NOTES AND CORRESPONDENCE

On Energy Flux and Group Velocity of Waves in Baroclinic Flows

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ABSTRACT

A modified energy flux is defined by adding a nondivergent term that involves β to the traditional energy flux. The resultant flux, when normalized by the total eddy energy, is exactly equal to the group velocity of Rossby waves on a β plane with constant zonal flow. In this study, we computed the normalized energy flux for *linear* wave packets in baroclinic basic states with different vertical profiles. The results show that the normalized energy flux is a good approximation to the group velocity of all parts of the wave packet for the basic states examined.

The extension to the nonlinear case is briefly discussed. The magnitude of the fluxes of a downstream developing wave group over the wintertime northern Pacific storm track defined by a regression analysis is computed, and the group velocity defined by the energy fluxes is found to be comparable to the group velocity of propagation of the observed wave packet. The results indicate a very strong component of downstream energy radiation, suggesting that downstream energy dispersion is very important in the evolution of waves in the storm track.

1. Introduction

Midlatitude tropospheric disturbances have been found to have the tendency to propagate as baroclinic wave groups (e.g., see Blackmon et al. 1984a; Blackmon et al. 1984b; Lim and Wallace 1991; Chang 1993a). While the propagation of long waves is believed to be described very well by the theory of barotropic Rossby wave downstream dispersion (e.g., Blackmon et al. 1984b; Simmons et al. 1983), the propagation of synoptic-scale waves is a bit more problematic, since those waves are highly unstable, and the baroclinic nature of the waves and the background flow must be taken into account. Using a regression analysis, Chang (1993a) found that midlatitude synoptic-scale baroclinic waves also exhibit the characteristics of downstream development and group propagation, and that the propagation is predominantly zonal over the storm track regions.

Orlanski and Chang (1993) studied the energetics of downstream developing baroclinic waves and found that the downstream dispersion of wave energy via the ageostrophic geopotential fluxes (or more generally, the traditional energy flux) is the triggering mechanism behind the growth of new disturbances downstream. Chang and Orlanski (1993) found that this redistribution of wave energy from upstream disturbances toward downstream disturbances can lead to the extension of the storm track deep into regions of weak baroclinicity and thus is very important in the understanding of the extension of the storm track. Orlanski and Chang (1993) computed the value of the energy flux, which, when normalized with the total eddy energy, appears to be a good approximation to the rate at which the disturbance is spreading out; hence, they postulated that the total energy flux should be a good approximation to the group velocity of the disturbances.

It has been pointed out in the literature that the energy cycle may not be the best diagnostic tool to examine the dynamics of waves (e.g., Plumb 1983), since the conversion terms are not uniquely defined, and even the sense of the conversion can be reversed simply by adding a constant term to all the conversions. It is also well known that the energy flux appears in the energetics equation only as a divergent term, and any nondivergent quantity can be added to the flux without affecting the energetics, thus introducing an ambiguity in the definition of the energy flux. In fact, Longuet-Higgins (1964) showed that for the case of barotropic Rossby waves on a β plane, the traditional energy flux is not parallel to the group velocity.

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Pedlosky (1987) suggested a form of the energy flux that "naturally arises when reference is made only to the governing equation of motion" and "reduces to the product of the energy and the group velocity" in the WKB limit. This expression differs from the traditional expression of the energy flux ($\mathbf{u}p$) by a nondivergent vector. In section 2, we will review this expression of the energy flux and relate it to the energy flux discussed in Orlanski and Chang (1993). In section 3, we will compare this energy flux with the group velocity of linear wave packets in baroclinic basic states, and in section 4 we will discuss the extension to nonlinearity.

2. Expression for the "modified energy flux"

Pedlosky (1987, section 6.10) derived several expressions for the energy flux vector and suggested the following form to be preferable due to the aforementioned reasons:

$$\mathbf{S} = \begin{bmatrix} u_g E - \rho_s \psi \dfrac{dv_g}{dt} - \rho_s \dfrac{\beta \psi^2}{2} \\ v_g E + \rho_s \psi \dfrac{du_g}{dt} \\ -\dfrac{\rho_s \psi}{S} \dfrac{d\theta_0}{dt} \end{bmatrix}, \tag{1}$$

where the standard quasigeostrophic scaling has been assumed. Here ψ is the streamfunction and is equal to ϕ/f_0, where ϕ is the geopotential; E is the product of the background density (ρ_s) and energy density. The derivation of this expression is straightforward and will not be repeated here. Since time derivatives are not readily available from observation, here we will rewrite (1) using the ageostrophic velocities. The ageostrophic velocity is defined by

$$\frac{d\mathbf{u}_g}{dt} - \beta y \mathbf{k} \times \mathbf{u}_g - f_0 \mathbf{k} \times \mathbf{u}_a = 0, \tag{2a}$$

$$\frac{d\theta_0}{dt} + w_a \frac{\partial \theta}{\partial z} = 0. \tag{2b}$$

Using (2), (1) can be rewritten as

$$\mathbf{S} = \begin{bmatrix} u_g E + \rho_s u_a \phi - \dfrac{\partial}{\partial y}\left(\dfrac{\rho_s \beta y \psi^2}{2}\right) \\ v_g E + \rho_s v_a \phi + \dfrac{\partial}{\partial x}\left(\dfrac{\rho_s \beta y \psi^2}{2}\right) \\ \rho_s w_a \phi \end{bmatrix}. \tag{3}$$

The notation used here is standard. Hence the vector $\mathbf{S}$ differs from the traditional energy flux by the vector $\mathbf{k} \times \nabla(\rho_s \beta y \psi^2/2)$, which, as discussed in Longuet-Higgins (1964), represents circulation of energy around high and low centers. Using arguments similar to those in Hayes (1977), it is easy to show that the vector defined by (3) is equal to the product of the group velocity and the eddy energy in the limit of a uniform zonal flow when eddy energy is conserved. The argument does not work for the traditional energy flux because the ageostrophic velocity as defined in (2) involves y explicitly. This explicit dependence on y is removed in (3) by the vector $\mathbf{k} \times \nabla(\rho_s \beta y \psi^2/2)$.

While examining the nonlinear evolution of wave packets, Orlanski and Chang (1993) used the traditional energy flux vector to depict the transfer of energy between energy centers and found that they had to (arbitrarily) remove a nondivergent component from the energy flux in order to show clearly the divergent transfer from one energy center toward another. More recently, Orlanski and Sheldon (1993) used an energy flux vector equal to (3) to leading order in a case study of downstream baroclinic development over western North America and found that the resultant vector depicts the energy transfer very clearly even for highly nonlinear waves, and the transfer shows up much better when compared to using the traditional energy flux. As discussed above, the main difference between the two fluxes is that for the modified flux defined in (3), the nondivergent circulation of energy around high and low centers is removed.

3. Group velocity of wave packets in a baroclinic flow

While the vector defined by (3) is exactly equal to the product of the group velocity and energy density of wave groups propagating in a uniform zonal flow, it is not so clear that the relation still holds in a medium with shear. The numerical results of Orlanski and Chang (1993) suggest that the energy flux at the leading edge, when normalized by the energy density, is approximately equal to the speed that a linear wave packet spreads out. But can we make that statement more precise and independent of possible errors and uncertainties inherent in analyzing results of numerical experiments?

In any case, we need to find a working definition for the group velocity of wave groups propagating in a medium that has vertical shear and supports instability. The theory of absolute and convective instability (Merkine and Shafranek 1980; Pierrehumbert 1984) gives us such a definition. According to the theory, the asymptotic form of any part of a wave packet moving with a speed $x/t = V$ can be determined by solving the two equations

$$D(\omega, k) = 0 \tag{4}$$

and

$$\frac{\partial \omega}{\partial k} = V, \tag{5}$$

where $D(\omega, k)$ is the dispersion relation. Lin and Pierrehumbert (1993) outlined a (numerical) procedure on how to solve (4) and (5), and Swanson and Pierrehumbert (1994) gave examples on how the structures of various parts of a wave packet vary with the velocity V. Clearly, V is the group velocity of that part of the wave packet, and by being able to solve for the structure that is moving with the group velocity V, we will be able to compute the energy fluxes and compare it to the group velocity. Obviously, this procedure works only for linear wave groups. We will return to the nonlinear case in the next section.

For simple models such as the Eady model, (4) and (5) can be solved analytically. But for more general cases involving variable shear, a background β, variable stability or density, surface friction, etc., solutions to (4) and (5) would require the use of numerical solutions [as outlined in Lin and Pierrehumbert (1993)]. But the Eady model is invaluable in that the numerical technique can be checked with the analytic results. We have computed the value of [S]/[E] (here [ ] represents averaging over half a wavelength) for many different models, but because of limitation in space, we will show only the results from two cases here. The first is a modified Charney model (with a rigid lid at 10 km, profile A), and the second is a slightly more realistic (while still idealized) vertical profile representing a jet centered at around 12.5 km near the tropopause [profile B; this profile is similar to one used by Kuo (1980)]. The vertical profiles of the nondimensionalized zonal wind and stability are shown in Fig. 1. The density-scale height is taken to be 10 km; for typical midlatitude tropospheric conditions, nondimensional $\beta = \beta{*}H/\epsilon\Lambda \sim 0.6$ and the nondimensional meridional wavenumber $l = l{*}H/\epsilon^{1/2} \sim 1$ have been assumed. Here Λ is the vertical shear ($\sim$3 m s^{-1}/km), and $\epsilon = f^2/N^2$. For profile B, we have also incorporated the effects of an Ekman boundary layer with $\nu \sim 10$ m^2 s^{-1}.

Figure 2 shows the values of [S]/[E] (zonal component) computed for the modified Charney model (profile A). For this model, the (lower and upper) limiting velocities for the expansion of an initially localized wave packet are 0.00 and 0.82, respectively. We see that the value of [S]/[E] corresponds very well to the group velocity over the entire wave packet, differing by at most 0.03 (the maximum jet speed being 1.0). Also shown in Fig. 2 are the contributions from the advection part of S. We see that the advection part generally overestimates the group velocity at the trailing end and underestimates the group velocity at the leading end. As discussed in Orlanski and Chang (1993), over at the leading (trailing) edge the wave packet is concentrated in the upper (lower) levels, where the ageostrophic fluxes are predominantly directed downstream (upstream). Thus, the contribution from the ageostrophic fluxes systematically corrects the "errors" of the advection fluxes toward the group velocity. Near the leading edge of the wave packet, advection contributes to about 70% of the total fluxes, and ageostrophic geopotential fluxes make up the remaining 30%.

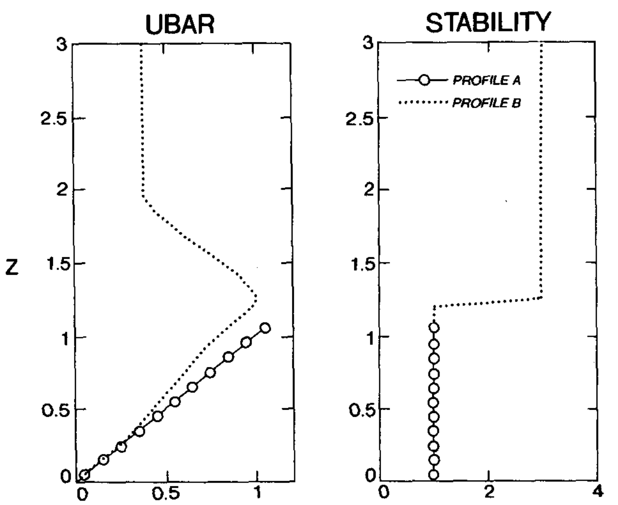

FIG. 1. The vertical profile of nondimensionalized zonal-mean wind (left) and stability (right) for profiles A (line with circles) and B (dotted line).

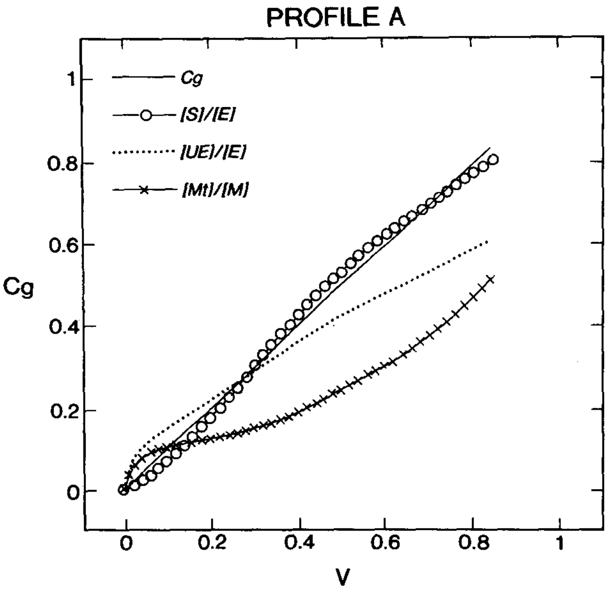

FIG. 2. Plots of [S]/[E] (line with circles), [$\bar{U}E$]/[E] (dotted line), and [M_T]/[M] (line with crosses) against V for profile A. The actual group velocity (equals V) is also plotted (solid line) for reference.

The results for profile B are shown in Fig. 3. For this case, the limiting velocities for a wave packet are about 0.04 and 0.70. Note that the ageostrophic geopotential fluxes again correct the contributions from the advection term toward the group velocity. The correspondence between [S]/[E] and the group velocity is slightly worse than that for profile A, but the error is everywhere less than 8% of the maximum jet speed. We have computed [S]/[E] for various other profiles and found qualitatively (and quantitatively) similar results.

The energy flux is not the only eddy flux that is parallel to the group velocity in the WKB limit. During

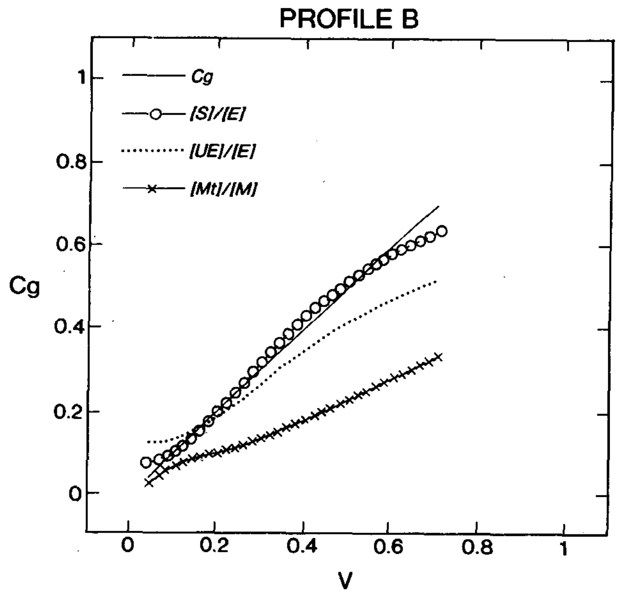

FIG. 3. Same as Fig. 2 but for profile B.

the past decade or so, several fluxes have been proposed for diagnosing wave propagation. Among them are the **E** vector of Hoskins et al. (1983), the **F** vector of Andrews (1983), the $\mathbf{E}_T$ vector of Plumb (1985), and the $\mathbf{M}_T$ vector of Plumb (1986). As discussed in Plumb (1986), the **E** vector of Hoskins et al. (1983) is actually not parallel to the group velocity, and the $\mathbf{M}_T$ vector of Plumb (1986) is a more suitable generalization. While the **F** vector of Andrews (1983) and the $\mathbf{E}_T$ vector of Plumb (1985) are based on conservation laws (conservation of the eddy activity $A = E + \rho_s q'^2/2\Gamma$, where $\Gamma = \partial\bar{Q}/\partial\Psi$, and q' and Q are perturbation and mean potential vorticity, respectively), the interpretation of $[\mathbf{F}]/[A]$ (or alternatively $[\mathbf{E}_T]/[A]$) as a velocity of wave propagation is ambiguous, since the wave activity A (a quantity similar to the pseudoenergy) is not positive definite (because Γ is in general negative) and in fact changes sign over different regions of a wave. The $\mathbf{M}_T$ vector of Plumb (1986) is based on an approximate conservation law, and the conserved quantity M (defined below) is positive definite. Here we will compute $[\mathbf{M}_T]/[M]$ for the two profiles and compare the results to $[\mathbf{S}]/[E]$ discussed above.

Plumb (1986) showed that assuming the mean potential vorticity gradient is slowly varying relative to the length scale of the transient eddy statistics, the following conservation law is satisfied:

$$\frac{\partial M}{\partial t} + \nabla\cdot\mathbf{M}_T = S_M, \tag{6}$$

where

$$M = \rho_s \frac{q'^2}{2|\nabla_H \bar{Q}|}, \tag{7}$$

$$\mathbf{M}_T = \bar{\mathbf{U}}M + \frac{\rho_s}{|\bar{\mathbf{U}}|}\begin{bmatrix} \bar{U}(v'^2 - e) - \bar{V}u'v' \\ \bar{V}(u'^2 - e) - \bar{U}u'v' \\ \dfrac{f_0}{\partial\bar{\Theta}/\partial z}(\bar{U}v'\theta' - \bar{V}u'\theta') \end{bmatrix} \tag{8}$$

for pseudowestward flow, and S_M is a nonconservative term. Here e is the energy density. The zonal component of $[\mathbf{M}_T]/[M]$ for the two profiles A and B are also shown in Figs. 2 and 3. We see that for both cases (and others considered, which are not shown), over much of the wave packet, $[\mathbf{M}_T]/[M]$ systematically underestimate the group velocity. As discussed above, over their leading portion (and also true for the case of barotropic Rossby waves), wave groups in general propagate faster than advection by the mean flow. However, unlike the ageostrophic fluxes, the correction term in $[\mathbf{M}_T]$, which is proportional to $v'^2 - e$, is in general negative (except for the case of barotropic Rossby wave, where it is positive because of the absence of potential energy). Thus, we see that for a flow with vertical shear, $[\mathbf{S}]/[E]$ is a much better approximation to the group velocity than $[\mathbf{M}_T]/[M]$, even though both are exactly equal to the group velocity for the case of a uniform zonal flow. As pointed out by Plumb (1985), the nonuniqueness of fluxes defined on the basis of conservation laws where only the flux divergence is involved raises general questions as to whether one may interpret such fluxes in general as indicators of the direction of eddy activity propagation. Currently, we have to resort to empirical evidence to settle this point.

4. Discussions

a. Nonlinear wave packets

The discussion in the above section is based on linear wave packets, since we do not yet have a complete theory of nonlinear wave packets. Linear wave packets that are initially localized will eventually expand to fill the whole domain. Observed wave packets in the atmosphere (e.g., Lee and Held 1993; Chang 1993a) obviously do not behave in that manner. Thus, one may question whether the results of the previous section are relevant to the real world at all, even if we assume that WKB descriptions such as group velocities are relevant for the atmosphere, which has a spatially varying basic state.

The evolution of a nonlinear wave packet on a baroclinically unstable jet has been studied numerically by Swanson and Pierrehumbert (1994). In particular, they found that "linear wave packet theory accurately bounds the upstream and downstream development of the synoptic disturbance throughout the nonlinear evolution," that "the waves that reach the largest amplitude are the waves at the leading edge of the wave packet," and that the group velocity of the localized nonlinear wave packet that eventually forms is equal to the speed of the downstream fringe predicted by the linear theory. In section 3, we have seen that the expression [S]/[E] approximates the speed of all parts of a linear wave packet quite well. In particular, [S]/[E] near the downstream fringe of the wave packet is a good approximation to the wave packet speed at the downstream edge. Since this speed has been shown by Swanson and Pierrehumbert (1994) to be the group velocity of a nonlinear wave packet, we have reasons to believe that [S]/[E] is also a good approximation to the group velocity of a nonlinear wave packet.

b. Application to observed wave packets

In order to test whether [S]/[E] is a good approximation to the group velocity of nonlinear wave packets, we have compared its value to the group velocity of an observed nonlinear wave packet. To generalize S from (3), which is based on quasigeostrophic theory, we have used the following expression suggested by Orlanski and Sheldon (1993):

$$\mathbf{S} = \mathbf{u}E + \rho\left(\mathbf{u}\phi - \mathbf{k} \times \nabla \frac{\phi^2}{2f(y)}\right). \qquad (9)$$

It is easy to show that this expression is equivalent to (3) to leading order.

Based on examination of data analyzed by the European Centre for Medium-Range Weather Forecasts (ECMWF), Chang (1993a) found that synoptic-scale waves over the wintertime Pacific storm track region propagate as downstream developing wave groups. Figure 4, taken from Chang (1993a), shows a Hovmöller diagram of the 300-mb v', computed from time-lagged correlation of the 300-mb v' (averaged between 30° and 60°N) based on the time series at the date line. In this figure, we can see a wave group propagating across the Pacific. From Fig. 4, we can estimate the group velocity of the wave group to be about 35 m s^{-1}.

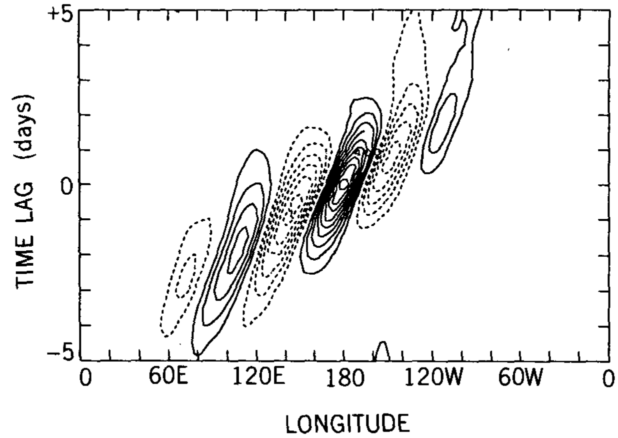

FIG. 4. Longitude–time plot of one-point lag correlation of ECMWF 300-mb v' averaged over 30° to 60°N for seven winter seasons (1980/81–86/87). The time series at the date line is taken to be the reference for the correlation. Contour interval is 0.1. Positive contours shown with solid lines and negative with dotted lines. The zero contour is omitted for clarity. Taken from Chang (1993a).

Using the wave group located over the Pacific storm track region analyzed by Chang (1993a) based on a regression analysis, we can compute group velocities as defined by [S]/[E]. For details of what individual fields of u', v', and T' of the wave group look like, please see Chang (1993a). Using the data, the value of the expression [S]/[E] is computed to be 34.6 m s^{-1}, very close to the group velocity of about 35 m s^{-1} estimated from Fig. 4. In addition, the contribution from the advective fluxes to [S]/[E] is found to be 24.6 m s^{-1}, or about 71% of the group velocity (the rest coming from the ageostrophic geopotential fluxes). This 7:3 split between the advective flux and the ageostrophic geopotential flux is very similar to the ratio found at the leading edge of the linear wave packets discussed previously in section 3.

We have not computed the value of [$\mathbf{M}_T$]/[M] from observations. Judging from the results shown in Plumb (1986), the nonadvective part of [$\mathbf{M}_T$]/[M] is more often westward than eastward, as is for the case of the linear wave packets discussed in section 3. Since the velocity computed from the advective part of [$\mathbf{M}_T$]/[M] should be similar to the advective part of [S]/[E], we expect that the value of [$\mathbf{M}_T$]/[M] will only be about 70% (or less) of the group velocity of the nonlinear wave packet.

While we have only one case from observation, we have also computed the value of [S]/[E] from numerical simulations of nonlinear wave packets on various basic states with different background zonal flow. The results confirm that [S]/[E] is a good approximation to the group velocity of nonlinear wave packets for all cases we have considered. Since the results are very similar to those discussed above for the observed wave packet, details will not be presented here [see Chang (1993b) for details].

5. Conclusions

In this paper, we have shown that the eddy energy flux, when modified to take the effects of a variable Coriolis parameter into account, gives a very good approximation to the group velocity of wave propagation when the flux is normalized by the total eddy energy. The result is exact for Rossby waves in a constant basic flow. For waves in a baroclinic basic state on a β plane, we have shown that this relationship is not exact, but the approximation is quite accurate as long as typical tropospheric parameters are considered.

For a background flow with shear, the total eddy energy does not satisfy a conservation law such as (6), even under linear, nondissipative conditions. Hence, to understand the evolution of the energetics of a system, we have to consider not only the fluxes but also the conversion between the eddy and the mean flow. It has been pointed out by various authors that the energy cycle is not uniquely defined and that the energy flux term, which can be interpreted as the redistribution of eddy energy by fluxes, is only defined up to a nondivergent term; that is, as far as the eddy energy equation is concerned, we can add any arbitrary nondivergent vector to the energy flux. Here, we have found that the modified energy flux (3) is approximately equal to the *total group velocity times the total eddy energy*; hence, its interpretation as a flux that spatially redistributes eddy energy is unambiguous. We do not advocate abandoning the diagnoses based on eddy fluxes that represent fluxes of conserved eddy activities and only consider the eddy energy fluxes. We believe that these different diagnostic tools are complementary. The "conserved" fluxes show more clearly the sources and sinks of eddy activity and eddy–mean flow interactions, while the eddy energy fluxes appear to have the advantage of more closely approximating the propagation of the eddies in a baroclinic basic state.

While the discussion in section 3 is limited to the linear case, in section 4 we have found empirical evidence that the results also apply to the nonlinear case. With $[\mathbf{S}]/[E]$ being so closely related to the group velocity of wave packets, it is not surprising that the ageostrophic geopotential fluxes have been found to be a good diagnostic of where the next new development is going to occur (Orlanski and Katzfey 1991; Orlanski and Sheldon 1993). These results from realistic case studies also suggest that the concepts introduced here should also be relevant even when there is zonal asymmetry in the basic state. Whether we can define a *local* group velocity in a zonally varying basic state using the modified energy fluxes will be left as a future extension of this research.

In section 4b, we found that the wave packet in the Pacific storm track displays very strong relative downstream flux, equivalent to a relative group velocity of about 10 m s^{-1} (relative to the advection speed). This shows that waves over the wintertime Pacific storm track region radiate a lot of energy toward the downstream direction. Whether this energy ends up in continued downstream propagation of the wave group, being dissipated by nonconservative processes, converted into energy of the mean flow, or feeding other waves via nonlinear wave–wave interaction obviously affects the weather and climate of the eastern Pacific and western North America. As such, more detailed analysis of the energetics and life cycle of waves over the Pacific storm track and its downstream side is warranted in order to understand how the downstream developing wave groups over the central North Pacific are related to the climate of the west coast of the United States.

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REFERENCES

Andrews, D. G., 1983: A conservation law for small-amplitude quasi-geostrophic disturbances on a zonally asymmetric basic flow. *J. Atmos. Sci.*, **40,** 85–90.

Blackmon, M. L., Y. H. Lee, and J. M. Wallace, 1984a: Horizontal structure of 500-mb height fluctuations with long, intermediate, and short time scales. *J. Atmos. Sci.*, **41,** 961–979.

——, ——, ——, and H. H. Hsu, 1984b: Time variation of 500-mb height fluctuations with long, intermediate, and short time scales as deduced from lag-correlation statistics. *J. Atmos. Sci.*, **41,** 981–991.

Chang, E. K. M., 1993a: Downstream development of baroclinic waves as inferred from regression analysis. *J. Atmos. Sci.*, **50,** 2038–2053.

——, 1993b: Downstream development of baroclinic waves. Ph.D. dissertation, Princeton University, 255 pp.

——, and I. Orlanski, 1993: On the dynamics of a storm track. *J. Atmos. Sci.*, **50,** 999–1015.

Hayes, M., 1977: A note on group velocity. *Proc. Roy. Soc. London A*, **354,** 533–535.

Hoskins, B. J., I. N. James, and G. H. White, 1983: The shape, propagation, and mean-flow interaction of large-scale weather systems. *J. Atmos. Sci.*, **40,** 1595–1612.

Kuo, H. L., 1980: Baroclinic instabilities of linear and jet profiles in the atmosphere. *J. Atmos. Sci.*, **36,** 2360–2378.

Lee, S., and I. M. Held, 1993: Baroclinic wave packets in models and observations. *J. Atmos. Sci.*, **50,** 1413–1428.

Lim, G. H., and J. M. Wallace, 1991: Structure and evolution of baroclinic waves as inferred from regression analysis. *J. Atmos. Sci.*, **48,** 1718–1732.

Lin, S. J., and R. T. Pierrehumbert, 1993: Is the midlatitude zonal flow absolutely unstable? *J. Atmos. Sci.*, **50,** 505–517.

Longuet-Higgins, M. S., 1964: On group velocity and energy flux in planetary wave motions. *Deep-Sea Res.*, **11,** 35–42.

Merkine, L., and M. Shafranek, 1980: The spatial and temporal evolution of localized unstable baroclinic disturbances. *Geophys. Astrophys. Fluid Dyn.*, **16,** 174–206.

Orlanski, I., and J. Katzfey, 1991: The life cycle of a cyclone wave in the Southern Hemisphere. Part I: Eddy energy budget. *J. Atmos. Sci.*, **48,** 1972–1998.

——, and E. K. M. Chang, 1993: Ageostrophic geopotential fluxes in downstream and upstream development of baroclinic waves. *J. Atmos. Sci.*, **50,** 212–225.

——, and J. Sheldon, 1993: A case of downstream baroclinic development over western north America. *Mon. Wea. Rev.*, **121,** 2929–2950.

Pedlosky, J., 1987: *Geophysical Fluid Dynamics.* 2nd ed. Springer-Verlag, 710 pp.

Pierrehumbert, R. T., 1984: Local and global baroclinic instability of zonally varying flow. *J. Atmos. Sci.*, **41,** 2141–2162.

Plumb, R. A., 1983: A new look at the energy cycle. *J. Atmos. Sci.*, **40,** 1669–1688.

——, 1985: An alternative form of Andrew's conservation law for quasi-geostrophic waves on a steady, nonuniform flow. *J. Atmos. Sci.*, **42,** 298–300.

——, 1986: Three dimensional propagation of transient quasi-geostrophic eddies and its relationship with the eddy forcing of the time-mean flow. *J. Atmos. Sci.*, **43,** 1657–1678.

Simmons, A. J., J. M. Wallace, and G. W. Branstator, 1983: Barotropic wave propagation and instability, and atmospheric teleconnection patterns. *J. Atmos. Sci.*, **40,** 1363–1392.

Swanson, K., and R. T. Pierrehumbert, 1994: Nonlinear wave packet evolution on a baroclinically unstable jet. *J. Atmos. Sci.*, **51,** 384–396.