speed was analyzed using 10 yr of 1-min-averaged hourly surface reports for a group of stations in the northeastern United States and another group near the Great Lakes.

The fast Fourier transform technique was used to estimate the spectrum between periods of about 2 hr and 2 yr. The results were compared with the earlier work of Van der Hoven (1957) for Brookhaven, Long Island.

It was found that most of the effects of aliasing are confined to frequencies between the folding frequency of 1 cycle/2 hr and 1 cycle/12 hr. After assuming a reasonable shape for the small-scale turbulence maximum in the

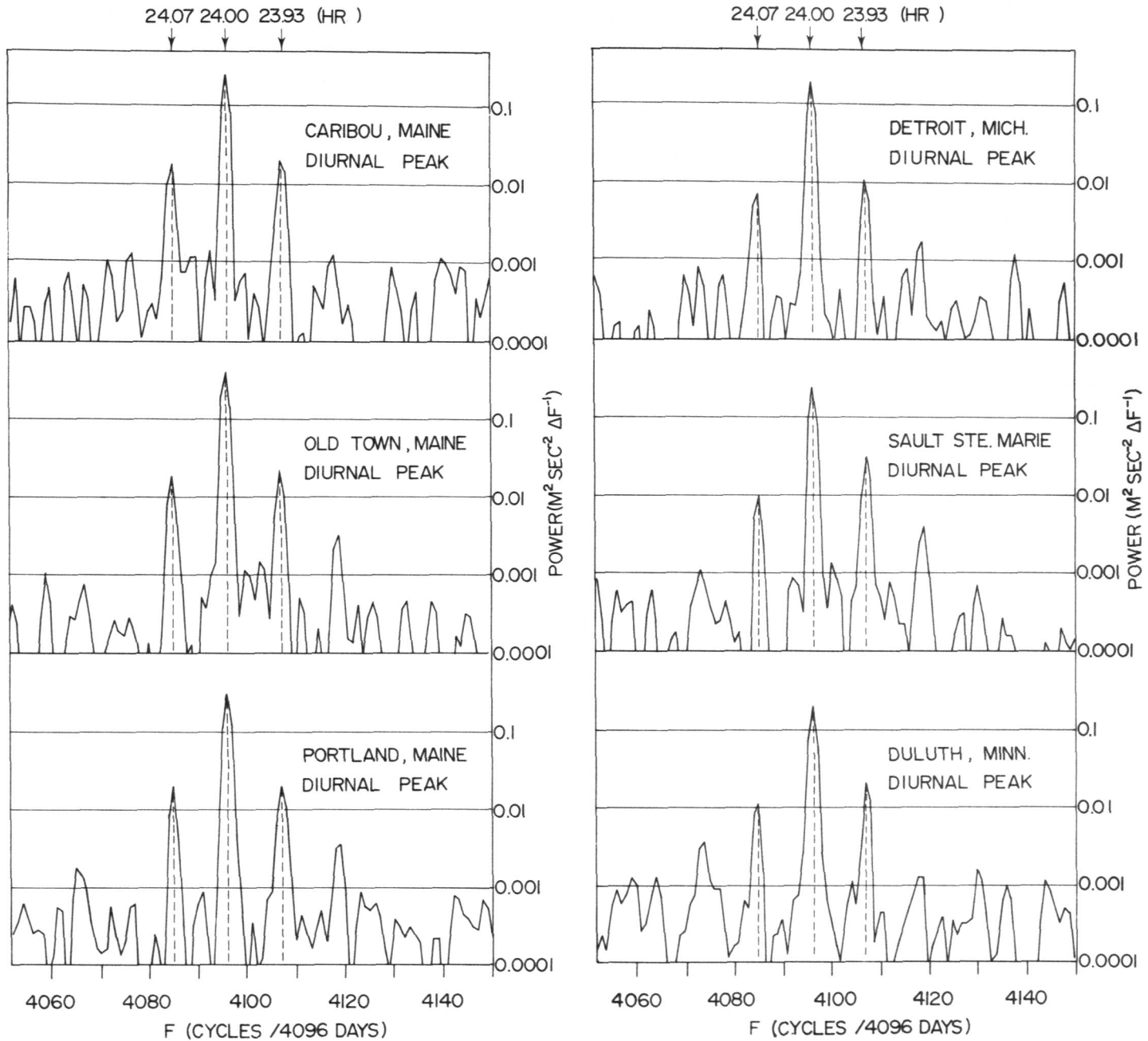


FIGURE 10.—Plot of the individual power estimates ($m^2 \text{ sec}^{-2}$ per elementary frequency interval) versus frequency near the diurnal period

minute range, the spectrum for Caribou, Maine, was corrected for the aliasing from frequencies between 1 cycle/1 min and 1 cycle/2 hr.

With regard to the final form of the spectrum as shown in figure 4, the following comments can be made:

1) Any reasonable method of correcting for the effects of aliasing seems to lead to quite small values for the spectrum near the folding frequency of 1 cycle/2 hr. This finding is compatible with the existence of a wide spectral gap between the small-scale turbulence maximum near a period of 2 min and the mesoscale phenomena at periods of a few hours. The existence of such a gap in the spectrum

was first discussed by Van der Hoven (1957) and has been most extensively documented by the atmospheric turbulence group at The Pennsylvania State University under Professor Hans A. Panofsky. The present study shows that there is some evidence for a spectral gap even if one uses a very long time series.

2) Most of the variance of the wind speed is explained by the activity of traveling cyclones and anticyclones at periods roughly between 2 and 7 days. This agrees quite well with Van der Hoven's results.

3) A large peak in the kinetic energy spectrum was found at the diurnal period and a minor peak at the semi-

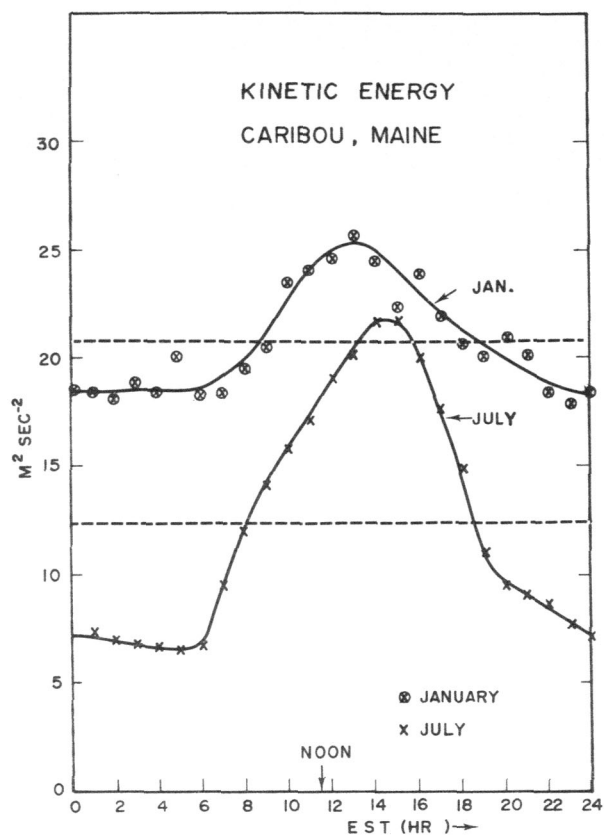


FIGURE 11.—Ten-month mean kinetic energy for January and for July as a function of local time at Caribou, Maine.

TABLE 3.—Data on the power spectrum of the horizontal wind speed at Caribou, Maine. Hourly data cover the period Jan. 1, 1949, through Dec. 31, 1958.

Band	N	\overline{PE} (days)	$\text{Log}_e \overline{F}$	\overline{P}	$\overline{P} \times \overline{F}$ ($\text{m}^2 \text{sec}^{-2}$)	DF
1	8630	0.09	10.75	0.03	1.33	4346
2	7710	.10	10.64	.03	1.22	3923
3	6900	.11	10.53	.03	1.13	3287
4	6180	.12	10.42	.03	1.00	2929
5	5520	.14	10.30	.04	.99	2796
6	4950	.15	10.19	.04	1.02	2388
7	4420	.17	10.08	.04	1.08	2236
8	3960	.19	9.97	.05	.98	1945
9	3540	.21	9.86	.05	.90	1752
10	3170	.24	9.75	.05	.94	1567
11	2840	.27	9.64	.06	.86	1351
12	2530	.30	9.53	.07	.91	1284
13	2270	.33	9.42	.08	1.00	989
14	2040	.37	9.31	.08	.92	996
15	1810	.42	9.19	.08	.83	883
16	1630	.47	9.08	.12	1.03	777
17	1450	.52	8.97	.19	1.47	243
18	1310	.58	8.86	.14	.96	614
19	1160	.65	8.75	.17	1.04	618
20	1040	.73	8.64	.19	1.07	508
21	940	.81	8.53	.28	1.41	476
22	830	.91	8.42	.29	1.30	388
23	442	1.03	8.29	1.45	5.79	6
24	396	1.15	8.18	.45	1.61	201
25	354	1.29	8.07	.45	1.42	182
26	317	1.44	7.96	.55	1.56	148
27	284	1.61	7.85	.58	1.49	129
28	253	1.80	7.73	.77	1.76	142
29	227	2.01	7.62	.93	1.89	104

TABLE 3.—Continued

Band	N	\overline{PE} (days)	$\text{Log}_e \overline{F}$	\overline{P}	$\overline{P} \times \overline{F}$ ($\text{m}^2 \text{sec}^{-2}$)	DF
30	204	2.24	7.51	1.07	1.96	99
31	181	2.51	7.40	1.38	2.26	98
32	163	2.80	7.29	1.71	2.51	82
33	145	3.13	7.18	2.32	3.04	65
34	131	3.50	7.07	2.05	2.41	66
35	116	3.91	6.96	3.08	3.23	63
36	104	4.37	6.84	2.61	2.45	55
37	94	4.88	6.73	2.46	2.06	55
38	83	5.46	6.62	4.10	3.08	44
39	75	6.10	6.51	3.15	2.12	32
40	67	6.82	6.40	2.88	1.73	34
41	59	7.62	6.29	4.03	2.17	35
42	54	8.52	6.18	2.90	1.40	29
43	48	9.52	6.06	4.29	1.85	25
44	42	10.6	5.95	2.57	.99	22
45	39	11.9	5.84	3.05	1.05	21
46	34	13.3	5.73	3.28	1.01	23
47	31	14.9	5.62	2.78	.77	21
48	27	16.6	5.51	2.92	.72	11
49	25	18.6	5.40	2.96	.66	17
50	22	20.8	5.29	1.68	.33	11
51	19	23.2	5.18	1.49	.27	11
52	18	25.9	5.06	2.22	.35	10
53	16	29.0	4.95	3.14	.44	8
54	14	32.4	4.84	4.61	.58	9
55	12	36.1	4.74	2.58	.29	5
56	12	40.4	4.62	3.53	.36	8
57	10	45.3	4.50	2.33	.21	7
58	9	50.6	4.39	4.65	.38	5
59	8	56.6	4.28	5.42	.39	4
60	7	63.1	4.17	3.44	.22	4
61	7	70.7	4.06	4.84	.28	5
62	5	78.8	3.95	4.93	.26	4
63	5	87.2	3.85	2.91	.12	5
64	5	97.6	3.74	5.17	.22	4
65	4	109.	3.62	4.54	.17	-----
66	4	122.	3.51	2.11	.07	-----
67	3	137.	3.40	3.86	.12	-----
68	3	152.	3.29	1.83	.05	-----
69	3	171.	3.18	2.34	.06	-----
70	2	191.	3.07	10.70	.23	-----
71	2	210.	2.97	9.80	.19	-----
72	2	234.	2.86	.65	.11	-----
73	2	265.	2.74	2.22	.03	-----
74	2	304.	2.60	9.50	.13	-----
75	1	341.	2.48	48.94	.59	-----
76	1	372.	2.40	80.85	.89	-----
77	1	410.	2.30	7.97	.08	-----
78	1	455.	2.20	6.01	.05	-----
79	1	512.	2.08	3.40	.03	-----
80	1	585.	1.95	2.51	.02	-----
81	1	683.	1.79	10.86	.01	-----

N = number of spectral estimates in frequency band.
 \overline{PE} = mean period in band (days).
 \overline{F} = mean frequency in band (cycles per 4,096 days).
 \overline{P} = mean power density in band ($\text{m}^2 \text{sec}^{-2}$ per elementary frequency interval, where elementary frequency interval = 1 cycle/4,096 days).
 $\overline{P} \times \overline{F}$ = mean spectral intensity in band ($\text{m}^2 \text{sec}^{-2}$).
 $\text{Log}_e \overline{F}$ = natural logarithm of mean frequency in band.
 DF = number of equivalent degrees of freedom for estimate of \overline{P} in band and $= N \overline{P}^2 / \overline{P}^2$ (see Blackman and Tukey, 1958, p. 24).

Note that the following relation should hold:

$$\text{total variance wind speed} = \sum_{j=1}^{81} N_j \times \overline{P}_j = 0.111 \sum_{j=1}^{81} (\overline{P} \times \overline{F})_j$$
 where the constant 0.111 represents the band width in $\text{log}_e F$ units.

diurnal period. Van der Hoven (1957) did not find the diurnal peak, probably because his data were taken near the top of the surface layer (about 100 m). Blackadar's

theory (1959) adequately explains the differences between Van der Hoven's and our results.

4) At low frequencies in the spectrum beyond the cyclone rise, most activity is found in the annual and semi-annual periods.

The stations used in this study are confined to a latitude belt between 42° N. and 47° N. One may expect that there will be a general shift in importance of the diurnal and annual peaks and of the cyclone rise, if one would study stations at a different latitude.

Through a comparison with the spectra of the zonal and meridional wind components, it is shown that the diurnal cycle in the wind speed is not accompanied by a similar cycle in the wind direction. The diurnal variation in the kinetic energy near the surface with a maximum between 2 and 3 p.m. is in evidence at each of the six stations and for each of the 10 yr.

A closer look at the individual estimates that make up the diurnal peak in the spectrum shows, in addition to the mean spike at 24 hr, side lobes which are probably due to the annual modulation of the diurnal cycle. The diurnal cycle is found to be more pronounced in July than in January. This is what one might expect with a more intense vertical exchange of kinetic energy between the surface and upper levels during the summer.

APPENDIX

Table 3 supplies more detailed information on the spectral estimates and their accuracy for Caribou, Maine. Table 4 gives the average kinetic energy as a function of time of the day for the six stations and the 10 yr studied.

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Copies of the data tapes used in this study were kindly supplied to us by Mr. Howard M. Frazier of the Travelers Research Center, Inc. We would like to acknowledge the help of the following people at the Geophysical Fluid Dynamics Laboratory, ESSA: Mr. Henry Stambler for writing the unpacking program for the data tapes, Mr. Richard Stoner for drawing the figures, and Mrs. C. T. Morgan for typing the manuscript.

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CORRECTION NOTICE

Vol. 97, No. 3, March 1969, p. 286, next to the last sentence: "Moscow Airport in Idaho reported -50°F, on the 30th, the coldest December temperature of record in the State." is incorrect and should be deleted.