Droplet nucleation: Physically-based parameterizations and comparative evaluation

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One of the greatest sources of uncertainty in simulations of climate and climate change is the influence of aerosols on the optical properties of clouds. The root of this influence is the droplet nucleation process, which involves the spontaneous growth of aerosol into cloud droplets at cloud edges, during the early stages of cloud formation, and in some cases within the interior of mature clouds. Numerical models of droplet nucleation represent much of the complexity of the process, but at a computational cost that limits their application to simulations of hours or days. Physically-based parameterizations of droplet nucleation are designed to quickly estimate the number nucleated as a function of the primary controlling parameters: the aerosol number size distribution, hygroscopicity and cooling rate. Here we compare and contrast the key assumptions used in developing each of the most popular parameterizations and compare their performances under a variety of conditions. We find that the more complex parameterizations perform well under a wider variety of nucleation conditions, but all parameterizations perform well under the most common conditions. We then discuss the various applications of the parameterizations to cloud-resolving, regional and global models to study aerosol effects on clouds at a wide range of spatial and temporal scales. We compare estimates of anthropogenic aerosol indirect effects using two different parameterizations applied to the same global climate model, and find that the estimates of indirect effects differ by only 10%. We conclude with a summary of the outstanding challenges remaining for further development and application.

1. Introduction

One of the greatest sources of uncertainty in projections of future climate change is the influence of anthropogenic aerosol on the optical properties of clouds [Forster et al., 2007]. By serving as the seeds (Cloud Condensation Nuclei, CCN) of cloud droplets, anthropogenic aerosol particles can increase droplet number concentration, thereby increasing total droplet surface area and hence cloud albedo if liquid water content is not changed [Twomey, 1974, 1977]. By reducing mean droplet size, drizzle production can be inhibited under certain conditions, leading to increased liquid water content, further enhancing cloud albedo [Albrecht, 1989]. These and other mechanisms by which aerosols affect clouds and climate through their influence on droplet number are referred to collectively as the aerosol...
indirect effect on climate [Haywood and Boucher, 2000; Lohmann and Feichter, 2005; Stevens and Feingold, 2009].

The root of this influence is the droplet nucleation process. Droplet nucleation involves the simultaneous condensational growth of an aerosol population in a cooling air parcel until maximum supersaturation is achieved and some of the wet particles are large enough to grow spontaneously into cloud droplets. Droplet nucleation also has important effects on the aerosol population, as nucleation scavenging of aerosol particles (i.e., when particles activated to form cloud droplets are subsequently removed from the atmosphere by precipitation from the cloud) is the dominant removal mechanism for accumulation mode (0.05–0.2 micron radius) aerosol [Jensen and Charlson, 1984; Flossmann et al., 1985]. In addition, aqueous phase oxidation of sulfur in cloud droplets is a major source of sulfate after particles are activated when droplets form and are then subsequently resuspended when cloud droplets evaporate [Hoppel et al., 1986; Meng and Seinfeld, 1994; Rasch et al., 2000]. Similarly, recent work [Sorooshian et al., 2007; Ervens et al., 2008; Perri et al., 2009] suggests that aqueous phase chemistry in cloud droplets is also an important source of secondary organic aerosol. The activation process determines which particles gain sulfate and organic matter within cloud droplets.

The first attempts to represent droplet nucleation in climate models [Jones et al., 1994; Jones and Slingo, 1996; Lohmann and Feichter, 1997] relied on empirical relationships between droplet number and measures of the aerosol such as sulfate mass concentration [Leaitch et al., 1992; Leaitch and Isaac, 1994; Boucher and Lohmann, 1995] or aerosol number [Jones et al., 1994; Martin et al., 1994]. These relationships do not account for the dependence of the droplet nucleation on size distribution, composition, or updraft velocity, and hence are extremely limited in their applicability to the wide variety of conditions controlling droplet formation.

Recognition of these limitations has driven the development of physically-based schemes that can more completely represent the dependence of the process on all of the key controlling parameters. These schemes have the added benefit of diagnosing the maximum supersaturation in updrafts and the partitioning of the aerosol into cloud-borne and interstitial phases so that aqueous phase chemistry and nucleation scavenging can be represented more realistically.

The theory of droplet nucleation is founded on seminal work by Köhler [1921, 1926], who determined the equilibrium radius \( r \) of particles as a function of dry radius \( r_d \) and relative humidity \( RH \). For supersaturated conditions the wet size generally dominates the dry size and the Köhler equilibrium can be approximated in terms of supersaturation \( S \) (defined as \( RH \)) as [Seinfeld and Pandis, 1998]

\[
S_{eq} = \frac{A}{r} - \frac{kr_d^3}{r^4}
\]

where the Kelvin coefficient \( A \) and hygroscopicity parameter \( \kappa \) are defined in Appendix A. Solutions to equation (1) for ammonium sulfate particles of four different dry radii are illustrated in Figure 1. For each dry particle size there is a maximum supersaturation in equilibrium with the wet radius. The maximum supersaturation is called the critical supersaturation \( S_c \) for the particle, because under most ambient conditions if the supersaturation in a cooling air parcel exceeds \( S_c \) the particle radius will grow beyond the equilibrium size at the maximum supersaturation, and the particles will continue to grow spontaneously until the supersaturation is reduced to a value at or below equilibrium. The critical supersaturation can be found by solving for the maximum of equation (1),

\[
S_c = \left( \frac{4A^3}{27k^2r_d^6} \right)^{1/2}
\]

Figure 1. Supersaturation as a function of equilibrium wet radius (solid curves) and the wet radius every 1 s for a rising air parcel (individual points) according to dynamic Köhler theory for ammonium sulfate particles with four different dry radii.
and the critical radius $r_c$ for activating particles (corresponding to the equilibrium wet size at the maxima of the Köhler curve for the particle) can be written

$$r_c \equiv \left( \frac{3k \rho_a}{A} \right)^{1/2}.$$

(3)

The equilibrium theory can determine which particles form droplets in an air parcel if its supersaturation history is known. The latter requires knowledge of the aerosol size distribution and the dynamical forcing (mixing, radiative or expansion cooling) that generates supersaturation; this is addressed with the more general dynamic Köhler theory [Pruppacher and Klett, 1997; Seinfeld and Pandis, 1998]. This theory consists of equations for the mass balance of each particle $j$ and the supersaturation balance of the air parcel, which under the assumptions of adiabaticity (no exchange of mass or enthalpy with the environment - this simplification will be addressed in section 2) and expansion cooling can be expressed as [Howell, 1949]

$$\frac{dr_j}{dt} = \frac{G}{r_j} \left( S - \frac{A}{r_j} + \frac{kr_a^3}{r_j^3} \right)$$

(4)

$$\frac{dS}{dt} = zw - \gamma \frac{dW}{dt}$$

(5)

where the growth coefficient $G$ and the coefficients $\alpha$ and $\gamma$, which are weak functions of temperature and pressure, are defined in Appendix A, $w$ is the updraft velocity, and the condensation rate can be expressed in terms of the growth rate of all particles,

$$\frac{dW}{dt} = \frac{4\pi \rho_a}{\rho_a V} \sum_i N_i r_i^2 \frac{dr_i}{dt}$$

(6)

where $W$ is the liquid water mass mixing ratio, $\rho_a$ is the density of air and the sum is over all particles in a parcel of volume $V$ large enough to contain a representative number of particles but small enough that the updraft velocity can be considered uniform within the parcel. Note that (1) is simply the equilibrium form of (4). Although the effect of the cooling rate on supersaturation in (5) is expressed in terms of updraft velocity, Ghan et al. [1993] show that it can be expressed more generally to account for radiative cooling as well.

A key parameter controlling the nucleation process is the updraft velocity. In cloud-resolving models the grid is fine enough to explicitly resolve spatial variability in the updraft velocity, but in large-scale models (with grid cells much larger than the 100 m size of turbulent eddies in the boundary layer), it is important to account for the subgrid variability and its influence on droplet nucleation. This issue is addressed in section 4.

According to (5) and (6) the supersaturation history depends on the history of the updraft velocity and the growth of all particles. Each particle competes with all others for water, with condensation on each particle affecting the supersaturation according to (5) and hence the growth of all particles according to (4). The complexity of the coupled equations (4)–(6) makes analytic solution impossible without approximations.

Numerical models of droplet nucleation are capable of representing much of the complexity of the process. Most numerical models [Warner, 1973; Fitzgerald, 1974; Jensen and Charlson, 1984; Flossmann et al., 1985; Abdul-Razzak et al., 1998; Nenes et al., 2001] represent the aerosol in terms of a large number (order 100) of sections of the size distribution, with each section assumed to be composed of an internal mixture of identical particles with the same size and composition. Equation (6) then becomes

$$\frac{dW}{dt} = \frac{4\pi \rho_a}{\rho_a V} \sum_i N_i r_i^2 \frac{dr_i}{dt}$$

(7)

where $N_i$ is the number concentration of particles in section $i$. More general formulations [Russell and Seinfeld, 1998] represent the aerosol in terms of sections with multiple externally-mixed populations. Particle-resolved representations of the aerosol [Andrejczuk et al., 2008; Shima et al., 2009] have also been applied to the droplet nucleation process using (6) rather than (7). Figure 2 shows a numerical simulation of supersaturation for a rising air parcel. Initially the supersaturation rises as supersaturation production by adiabatic cooling (the $zw$ term in equation (5)) dominates supersaturation loss by condensation on droplets and unactivated aerosol (the $-\gamma \frac{dW}{dt}$ term in equation (5)). As the particles grow their

![Figure 2](image-url)  
Figure 2. Numerical solution for supersaturation in an air parcel rising at a rate of 1 m s$^{-1}$ with a lognormal size distribution of ammonium sulfate aerosol with total number concentration 1000 cm$^{-3}$, number mode radius 0.05 μm, and geometric standard deviation 2. At time zero the aerosol is assumed to be in equilibrium with a 100% relative humidity. A total of 144 sections were used with size ranges such that an equal number of particles are in each section, with the middle section corresponding to the number mode radius.
surface area increases, which together with a rise in supersaturation increases the condensation rate. Eventually (here, after 10 s) droplets grow large enough and supersaturation becomes great enough that the condensation rate exceeds supersaturation production, and the parcel supersaturation begins to decrease.

According to equilibrium Köhler theory a particle remains in equilibrium with parcel supersaturation while \( S < S_c \), and instantaneously activates when \( S \geq S_c \). Although this often is a good approximation of particle behavior, it does not always hold. Characteristic examples of the dynamical behavior of four different particle sizes for this case are illustrated in Figure 1. Initially all four particle sizes grow, but the inertial kinetic limitation mechanism [Nenes et al., 2001] limits the growth of the larger particles to sizes smaller than expected from equilibrium Köhler theory. According to equation (4) this lag in growth actually enhances condensation as the supersaturation exceeds the equilibrium supersaturation for the particle size. Although the growth of the smallest of the four particle sizes (dry radius 0.02 \( \mu \text{m} \)) follows equilibrium Köhler theory, the supersaturation of the air parcel never exceeds the critical supersaturation for that particle size, so those particles lose water when the supersaturation declines. In contrast, the larger particles exceed their critical size and hence continue to grow beyond the point of maximum supersaturation. Thus, the activation process separates the aerosol into a population that forms cloud droplets and the remainder that do not (often referred to as interstitial aerosol). The smallest of those that form droplets typically activate last in the cloud parcel, as they have a critical supersaturation close to the parcel maximum supersaturation; larger particles have a lower \( S_c \), are activated sooner and grow beyond their critical size before maximum supersaturation occurs. The largest particles are typically subject to the inertial kinetic limitation mechanism, during which \( r_c \) is not attained before maximum supersaturation is achieved (e.g., dry radius 0.2 \( \mu \text{m} \) in Figure 1). Although not strictly activated, these inertially-limited particles are indistinguishable from activated droplets, because they exhibit comparable sizes and continue to grow (as their \( S_{eq} \) is very small). The time for which \( S > S_c \) may be insufficient for particles with \( S_c \sim S_{\text{max}} \) to grow beyond their \( r_c \) and activate; slightly larger particles may initially activate, but subsequently deactivate because \( S \) may drop below \( S_{eq} \) and evaporate the particle. Both of these kinetic limitation mechanisms (identified by Nenes et al. [2001] as the deactivation and evaporation mechanisms, respectively) appreciably affect droplet number under highly polluted conditions [Nenes et al., 2001]. For all other atmospherically-relevant conditions, it is sufficient to state that particles for which \( S_c \leq S_{\text{max}} \) will nucleate cloud droplets.

Numerical models of droplet nucleation are computationally expensive, because of the need to discretize the aerosol size distribution, resolve the short time scales of the condensation process, and integrate over time until maximum supersaturation is achieved. This limits their application to exploration of parameter space with parcel models or to simulations of hours to days with three-dimensional models [Kogan, 1991; Khairoutdinov and Kogan, 1999]. Even for such short simulations there are challenges due to discretization errors in Eulerian representations of water and temperature transport and the nonlinear dependence of supersaturation on temperature and water vapor [Clark, 1974; Stevens et al., 1996; Morrison and Grabowski, 2008].

This concern has led to the development of a Lagrangian particle-based representation of the aerosol and cloud droplets [Andrejezun et al., 2008, 2010], but at a considerable computational expense. Thus, although numerical models provide valuable benchmark simulations of the nucleation process and can be used in short cloud simulations, they are not practical for global simulations of decades or centuries.

Physically-based parameterizations of droplet nucleation are designed to quickly diagnose the number nucleated as a function of the primary controlling parameters: the cooling rate and the size distribution of aerosol number and hygroscopicity. This permits treatment of droplet nucleation for a spectrum of updraft velocities within each grid cell in long global simulations [Ghan et al., 1997]. Thus, parameterizations have been widely used in global models to estimate aerosol indirect effects, and will be relied on for future multi-century simulations of climate change.

In this review article we compare and contrast the key assumptions and approximations used in developing each of the most popular parameterizations and compare their performances under a variety of conditions. The parameterizations are summarized in Table 1. We then discuss the various applications of the parameterizations to cloud-resolving, regional and global models to study aerosol effects on clouds at a wide range of spatial and temporal scales, and compare estimates of anthropogenic aerosol indirect effects with two parameterizations applied to the same model. We conclude with a summary of the outstanding challenges remaining for further development.

## 2. Parameterization Descriptions

Given the fact that the equilibrium Köhler theory accurately diagnoses activation of particles provided the maximum supersaturation is known, the crux of the parameterization problem is the determination of the maximum supersaturation in a cloudy parcel. However, equations (4)–(6) are too complex for analytic solutions without approximations. Most parameterizations therefore rely on the following assumptions.

1. No cloud droplets are present before cooling begins. Although ice crystals might be present, we assume their influence on supersaturation is too slow to affect aerosol activation.
2. Adiabatic conditions.
3. The aerosol population can be represented in terms of the distribution of number with size, which can be described by a power law, by multiple sections, or by lognormal modes, each with a uniform bulk hygroscopicity.

4. Particles are composed of internal mixtures of salts and insoluble components within each section or mode.

5. The number of nucleated droplets is determined by the number of particles with critical supersaturation less than the maximum supersaturation.

6. Particles grow in equilibrium with relative humidity until the supersaturation exceeds the particle critical value for activation.

7. Beyond the point of activation, particle growth rates are not significantly influenced by droplet curvature and solute effects.

Some of these approximations have been relaxed in a few parameterizations.

Assumption 1 restricts the theory to droplets forming near the base of existing clouds or as a result of diabatic or diabatic cooling in clear air. Thus, the condensation rate can be approximated in terms of a power law [Twomey, 1959],

$$\frac{dN}{dr_c} = a r_c^{-b}$$  \hspace{1cm} (8)

or multiple lognormal distributions [Abdul-Razzak and Ghan, 2000; Fountoukis and Nenes, 2005],

$$\frac{dN}{dr_c} = \frac{N_i}{r_i - r_{i-1}}$$  \hspace{1cm} (9)

or multiple lognormal distributions [Abdul-Razzak and Ghan, 2000; Fountoukis and Nenes, 2005],

$$\frac{dN}{dr_c} = \frac{N_m}{\sqrt{2\pi}\ln\sigma_m} \exp\left(-\frac{\ln^2(r_d/r_m)}{2\ln^2\sigma_m}\right)$$  \hspace{1cm} (10)

where $a$ and $b$ are constants defining the size distribution, $N_i$ is the number concentration in section $i$, $r_i$ is the dry radius at the upper boundary of section $i$, $N_m$ is the number concentration in mode $m$, and $r_m$ and $\sigma_m$ are the mode radius and geometric standard deviation for mode $m$. If the wet radius of the particle and its growth rate are known then the condensation rate can be expressed in terms of the size distribution,

$$\frac{dN}{dr_c} = \frac{4\pi\rho_w}{\rho_a} \int_0^{\infty} r^2 dN dr_c dr.$$  \hspace{1cm} (11)

It is helpful to express the aerosol number distribution with size in terms of critical supersaturation,

$$\frac{dN}{d\ln S_c} = \frac{dN}{d\ln r_d} \frac{d\ln r_d}{d\ln S_c}.$$  \hspace{1cm} (12)

Noting that from (2)

$$\frac{d\ln r_d}{d\ln S_c} = -\frac{2}{3}$$  \hspace{1cm} (13)

equations (8)–(11) become

$$\frac{dN}{dS_c} = -\frac{2a}{3S_c} \left(\frac{4A^3}{27kS_c}\right)$$  \hspace{1cm} (14)

$$\frac{dN}{dS_c} = \frac{N_i}{S_i - S_{i-1}}$$  \hspace{1cm} (15)
\[
\frac{dN}{d\ln S_c} = -\sum_m \frac{2N_m}{3\sqrt{2\pi} \ln \sigma_m} \exp \left( -\frac{\ln^2(S_m/S_c)}{2 \ln^2 \sigma_m} \right) \tag{16}
\]

\[
\frac{dW}{dt} = \frac{4\pi\rho_w}{\rho_a} \int_0^\infty r^2 \frac{dN}{ds_c} \frac{dr}{ds_c} ds_c \tag{17}
\]

where \(S_i\) denotes the critical supersaturation at the boundary of bin \(i\) and \(S_m\) is the critical supersaturation for the mode radius of mode \(m\). Then if the maximum supersaturation \(S_{\text{max}}\) is known, the number nucleated can be determined by integrating (14)–(16) from zero to \(S_{\text{max}}\) yielding

\[
N_{\text{act}} = cS_{\text{max}}^k \tag{18}
\]

\[
N_{\text{act}} = \sum_{j=1}^{i-1} N_j + N_i \frac{S_{\text{max}} - S_i}{S_{i+1} - S_i} \tag{19}
\]

\[
N_{\text{act}} = \frac{1}{2} \sum_M N_m \left[1 - \text{erf}(z_m)\right] \tag{20}
\]

where

\[
k \equiv \frac{2}{3} (b - 1) \tag{21}
\]

\[
z_m = 2 \ln(S_m/S_{\text{max}}) / \left(3\sqrt{2\ln \sigma_m}\right) \tag{22}
\]

A useful approximation to (20) is to replace the error function \(\text{erf}(z)\) with the hyperbolic tangent \(\text{tanh}(2z/\sqrt{\pi})\) [Ghan et al., 1993; Khvorostyanov and Curry, 2006], so that (20) becomes

\[
N_{\text{act}} = \sum_m N_m \left[1 + \left(S_m/S_{\text{max}}\right)^{c_m}\right] \tag{23}
\]

where \(c_m \equiv 9/(3\sqrt{2\pi} \ln \sigma_m)\). The distribution of number with supersaturation that is consistent with (23) can be written [Khvorostyanov and Curry, 2006]

\[
\frac{dN}{ds_c} = \sum_m c_m N_m S_m^{c_m} \left[1 + \left(S_m/S_{\text{max}}\right)^{c_m}\right] \left[S_{\text{min}}^{c_m} + c_m S_{\text{max}}^{c_m} / S_m^{c_m}\right]^{1/2} \tag{24}
\]

For small \(S_c\) with respect to \(S_{\text{max}}\), (24) reduces to a sum of multiple power laws. For the more general case (24) is a multimode version of the extension of the power law size distribution proposed for a single mode by Cohard et al. [1998, 2000] and extended to the multimode case by Shipway and Abel [2010]. This provides a connection between parameterizations based on log-normal size distributions and those based on power laws. In addition, (23) is much faster to compute than the error function in (20), and is a good approximation under most conditions [Ghan et al., 1993; Khvorostyanov and Curry, 2006].

Under assumption 4, the hygroscopicity of each section or mode is given by the volume mean of the hygroscopicity of the components comprising the section or mode, as described in Appendix A.

Assumption 5 has already been invoked in equation (1). In addition, as we shall see later this assumption simplifies the treatment of the effects of surfactants on droplet nucleation.

Assumption 6 permits the number activated to be determined from the CCN spectrum and the maximum supersaturation, thus reducing the problem to that of determining the maximum supersaturation. This assumption neglects kinetic limitation mechanisms for droplet nucleation. As noted earlier, such an approximation performs well for all but extremely polluted conditions.

Assumption 7 applies an equilibrium growth approximation to determine the droplet size distribution at maximum supersaturation. This is a critical step in determining the maximum supersaturation. To see this, consider the combination of equations (5) and (17) at maximum supersaturation:

\[
aw = \frac{4\pi\rho_w v_i^2}{\rho_a} \int_0^{S_{\text{max}}} r^2 \frac{dN}{ds_c} \frac{dr}{ds_c} \frac{dS_c}{dt} \tag{25}
\]

where

\[
\gamma^* = \frac{4\pi\rho_w v_i^2}{\rho_a} \tag{26}
\]

and the upper limit of the integral is \(S_{\text{max}}\) because no particles with \(S_c > S_{\text{max}}\) can be activated.

The path forward is to use equation (25) to determine \(S_{\text{max}}\). To do so the radius and its growth must be related to \(S_{\text{max}}\). Assumption 7 can be used in combination with equations (1) and (4) to do this. First write equation (4) as

\[
\frac{dr}{dt} = \frac{G}{r} (S - S_{eq}) \tag{26}
\]

where \(S_{eq}\) is given by (1). We apply assumption 7 to (26) and integrate from the time of activation \(t_{\text{act}}\) to the time of maximum supersaturation \(t_{\text{max}}\):

\[
r^2(t_{\text{max}}) = r^2(t_{\text{act}}) + 2G \int_{t_{\text{act}}}^{t_{\text{max}}} (S - S_{eq}) dt \tag{27}
\]

where \(r^2(t_{\text{act}})\) is often assumed to be given by (3). Note that according to Figure 1, assumption 6 is not appropriate for larger particles, specifically for particles with radius 0.1 \(\mu\)m or larger for the conditions of Figure 1.

Assumption 8 neglects \(S_{eq}\) in (26) and (27). Then (26) at maximum supersaturation and (27) become
\[
\frac{dr}{dt} = GS_{\text{max}} \tag{28}
\]

\[
r^2(t_{\text{max}}) = r^2(t_{\text{act}}) + 2G \int_{t_{\text{act}}}^{t_{\text{max}}} Sdt. \tag{29}
\]

As is evident in Figure 1, assumption 8 is a reasonable approximation for particles that activate well before maximum supersaturation, but not for particles that activate just before maximum supersaturation. Thus, (28) may overestimate the growth rate of particles with \( S_c \) near \( S_{\text{max}} \). However, unless inertial kinetic limitations are important [Barahona et al., 2010], (29) still accurately determines the droplet size at \( t_{\text{max}} \) because \( t_{\text{act}} \) is near \( t_{\text{max}} \) for those particles.

Substituting (28) and (29) into (25) yields

\[
ZW = \gamma^* GS_{\text{max}} \left[ \frac{1}{2} \int_{0}^{t_{\text{act}}} \left( r^2(t_{\text{act}}) + 2G \right) Sdt \right] \tag{30}
\]

Solution of (30) is facilitated by splitting the aerosol population into two parts [Nenes and Seinfeld, 2003]: (a) particles with \( S_c \) near \( S_{\text{max}} \), such that

\[
r^2(t_{\text{act}}) >> 2G \int_{t_{\text{act}}}^{t_{\text{max}}} Sdt \tag{31}
\]

and (b), particles with \( S_c \) much less than \( S_{\text{max}} \), such that

\[
r^2(t_{\text{act}}) < < 2G \int_{t_{\text{act}}}^{t_{\text{max}}} Sdt \tag{32}
\]

Then (30) can be reduced to

\[
ZW = \gamma^* \sqrt{GS_{\text{max}} \left[ \frac{1}{2} \int_{0}^{t_{\text{act}}} \left( 2G \int_{t_{\text{act}}}^{t_{\text{max}}} Sdt \right) \frac{dN}{dS_c} dS_c + \int_{t_{\text{act}}}^{t_{\text{max}}} r(t_{\text{act}}) \frac{dN}{dS_c} dS_c \right]} \tag{33}
\]

where \( S_{\text{part}} \) is a partitioning supersaturation [Nenes and Seinfeld, 2003] that distinguishes between particles that grow substantially between \( t_{\text{act}} \) and \( t_{\text{max}} \) and those that do not.

We are now prepared to distinguish between the various different parameterizations of droplet nucleation.

The first parameterization of droplet nucleation was formulated by Twomey [1959]. Twomey’s scheme assumes \( S_{\text{part}} = S_{\text{max}} \) so that the second term in (33) is neglected. This underestimates the size of particles with \( S_c \) near \( S_{\text{max}} \) but the growth rate of those particles should be slow according to (26). Twomey assumes a power law for the CCN spectrum (i.e., equation (18)). By approximating the integral of supersaturation in (33) in terms of \( S_{\text{max}} \)

\[
\int_{t_{\text{act}}}^{t_{\text{max}}} Sdt \approx \frac{S_{\text{max}}^2 - S_c^2}{2\pi W} \tag{34}
\]

Twomey is able to solve (33) for \( S_{\text{max}} \) in terms of \( w \) and the parameters of the CCN spectrum. However, Twomey’s solution is unbounded at high updraft velocity, and hence produces unphysical results (droplet number exceeding aerosol number) for large updraft velocity or low aerosol concentrations [Ghan et al., 1993]. Cohard et al. [1998, 2000] have extended the Twomey solution by replacing the power law size distribution with a more general expression that limits the number activated at high supersaturation. Shipway and Abel [2010] extended that solution to the multimode case.

The second term in (33) was introduced by Ghan et al. [1993], who assumed \( S_{\text{part}} = 0 \) so that the first term is neglected and when \( S = S_{\text{max}} \) the particles are at their critical size for activation. Thus, under some conditions droplet size is underestimated for particles that grow after their radius reaches \( r_c \), i.e., particles with \( S_c << S_{\text{max}} \). However, as is evident from Figure 1, growth of the larger droplets does not follow their Koehler curves, and their size at maximum supersaturation is not far from their critical size for activation. Ghan et al. [1993] also departed from Twomey by retaining \( S_{\text{eq}} \) in (26), and by assuming a single lognormal aerosol size distribution. The integrals in (33) are then performed analytically, and the resulting function of \( S_{\text{max}} \) is approximated to permit a simple analytic expression for the number activated: \( N_{\text{act}} = wN_f/(w+dN_a) \) where the parameter \( d \) depends on the width of the aerosol size distribution. Ghan et al. [1995] extended the scheme to the case of multiple lognormal size distributions. Chuang et al. [1997] introduced empirical expressions for the parameter \( d \) to account for dependence on the updraft velocity and mean aerosol particle size. Lin and Leaitch [1997] estimated the parameter \( d \) from observations.

A major weakness of the Ghan et al. [1993] scheme is that it does not account for the dependence of the number nucleated on the mean radius for each mode. Under most conditions the scheme underestimates the droplet size and hence the condensation rate, which leads it to overestimate the maximum supersaturation and the number nucleated by as much as a factor of three. Abdul-Razzak et al. [1998] and Abdul-Razzak and Ghan [2000] combined the treatments of Twomey and Ghan et al., integrating both terms in (33) from 0 to \( S_{\text{max}} \) but neglecting the \( S_{eq} \) term in (26), i.e., neglecting the curvature and solute terms beyond activation. This yields a complex function of \( S_{\text{max}} \) that cannot be solved analytically. However, Abdul-Razzak et al. used numerical simulations for a single lognormal size distribution to find that the ratio \( S_{\text{act}}/S_{\text{max}} \) is
proportional to simple functions of $S_m$ and two non-dimensional parameters,

$$\eta_m \equiv \frac{2(xw/G)^{3/2}}{\tau^* N_m^{3/2}}$$  \hspace{1cm} (35)$$

$$\zeta \equiv \frac{2A}{5} \left( \frac{xw}{G} \right)^{1/2}$$  \hspace{1cm} (36)$$

Since neither of these parameters depends on the width of the size distribution, Abdul-Razzak et al. parameterized the dependence of $S_{\text{max}}$ on $\sigma$ from a large number of numerical solutions. The result for the multiple lognormal case [Abdul-Razzak and Ghan, 2000] is

$$S_{\text{max}}^2 = 1/\sum \frac{1}{\eta_m} \left[ f_m \left( \eta_m^{3/2} \right) + g_m \left( \frac{S_{\text{max}}^2}{\eta_m + 2\zeta} \right)^{3/4} \right]$$  \hspace{1cm} (37)$$

where

$$f_m \equiv 0.5 \exp\left( 2.5 \ln^2 \sigma_m \right)$$  \hspace{1cm} (38)$$

$$g_m \equiv 1 + 0.25 \ln \sigma_m$$  \hspace{1cm} (39)$$

Although (37) diagnoses $S_{\text{max}}$ accurately under a wide variety of conditions, it should be noted that it does not represent kinetic limitations on droplet nucleation, which can be important under highly polluted conditions [Nenes et al., 2001]. However, as we shall see, it performs surprisingly well under many conditions.

Note also that the Abdul-Razzak scheme is tuned to numerical simulations using a particular value (1.0) of the condensation coefficient $\chi_c$ that is used to determine $G$ (see Appendix A), and that $G$ in (35) and (36) is estimated neglecting gas kinetic effects completely (i.e., an infinite radius in equations (A5) and (A6)). The value of $\chi_c$ is uncertain to within a factor of three for pure water, and is further reduced by the presence of organic films on the particle surface [Chuang, 2003]. The treatment of $G$ in the Abdul-Razzak scheme limits its ability to account for such variations in $\chi_c$. However, the dependence on gas kinetic effects can be added to the scheme by approximating $G$ in (35) and (36) as

$$G = G_0 G_{\text{ac1}} / G_{\text{acl}}$$  \hspace{1cm} (40)$$

where $G_0$ is the value of $G$ estimated without gas kinetic effects, $G_{\text{ac1}}$ is estimated using the critical radius corresponding to the number mode radius and using the selected value of the condensation coefficient, and $G_{\text{acl}}$ is estimated using the same critical radius but a value of 1.0 for the condensation coefficient. This treatment of $G$ yields the same results as the original tuned parameterization when the condensation coefficient equals 1.0 (and thus preserves the tuning), but as we shall see it accounts for the dependence on the value of the condensation coefficient remarkably well.

The limitations of the Abdul-Razzak scheme were addressed by Nenes and Seinfeld [2003] for a sectional representation of the size distribution and by Fountoukis and Nenes [2005] for a lognormal representation. Considerable care was devoted to relating $S_{\text{part}}$ to $S_{\text{max}}$ particularly for conditions when kinetic limitations on droplet nucleation are expected. The extent of kinetic limitations is expressed in terms of the difference between $S_{\text{max}}^2$ and $2\zeta$. When kinetic limitations are expected, the relationship between $S_{\text{max}}$ and $S_{\text{part}}$ is determined empirically from numerical simulations for a range of conditions.

Another unique feature of the Nenes scheme is its ability to account for the influence of gas kinetics on the water vapor diffusivity. This influence depends on particle size and on the value of the condensation coefficient $\chi_c$ (see Appendix A). Fountoukis and Nenes [2005] found that an average value of the diffusivity over an appropriate size range can account for the influence of gas kinetics on droplet nucleation. By expressing the solution in terms of $\chi_c$, the scheme is applicable to a range of values of the condensation coefficient. This same approach was also adopted in the Shipway and Abel [2010] scheme.

Although, unlike the Abdul-Razzak scheme, the Nenes scheme does not approximate functions of $S_{\text{max}}$, and does not rely on empirical relationships (except when kinetic limitations are dominant throughout the CCN population), it does require iterations to solve for $S_{\text{max}}$. It is consequently a factor of 20–100 more computationally expensive than the Abdul-Razzak scheme, though it is still much less expensive than the full numerical integration.

Another parameterization that uses iteration is that developed by Ming et al. [2006]. Population splitting is not used; instead, the integration over $S_c$ in equation (33) is performed numerically using a sectional representation of the aerosol size distribution. Kinetic effects are treated by using an empirical modification of (33):

$$\int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} S_{\text{dr}} \approx \frac{S_{\text{max}}^2 - S_{\text{c1}}^2}{2xw}$$  \hspace{1cm} (41)$$

and $G$ is evaluated at the critical radius $r_c$. The value 2.4 for the exponent has been determined empirically from numerical simulations that treat kinetic effects. The expression for the particle growth rate used to estimate the condensation rate retains the $S_{\text{eq}}$ term in (26). Thus, under some conditions this scheme could be more accurate than the Nenes scheme. However, the use of numerical integration over size increases its computational cost considerably, although it is also much faster than explicit numerical integration over time.
Cohard et al. [1998, 2000] extended Twomey’s [1959] parameterization by adding a term to the power law CCN spectrum to limit activation at high supersaturation. Khvorostyanov and Curry [2006] showed how Cohard’s CCN spectrum is an approximation to that of a lognormal distribution. Although this valuable extension is limited to a single mode, Shipway and Abel [2010] recently extended it to the multimode case, related the aerosol physicochemistry to the spectrum parameters (using results of Khvorostyanov and Curry [2006]), and demonstrated agreement with numerical simulations of droplet formation to within 20%.

Several other droplet nucleation parameterizations have been proposed [Hänel, 1987; Khvorostyanov and Curry, 2008; Kivekas et al., 2008; Khvorostyanov and Curry, 2009]. We will not examine them in detail, but will briefly summarize them here.

Hänel [1987] adopted some of the same assumptions later adopted by Ghan et al. [1993], namely that at maximum supersaturation all activated particles are at their critical size, and retains the $S_{eq}$ term in (26). The resulting expression for supersaturation balance at maximum supersaturation

$$\tilde{\alpha} = \gamma' G \left[ \int_0^{S_{\text{max}}} (S_{\text{max}} - S_c) r_c \frac{dN}{dS_c} dS_c \right]$$

(42)

can be rearranged in the form

$$S_{\text{max}} = \left( \frac{\tilde{\alpha}}{\gamma' G} + \int_0^{S_{\text{max}}} S_c r_c \frac{dN}{dS_c} dS_c \right) / \int_0^{S_{\text{max}}} r_c \frac{dN}{dS_c} dS_c$$

(43)

Equation (43) can be solved by numerical integration over size and by iteration on $S_{\text{max}}$ in a manner similar to that of Ming et al. Errors in (43) due to the assumption that $r(t_{\text{max}}) = r_c$ are addressed by introducing an empirical correction factor based on numerical simulations with a parcel model. Roelofs et al. [2006] have applied this scheme to the ECHAM5-HAM global climate model.

Khvorostyanov and Curry [2008, 2009] use (23) to approximate the number nucleated as a function of $S_{\text{max}}$. This simplifies the solution for $S_{\text{max}}$ via a quick numerical solution for the time of maximum supersaturation. Comparisons of the nucleated with numerical integrations indicate agreement to within 10% for a modest range of conditions. So far the scheme has been limited to the case of a single lognormal aerosol mode.

Kivekas et al. [2008] use a large number of numerical simulations to parameterize the size of the smallest particle activated in terms of a simple function of updraft velocity, total submicron volume concentration, the soluble volume fraction, and a number to volume ratio, where the number is for particles larger than a cutoff radius (0.05 μm) and the volume is the total submicron aerosol volume. The total number nucleated can then be estimated from the number of particles larger than the smallest size activated. The scheme determines the number nucleated to within 15% for a range of conditions and is exceptionally fast.

Several extensions to the conditions considered above have also been introduced. Barahona and Nenes [2007] showed that the influence of entrainment on supersaturation and activation can be accounted for by generalizing the expression for the adiabatic expansion parameter $\alpha$ in (5):

$$\alpha \equiv \left( \frac{g_{\text{w}} - \frac{g_{\text{w}}}{RT}}{T} \right) \left( 1 - \frac{e}{e_c} \right)$$

(44)

where $e$ is the entrainment rate ($\text{m}^{-1}$) and $e_c$ is the critical entrainment rate (defined as the value of $e$ that completely inhibits cloud formation), given by

$$e_c \equiv \frac{g_{\text{w}} - \frac{g_{\text{w}}}{RT}}{(1 - RH) - \frac{M_{\text{w}} L}{RT} (T - T')}$$

(45)

where $T$ and $RH$ are the temperature and relative humidity of the cloud environment. Thus, if $e > e_c$, then $\alpha < 0$ and droplets do not nucleate. This treatment of the dependence of activation on entrainment can be applied to any physically-based parameterization that explicitly considers the cooling rate term $\tilde{\alpha} w$ in (5). Although it provides a simple treatment of the effects of entrainment on supersaturation in clouds, it does not address the potential effects of activation of aerosol entrained into the cloud from the lateral edges of the cloud [Fridlind et al., 2004].

A second extension is treatment of the influence of organic surfactants on the activation process. Organic surfactants lower the surface tension of the particle, which according to equation (2) reduces the critical supersaturation of the particle. But most organic surfactants have a lower hygroscopicity than that of sea salt and sulfate, which reduces the bulk hygroscopicity of the particle and hence increases the critical supersaturation. Thus, the net effect depends on the hygroscopicity of the surfactant and how strongly it influences surface tension.

The effects of organic surfactants on aerosol activation has been addressed by Abdual-Razzak and Ghan [2004]. They showed that, if the surface tension can be expressed analytically in terms of the surfactant concentration (which changes through dilution as the particle grows by condensation of water) then surfactant effects can be quantified using the surface tension estimated at the critical size corresponding to the number mode radius of the size distribution. In this case the expression for the critical radius must be generalized to account for the dependence of surface tension (and the parameter $A$ that depends on surface tension – see the Appendix) on the wet radius:

$$r_c^4 \frac{dS}{dr_c} = Ar_c^2 - 3kr_m^2 - r_c^3 \frac{dA}{dr_c} = 0$$

(46)
Equation (46) can be solved for \( r_c \) by iteration with the expression for the dependence of surface tension on surfactant concentration and hence \( r_c \). Once \( r_c \) is known, the parameter \( A \) can be calculated and the expression for the critical supersaturation for the mode radius, \( S_m \), can be evaluated and used in (37) to diagnose \( S_{max} \). The number activated follows from (20).

Rissman et al. [2004] also extended the Abdul-Razzak scheme to include surface tension effects consistent with Köhler theory, using a formalism similar to equation (46); a simplified explicit form was then derived and subsequently used to compute analytical sensitivities of nucleated droplet number to chemical, dynamical and microphysical parameters.

The parameterization frameworks of Nenes and Seinfeld [2003] and Ming et al. [2006] can also directly account for organic surfactant impacts on droplet formation, provided the routines used to compute the critical supersaturation spectrum (equation (5)) consider surface tension impacts (e.g., equation (46)). Nenes and Seinfeld [2003] demonstrate this capability by using surface tension data derived from Po Valley fogwater; a global modeling study on surface-tension impacts on cloud droplet number has not been carried out to date.

If giant CCN are present then Barahona et al. [2010] have shown that it is important to include an additional condensation rate term that accounts for condensation on inertia-limited large particles.

An alternative to parameterizations is the use of look-up tables based on a large number of numerical simulations. To reduce the dimensionality of the look-up table, Saleeby and Cotton [2004] accounted for the dependence on only four parameters: temperature, updraft velocity, CCN concentration, and median radius of the CCN size distribution. Segal and Khain [2006] tabulated droplet number for several combinations of updraft velocity, CCN concentration, median radius, and width of the size distribution. Such treatments can be computationally very fast and are appropriate for conditions when the composition and number of modes are uniform, but for global applications, which span a wide range of composition and size distributions, additional dimensions are required because, as we shall see, droplet nucleation depends on many parameters. Although look-up tables have been used in one global model [Ming et al., 2007], the large dimensionality of the tables limits applicability of the model to computing systems with sufficient local memory to support the large tables.

To summarize the description of the activation schemes, Table 1 lists the treatments of mass transfer, the condensation coefficient, the integration over size distribution, the solution method, the treatment of kinetic limitations for each scheme. We do not have computational timings of every scheme, but we have found that the iterative Nenes, Shipway and Ming schemes are a factor of 20–100 times slower than the analytic ARG scheme, and that the timing of the ARG scheme can be reduced by a factor of 10 by replacing the error function in the expression for activation (20) with the hyperbolic tangent approximation (23). The approximated ARG scheme is a factor of 200–1000 faster than the others. The timings of the Nenes and Shipway schemes are comparable, but the Ming scheme takes about three-fold more time than the Nenes and Shipway schemes because, in addition to iterating, it integrates numerically over the aerosol size distribution; consequently, in applications to a climate model [Ming et al., 2007] it is replaced with a lookup table. The Nenes scheme requires about 30 iterations to converge, which explains the much larger timing compared with the ARG scheme.

3. Parameterization Performance

Our evaluation will focus on the Abdul-Razzak and Ghan [2000, hereinafter ARG], Fountoukis and Nenes [2005, hereinafter Nenes], Ming et al. [2006, hereinafter Ming], and Shipway and Abel [2010, hereinafter Shipway] parameterizations. Those are the physically-based schemes that are being or will be used in global models, and hence merit the greatest attention. For the Nenes scheme the Barahona et al. [2010] extension for condensation on inertia-limited particles is included.

In principle one could use atmospheric measurements of updraft velocity, droplet number concentration and aerosol composition and size distribution to evaluate models of the activation process. Although this has been done successfully [Meskhidze et al., 2005; Fountoukis et al., 2007], the conclusions are constrained by the limited parameter space explored and uncertainties of measuring updraft velocity and measuring both the aerosol below cloud and the droplet number within cloud. We therefore turn to detailed numerical solutions to provide benchmark simulations of cloud droplet nucleation.

To evaluate the performance of the parameterizations we compare results with numerical solutions for a variety of conditions. The numerical model we are using is that described by Abdul-Razzak et al. [1998], which is based on the general dynamic Köhler theory described by Pruppacher and Klett [1997], i.e., equations (4), (5), and (7). The aerosol size distribution is divided into 100 internally-mixed bins, with the bin boundaries spaced such that each bin has the same number of particles, and the central bin corresponding to the mean size of the distribution. For cases with multiple aerosol modes, 100 bins are devoted to each mode. The model produces results virtually identical to the numerical model of Nenes et al. [2001], which produced results in the middle of a range of results from different numerical models [Kreidenweis et al., 2003]. However, it must be recognized that, as documented by Kreidenweis et al. [2003], inter-model differences in binning and the treatment of mass transfer of water vapor can lead to 30% differences in number nucleated. Indeed, the numerical model that the Ming scheme is based on produces results that differ significantly from those produced by this numerical model; those differences contribute to the differences between the...
overestimations in $S_{\text{max}}$ of water vapor on inertially-limited particles can lead to large overpredictions of droplet number. This differs from the droplet definition of Nenes et al. [2001] who consider inertially-limited particles as droplets. Although this does not affect the numerical parcel simulations, Barahona et al. [2010] demonstrated that neglecting condensational depletion of water vapor on inertially-limited particles can lead to large overestimations in $S_{\text{max}}$ and droplet number.

To explore the parameter space we first consider a baseline case and then evaluate the performance as selected parameters are varied from the baseline case. We first consider a single lognormal aerosol mode to establish the dependence of the performance on each of the parameters, and then address more general multimode cases that are more applicable to interpretation of the application to global models of aerosol effects on clouds. We have also compared the schemes using measured aerosol size distributions, but do not report those results here because they can be understood in terms of the multimodal cases.

The first baseline case is a single lognormal mode with a total aerosol number concentration of 1000 cm$^{-3}$, a number mode radius of 0.05 µm, and a geometric standard deviation of 2. The aerosol is assumed to be composed of ammonium sulfate, which has a density of 1.71 g cm$^{-2}$ and a hygroscopicity of 0.7. The condensation coefficient is assumed to be 1, and entrainment is neglected. The baseline updraft velocity is 0.5 m s$^{-1}$, which is typical of stratiform clouds that exhibit the greatest sensitivity to the aerosol.

Figure 3 shows the maximum supersaturation and number fraction activated as functions of updraft velocity. Although the parameterizations were designed for application to stratiform clouds, which usually have updraft velocities between 0.1 and 1 m s$^{-1}$ [Meskhidze et al., 2005], we show results for updrafts up to 10 m s$^{-1}$ to demonstrate their applicability to cumulus clouds. The Nenes and Shipway schemes diagnose maximum supersaturation in good agreement with the numerical solution for all updraft velocities. The ARG scheme underestimates $S_{\text{max}}$ for updrafts stronger than 1 m s$^{-1}$, while the Ming scheme overestimates maximum supersaturations for updrafts stronger than 2 m s$^{-1}$, which is beyond the updraft range that it is used for. The Ming scheme underestimates $S_{\text{max}}$ for updrafts weaker than 1 m s$^{-1}$, so it exaggerates the sensitivity to updraft velocity. Consistent with the performance for supersaturation, the Nenes and Shipway schemes diagnose the number fraction activated in excellent agreement with the numerical solution, except for updrafts weaker than 0.4 m s$^{-1}$, when the Nenes scheme underestimates activation by about 30%. The abrupt drop in number activated by the Nenes scheme for low updraft velocity arises when droplet activation is dominated by kinetic limitations and the expression used to derive $S_{\text{part}}$ changes; this feature is evident in other figures as the parameter space is explored. Smoothing this transition out further will be the subject of a future study. The ARG scheme consistently underestimates the number activated by 10–20%. The Ming scheme underestimates activation by up to 40% for updrafts weaker than 1 m s$^{-1}$, but diagnoses activation quite accurately for strong updrafts.

Figure 4 explores the performance as a function of aerosol number concentration for a fixed updraft velocity of 0.5 m s$^{-1}$. The ARG scheme consistently underestimates $S_{\text{max}}$ by about 10%, while the Nenes scheme diagnoses $S_{\text{max}}$ remarkably well for aerosol number concentrations less than 1000 cm$^{-3}$, but underestimates $S_{\text{max}}$ by about 20% for $N_a > 2000$ cm$^{-3}$. Ming underestimates $S_{\text{max}}$ by about 30% for $N_a > 2000$ cm$^{-3}$ but overestimates $S_{\text{max}}$ by up to 30% for low aerosol number concentrations. The Shipway scheme diagnoses $S_{\text{max}}$ well for number concentrations less than 3000 cm$^{-3}$, but overestimates $S_{\text{max}}$ for higher concentrations. Consequently, the ARG scheme consistently underestimates the number activated by about 10%, the Nenes scheme diagnoses the number activated remarkably well for $N_a < 1000$ cm$^{-3}$ but underestimates activation by about 20% for $N_a > 2000$ cm$^{-3}$, Ming underestimates activation for $N_a > 1000$ cm$^{-3}$ by up to 50% but overestimates activation for $N_a < 300$ cm$^{-3}$, and the Shipway scheme diagnoses the number activated well for $N_a < 3000$ cm$^{-3}$ but overestimates activation by up to 100% as $N_a$ approaches 10,000 cm$^{-3}$.

Figure 5 considers the dependence on the mode radius of the size distribution. As the distribution shifts to larger sizes more particles are activated earlier, limiting the supersaturation increase and resulting in smaller $S_{\text{max}}$. The ARG scheme diagnoses both $S_{\text{max}}$ and the number activated remarkably well for all sizes. The Nenes and Shipway schemes both perform well for mode radius up to 0.1 µm, but for larger sizes they overestimate both $S_{\text{max}}$ and the number activated significantly. The Ming scheme overestimates $S_{\text{max}}$ and the number activated for mode radius less than 0.03 µm or greater than 0.5 µm and underestimates $S_{\text{max}}$ and the number activated for intermediate mode radius. The surprisingly strong performance of the ARG scheme, which neglects kinetic limitations that one expects to be important at larger sizes, shows how empiricism (which the ARG scheme relies on more heavily than the other scheme) can...
perform better for some conditions. However, as we shall see the ARG scheme does not perform as well under other conditions.

Figure 6 examines the performance as a function of the width of the aerosol size distribution. The numerical solution shows a decrease in $S_{\text{max}}$ with increasing width because of the addition of larger particles that are activated earlier. The ARG and Ming schemes capture much of this dependence, although the Ming scheme underestimates $S_{\text{max}}$ for all widths. The Nenes and Shipway schemes, on the other hand, produce almost no dependence on the width. Consequently, the ARG and Ming schemes correctly yield decreasing activation with increasing width, but the Nenes and Shipway schemes do not except for very narrow distributions.

The dependence of the performance on composition can be evaluated by considering the performance as a function of the hygroscopicity parameter $\kappa$, shown in Figure 7. The dependence is captured quite well by the ARG, Nenes and Shipway schemes. As in the cases of sensitivity to updraft velocity, aerosol number concentration, and mode radius, the Ming scheme exaggerates the dependence of $S_{\text{max}}$ on hygroscopicity. However, sensitivity of number activated to hygroscopicity is underestimated by the Ming scheme, because the number activated depends on both the maximum supersaturation and the sensitivity of activation to the supersaturation (which depends on hygroscopicity).

The activation process also depends on the value of the condensation coefficient, which has values reported between 0.01 and 1 [Mozurkewich, 1986]. Figure 8 examines the dependence of activation on the value. All four schemes capture this dependence quite well.

The dependence of maximum supersaturation and number activated on the entrainment rate is evaluated in Figure 9. The entrainment rate is expressed in terms of the ratio of the entrainment rate to the critical value $e_c$. The Barahona and Nenes [2007] treatment of the influence of entrainment has been applied to each of the parameterizations. All parameterizations capture this dependence accurately.

The influence of surfactants is illustrated in Figure 10, which evaluates the ARG and Nenes parameterizations as a
function of surfactant fraction of the aerosol. Both schemes correctly diagnose the increase in $S_{\text{max}}$ and decrease in number activated with increasing surfactant fraction simulated by the numerical model. Abdul-Razzak and Ghan [2004] show that their treatment also performs well for the more realistic case of organic surfactants sampled in the Po Valley of Italy, which produces a very different dependence on organic surfactants.

A second class of cases to evaluate use a more general representation of the aerosol size distribution based on trimodal lognormal fits to measurements [Whitby, 1978]. Table 2 lists the lognormal parameters for several aerosol types.

Figure 11 evaluates the activation of the marine aerosol for updraft velocities between 0.1 and 10 m s$^{-1}$. The Nenes scheme diagnoses this more complex case remarkably well, but the ARG, Shipway and Ming schemes all produce biases. The Shipway scheme underestimates $S_{\text{max}}$ by up to 30% and the ARG scheme underestimates the maximum supersaturation by 40% for all updraft velocities. Consequently, the Shipway and ARG schemes underestimate the activation of the nuclei mode for strong updrafts and the accumulation mode for weak and moderate updrafts. Ming overestimates $S_{\text{max}}$ for strong updrafts (beyond those it is used for) and hence overestimates activation of the nuclei mode for strong updrafts, but estimates the activation of the other modes quite well.

Figure 12 examines the performance for the clean continental aerosol. The Nenes and Shipway schemes both perform remarkably well, and the biases in the ARG scheme are smaller than for the marine aerosol. Ming performs well for the weaker updrafts it was designed for. For weaker updrafts the performance of the ARG and Ming schemes is comparable.

The activation of the Whitby background aerosol is evaluated in Figure 13. The ARG scheme underestimates $S_{\text{max}}$ and activation of the accumulation mode by about 30% for all updraft velocities. The Nenes scheme performs very well for strong updrafts, but underestimates $S_{\text{max}}$ and activation of the accumulation mode by about 30% for updrafts weaker than 0.6 m s$^{-1}$. The Ming scheme underestimates $S_{\text{max}}$ and hence activation of the

![Figure 4](image)

**Figure 4.** As in Figure 3, but as a function of aerosol number concentration for a fixed updraft velocity of 0.5 m s$^{-1}$. The baseline number concentration is 1000 cm$^{-3}$.
accumulation mode by up to 50% for updrafts weaker than 2 m s\(^{-1}\). The Shipway scheme performs well for all updraft velocities.

For the Whitby urban aerosol (Figure 14), the Shipway scheme overestimates \(S_{\text{max}}\) and hence aerosol activation while the other schemes underestimate \(S_{\text{max}}\) and consequently aerosol activation. The Shipway scheme diagnoses \(S_{\text{max}}\) well for strong updrafts but for weak updrafts overestimates \(S_{\text{max}}\) by a factor of about two, and consequently overestimates activation for weak updrafts. The Nenes and Ming schemes diagnose \(S_{\text{max}}\) well for updrafts weaker than 0.5 m s\(^{-1}\), but the ARG scheme is more accurate for updrafts stronger than 2 m s\(^{-1}\). The Nenes, ARG and Ming schemes underestimate activation of the accumulation mode for strong updrafts, and the ARG scheme underestimates activation of the coarse mode for weak updrafts.

These results for the Whitby trimodal aerosol are consistent with those for the single mode aerosol. The Ming scheme overestimates \(S_{\text{max}}\) and the number activated for low aerosol concentrations such as for the marine aerosol but underestimates \(S_{\text{max}}\) and activation for increasingly high aerosol concentrations such as the urban aerosol. The Shipway and Nenes schemes perform well under most conditions, except in polluted conditions when Shipway overestimates \(S_{\text{max}}\). In some cases (dependence on width of distribution) the ARG scheme performs better than the Nenes and Shipway schemes because the former has been tuned to agree with the numerical simulations, but in other cases the Nenes and Shipway schemes perform better because they have more robust physics. The ARG scheme estimates lower \(S_{\text{max}}\) and activation than Nenes and Shipway for low aerosol concentrations and estimates more activation than Nenes for high aerosol concentrations, and hence is more sensitive to increases in aerosol concentration. Some of this difference is likely due to neglect of kinetic effects in the ARG scheme, which are more important for higher aerosol concentrations [Nenes et al., 2001]. As we shall see,

![Figure 5](image-url)
this has implications for effects of anthropogenic aerosol on clouds.

4. Applications

Physically-based droplet nucleation parameterizations have been applied to a variety of models. Applications are summarized in Table 3. In all applications the parameterizations are applied to double-moment models that predict droplet number concentration from the droplet number balance, in most models also including effects of collision/coalescence, collection and evaporation as well as nucleation \[\text{Ghan et al., 1997}\]. The first applications were to global models because of interest in quantifying the aerosol indirect effect on climate. In such models a critical element of the application of droplet nucleation schemes is the representation of the updraft velocity, which as we have seen has a strong and nonlinear influence on droplet nucleation. Given the coarse resolution of global models, updrafts are not adequately resolved. Subgrid variations in updraft velocity and droplet nucleation must be represented. Ghan et al. [1997] show how this subgrid variability can be expressed in terms of the subgrid probability distribution of updraft velocity, \(p(w)\):

\[
\tilde{N}_n = \int_0^\infty N_{act}(w)p(w)dw \tag{47}
\]

For turbulent boundary layers \(p(w)\) can be approximated by a Gaussian distribution with mean given by the resolved vertical velocity and the standard deviation of the distribution, \(\sigma_w\), related to the turbulence kinetic energy (\(e\)) by assuming the turbulence is isotropic: \(\sigma_w^2 = \frac{2}{3}e\) [Lohmann et al., 1999]. Alternatively, \(\sigma_w\) can be related to the eddy diffusivity \((K)\): \(\sigma_w = \frac{K}{l_c}\) [Morrison and Gettelman, 2008], where \(l_c\) is a prescribed mixing length. Since \(N_{act}\) is a complex function of updraft velocity the integral in (47) cannot be performed analytically. Some global models therefore integrate numerically [Ghan et al., 2001a, 2001b; Ghan and Easter, 2006], but this can be computationally expensive. Most models [e.g., Lohmann et al., 2007; Ming et al., 2007; Gettelman et al., 2008; Wang and Penner, 2009]
therefore approximate the integral by evaluating \( N_{\text{act}} \) at only one updraft velocity, often the sum of the resolved updraft velocity and \( \sigma_w \), within the grid cell. Morales and Nenes [2010] have explored this issue more deeply, and have shown that for a Gaussian \( p(w) \) use of a single characteristic updraft velocity given by \( 0.65 \sigma_w \) yields grid cell mean droplet numbers within 10% of those with numerical integration over \( p(w) \). In some models [Ghan et al., 1997] a minimum for \( \sigma_w \) is applied because the processes that drive turbulence are not represented well, and this minimum value is treated as a tuning parameter because it is not constrained well by measurements.

Since droplet nucleation is one of several terms in the droplet number balance, the estimate of the number nucleated must be converted to a droplet nucleation tendency. Several methods have been employed. Ghan et al. [1997] and Ovtchinnikov and Ghan [2005] distinguish between droplet nucleation in growing clouds and nucleation at the base of existing clouds:

\[
\frac{dN_k}{dt} = \frac{dN_k}{dt} N_n + \frac{\text{min}(f_k-f_{k-1},0)}{\Delta z} \int_{w_{\text{min}}}^{\infty} w N_{\text{act}}(w) dw \tag{48}
\]

where \( f_k \) is the cloud fraction in layer \( k \) and \( w_{\text{min}} \) is a minimum updraft velocity estimated by assuming stronger updrafts occur in the cloudy fraction of the grid cell. Nucleation in the interior of existing clouds is neglected. The factor \( f_k f_{k-1} \) in (48) is the clear sky fraction below layer \( k \), assuming maximum overlap.

A second method [Lohmann et al., 1999] to determine the tendency simply restores the droplet number toward the number nucleated:

\[
\frac{dN}{dt} = \frac{\text{max}(N_n-N_i,0)}{\Delta t} \tag{49}
\]

where \( N_i \) is the droplet number and \( \Delta t \) is the time step. This treatment is applied to all layers where cloud is present.

Figure 7. As in Figure 3, but as a function of hygroscopicity for a fixed updraft velocity of 0.5 m s\(^{-1}\). The baseline hygroscopicity is 0.7.
third method uses a prescribed activation timescale [Morrison and Gettelman, 2008] instead of the timestep in (49) to reduce the sensitivity to the timestep.

In most past applications the primary purpose of predicting droplet number has been to quantify the indirect effect of anthropogenic aerosol on the planetary energy balance through effects on droplet number, droplet effective radius, droplet collision/coalescence, cloud liquid water content, and cloud albedo [Chuang et al., 1997; Lohmann et al., 2000; Ghan et al., 2001b; Chuang et al., 2002; Takemura et al., 2005; Chen and Penner, 2005; Ghan and Easter, 2006; Ming et al., 2007; Storelvmo et al., 2006, 2008; Lohmann et al., 2007; Seland et al., 2008; Wang and Penner, 2009; Quaas et al., 2009; Hoose et al., 2009; Chen et al., 2010a, 2010b; Salzmann et al., 2010; Lohmann et al., 2010; Lohmann and Ferrachat, 2010]. More recent applications have involved coupled atmosphere-ocean simulations to estimate effects on climate [Chen et al., 2010b]. Many of the global models with these droplet number parameterizations are presently being used in climate change simulations for the fifth assessment of climate change by the Intergovernmental Panel on Climate Change.

In the last few years, droplet nucleation parameterizations have also been applied to cloud-resolving models to provide a computationally efficient alternative to explicit prediction of supersaturation. This has led to several studies investigating aerosol effects on cumulus clouds [Lee et al., 2008a, 2008b, 2009b, 2010], which have been neglected in most studies with global models. Other studies of such effects have been conducted using cloud models with size-resolved bin microphysics [Fridlind et al., 2004; Khain et al., 2004; Xue and Feingold, 2006; Fan et al., 2007, 2009; Li et al., 2008], which require explicit prediction of supersaturation and are much more computationally expensive, but offer the advantage of being able to treat droplet nucleation on the lateral edges and in the interior of the clouds.

Droplet nucleation parameterizations have also been applied to regional models that do not resolve cloud updrafts explicitly but provide a finer horizontal resolution.

Figure 8. As in Figure 3, but as a function of the condensation coefficient for a fixed updraft velocity of 0.5 m s$^{-1}$. The baseline condensation coefficient is 1.
than global models for a limited domain. Such models are
well suited for evaluating simulated sensitivity of clouds to
observed gradients in aerosol concentration [Gustafson
et al., 2007; Ivanova and Leighton, 2008; Bangert et al., 2011].

Finally, one scheme has been applied recently to a multi-
scale modeling framework [Wang et al., 2011a], in which a
cloud resolving model with double moment microphysics is
applied to each grid cell of a global model. That model has
been used to estimate global aerosol effects on cumulus as
well as stratiform clouds [Wang et al., 2011b].

5. Comparison of Parameterizations in a Global
Model

The initial purpose of droplet nucleation parameterizations
was to estimate aerosol effects on warm clouds. Although
different parameterizations have been applied to a variety of
models as summarized in Table 3, only recently have
different parameterizations been applied to the same model
so that their different influence on the estimated aerosol
indirect effects can be unambiguously compared. The
Abdul-Razzak and Ghan [2000] and Fountoukis and Nenes
[2005] schemes have both been applied to the Community
Atmosphere Model (CAM5), which has been released to the
public (http://www.cesm.ucar.edu/models/cesm1.0/cam/) with
the ARG scheme only. A detailed description of CAM5 is available at http://www.cesm.ucar.edu/models/
nces1.0/cam/docs/description/cam5_desc.pdf.

Figure 15 compares the annual mean column droplet
number simulated for present day emissions by CAM5 with
the ARG and Nenes schemes. Although the same treatment of
aerosol processes is used in each simulation, the aerosol
distributions are slightly different (to within less than 10%,
with no bias) because the droplet nucleation schemes pro-
duce slightly different simulations of nucleation scavenging
of the aerosol. The simulated column droplet number con-
centrations are remarkably similar, with the Nenes scheme
producing systematically larger concentrations by 0–20%.
This result is consistent with the tendency of the Nenes
scheme to diagnose higher activation fractions than the
ARG scheme for most conditions, as demonstrated in
Figures 3–14.

The comparison for preindustrial aerosol and precursor
emissions (but with present day ocean surface temperatures
and greenhouse gases), also shown in Figure 15, reveals a
greater tendency of the Nenes scheme to produce larger
droplet concentrations, by 20–50% compared with the ARG
scheme. This greater difference for preindustrial emissions (when aerosol number concentrations are lower) is not evident in Figure 4, which shows good agreement between the ARG and Nenes schemes for all number concentrations, but is apparent in comparing Figures 11–14, which clearly show greater differences in number activated for the Whitby marine aerosol and smaller differences for the increasingly polluted clean continental, background, and urban aerosol, the last indicating a reversal in the difference.

The greater difference in column droplet number concentrations from the two schemes for preindustrial conditions means smaller differences between present day and preindustrial column droplet number concentrations for the Nenes scheme than for the ARG scheme, which translates into a smaller estimate for aerosol indirect effects: the global mean difference between present day and preindustrial shortwave cloud forcing (a measure of aerosol indirect effects because experiments neglecting absorption by black carbon produce similar results) is $-1.60 \text{ W m}^{-2}$ with the Nenes scheme but is $-1.76 \text{ W m}^{-2}$ for the ARG scheme. This is a much smaller difference than the two-fold differences between indirect effects estimated by completely different GCMs [Penner et al., 2006; Forster et al., 2007; Lohmann et al., 2010], which suggests that differences in

Table 2. Aerosol Distribution Parameters (r_m, in μm, N_m, in cm$^{-3}$)$^a$

<table>
<thead>
<tr>
<th>Mode</th>
<th>Nuclei Mode</th>
<th>Accumulation Mode</th>
<th>Coarse Mode</th>
</tr>
</thead>
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<td></td>
<td>$r_1$, $\sigma_1$, $N_1$</td>
<td>$r_2$, $\sigma_2$, $N_2$</td>
<td>$r_3$, $\sigma_3$, $N_3$</td>
</tr>
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<td>Marine</td>
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<td>0.035, 2.0, 60</td>
<td>0.31, 2.7, 3.1</td>
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<td>Clean Continental</td>
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<td>0.034, 2.1, 800</td>
<td>0.46, 2.2, 0.72</td>
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<tr>
<td>Background</td>
<td>0.008, 1.7, 6400</td>
<td>0.038, 2.0, 2300</td>
<td>0.51, 2.16, 3.2</td>
</tr>
<tr>
<td>Urban</td>
<td>0.007, 1.8, 106000</td>
<td>0.027, 2.16, 32000</td>
<td>0.43, 2.21, 5.4</td>
</tr>
</tbody>
</table>

$^a$Whitby [1978].
other aspects of the treatments of clouds and aerosol in GCMS are producing most of the differences between estimates of aerosol indirect effects.

Given the large difference in the timings of ARG and Nenes schemes discussed in section 2, the timing difference in a GCM is also of interest. In CAM5, the Nenes scheme almost doubles the total run time compared to simulations with the ARG scheme. The difference would be larger if droplet nucleation was calculated for multiple updraft velocities rather than a single updraft velocity, or for all cloudy layers rather than just at cloud base and in growing clouds. It would be smaller if the error function is replaced by the hyperbolic tangent approximation.

6. Further Development

Although the droplet nucleation schemes provide robust physically-based representations of aerosol effects on droplet nucleation, further development is needed in several directions.
First, the influence of surfactants needs further development. Surfactant effects (that include bulk-surface partitioning of organics) should be applied to the lognormal Nenes scheme [Fountoukis and Nenes, 2005], and the influence needs to be connected to organic surfactants in application models. The influence of organics and particle phase state on droplet activation kinetics needs to be quantified and understood so that it can be represented in the parameterizations (e.g., parameterized as changes in the condensation coefficient). This issue remains an outstanding source of droplet number prediction uncertainty [Nenes et al., 2002]. Although secondary organic aerosol and highly aged ambient aerosol with high organic content tends to exhibit rapid activation kinetics similar to CCN composed of pure NaCl and (NH₄)₂SO₄ [Engelhart et al., 2008; Moore et al., 2008; Bougiatioti et al., 2009; Murphy et al., 2009; Asa-Awuku et al., 2010; Padro et al., 2010; Engelhart et al., 2011], an emerging body of evidence suggests that secondary organic CCN of low hygroscopicity can exhibit substantially slower activation kinetics than CCN composed of pure NaCl or (NH₄)₂SO₄ [Chuang, 2003; Lance, 2007; Ruehl et al., 2008; Sorooshian et al., 2008; Asa-Awuku et al., 2009; Murphy et al., 2009; Ruehl et al., 2009; Shantz et al., 2010].
Second, the hygroscopicity of insoluble particles (such as dust and volcanic ash) can originate from the presence of soluble salts in the particles, and, from the adsorption of water vapor on their surface. The relative importance of each, together with the dry particle size, controls their critical supersaturation \cite{Kumar et al., 2009a, 2011a; Lathem et al., 2011}. The combined effect of adsorption and solute can be comprehensively accounted for by implementing the unified activation theory of \textit{Kumar et al.} \cite{2011b} within the activation parameterizations presented here \cite[e.g.,][]{Kumar et al., 2009b}.

Third, although field measurements have been used to evaluate the Köhler theory and one of the parameterizations described here \cite[Meskhidze et al., 2005; Fountoukis et al., 2007], information about the particle composition in those evaluations has been limited. Further evaluation is needed using particle size and composition information now available from single-particle mass spectrometers mounted behind a counterflow virtual impactor that samples and evaporates cloud droplets, leaving behind the particle upon which the droplet formed.

<table>
<thead>
<tr>
<th>Model Name</th>
<th>Type</th>
<th>Parameterization</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
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<td>CCM1</td>
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<td>Ghan et al. [1997]</td>
</tr>
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<td>global</td>
<td>Chuang et al.</td>
<td>Chuang et al. [1997, 2002]</td>
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<td>ECHAMS-HAM</td>
<td>global</td>
<td>Hanel</td>
<td>Roelofs et al. [2006]</td>
</tr>
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<td>MIRAGE</td>
<td>global</td>
<td>Abdul-Razzak</td>
<td>Ghan and Easter [2006]</td>
</tr>
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<td>global</td>
<td>Abdul-Razzak</td>
<td>Gettelman et al. [2008]</td>
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<td>Abdul-Razzak</td>
<td>Storelmo et al. [2006, 2008]</td>
</tr>
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<td>CAM3-UMich</td>
<td>global</td>
<td>Abdul-Razzak</td>
<td>Song and Zhang [2011]</td>
</tr>
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<td>This study.</td>
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</tbody>
</table>
updrafts are so strong that the supersaturation production occurs above the base of deep cumulus clouds when
Segal et al. [2008] and Ghan et al. [2009] variation on the
clouds in a global model, and the relationship between aerosol and droplet number to cumulus
importance for deep cumulus clouds [Bretherton and Park, 2009; Park and Bretherton, 2009]. Evaluation of the entrainment effect formulation [Barahona and Nenes, 2007] by Morales et al. (manuscript in preparation, 2011) for shallow and moderate-size cumulus suggests that it substantially improves the prediction of droplet number (compared to using an adiabatic formulation).

Fourth, the schemes need to be applied to shallow cumulus clouds, including the effects of entrainment [Barahona and Nenes, 2007; Barahona et al., 2011]. This should be straightforward for models with shallow cumulus schemes that diagnose mass flux and cloud fraction and hence updraft velocity [Bretherton and Park, 2009; Park and Bretherton, 2009]. Evaluation of the entrainment effect formulation [Barahona and Nenes, 2007] by Morales et al. (manuscript in preparation, 2011) for shallow and moderate-size cumulus suggests that it substantially improves the prediction of droplet number (compared to using an adiabatic formulation).

Fifth, the schemes also should be applied to deep cumulus parameterizations. Although the representation of cloud microphysics in deep cumulus parameterizations has been crude for many years, recent developments [Lohmann, 2008; Song and Zhang, 2011] suggest it is time to apply double-moment microphysics schemes to cumulus parameterizations. Menon and Rotstain [2006] applied an empirical relationship between aerosol and droplet number to cumulus clouds in a global model, and Lohmann [2008] applied a variation on the Ghan et al. [2003] scheme to deep cumulus clouds. Although application of physically-based nucleation schemes to droplet formation at the base of cumulus clouds is straightforward in cumulus parameterizations that diagnose updraft velocity, secondary nucleation of droplets can also be important for deep cumulus clouds [Pinsky and Khain, 2002; Segal et al., 2003; Heymsfield et al., 2009]. Secondary nucleation occurs above the base of deep cumulus clouds when updrafts are so strong that the supersaturation production term in equation (5) drives supersaturation even in the presence of large liquid water contents. To see this, we note that in strong updrafts in the cloud interior the droplets are not in thermodynamic equilibrium ($S > S_0$), so that the expression for the condensation rate can be simplified to

$$\frac{dW}{dt} = 4\pi S\rho_w \sum_i r_i G.$$  (50)

Then, assuming the Lagrangian supersaturation tendency $dS/dt$ is small compared to supersaturation production in updrafts and supersaturation depletion by condensation, the supersaturation balance can be approximated [Korolev, 1995; Morrison et al., 2005; Ming et al., 2007] by applying (50) to (5):

$$\alpha w = \gamma^* GSN \tilde{r}$$  (51)

where the dependence of $G$ on hydrometeor size is neglected and the summation over hydrometeors in (50) has been expressed in terms of the number of hydrometeors and their mean radius $\tilde{r}$. Equation (50) can be directly solved for the supersaturation. The potential utility of this diagnostic method was explored by Dearden [2009] in a kinematic framework that neglects the reduction in droplet number due to collision/coalescence and precipitation. To consider the influence of these effects we have applied (49) to the updrafts and cloud microphysics simulated by a cloud-resolving model with explicit cloud microphysics. The results are illustrated in Figures 16 and 17, where the diagnosed supersaturation is compared with that predicted in a model simulation of convective clouds forming near Kwajalein Island during the Kwajalein Experiment, KWAJEX [Yuter et al., 2005]. In this example, a cloud-resolving model [Khairoutdinov and Randall, 2003] with spectral bin cloud microphysics [Khain et al., 2004] is run in a two-dimensional configuration using 320 columns and 144 levels, with uniform 100-m horizontal and vertical resolution below 10 km and a stretched vertical grid above. Boundary conditions and prescribed large-scale forcing are from Blossey et al. [2007] and the fields presented here are for 0930 UTC 17 August 1999. Although hydrometeor radius in the strong updrafts is generally much greater than droplet radius near cloud base, the number of hydrometeors is greatly depleted by collision and coalescence and precipitation fallout, so that the product of number and radius may be smaller in the updrafts than where primary nucleation occurs near cloud base. For the case of precipitating convective clouds shown here, both methods determine supersaturations up to 5%, which are high enough to activate much of the interstitial aerosol in the interior of the cloud. Although the diagnostic method does not explain all of the variations in simulated supersaturation because it neglects other terms in the supersaturation budget (such as the Lagrangian tendency, which is important near cloud base), it estimates supersaturation to within 30% at most points, which suggests it is certainly better than
neglecting supersaturation in deep cumulus clouds. However, further analysis under a wider variety of conditions is needed to determine whether extensions to this simple model are needed.

Finally, observational and modeling analysis [Korolev and Mazin, 1993; Korolev, 1994, 1995; Magaritz et al., 2009] suggests secondary nucleation also occurs in the interior of stratiform clouds when drizzle or evaporation depletes droplet surface area and the updraft velocity exceeds a threshold given by (49) such that the supersaturation exceeds the critical supersaturation of the most easily activated interstitial particles. Although analysis of trajectories by the cited

Figure 16. Vertical cross-sections of (a) liquid water mixing ratio, (b) vertical velocity and (c) supersaturation simulated by the SAM_SBM, and (d) diagnosed supersaturation derived from the supersaturation balance equation (49) using simulated updrafts and parameters of hydrometeor distributions. Only part of the model domain is shown to enhance details.
authors suggests secondary nucleation occurs frequently in rising air parcels that had previously descended and lost droplets to evaporation, it is not clear whether a net increase in droplet number due to secondary nucleation is sufficiently common in stratiform clouds that it needs to be represented in cloud models.

7. Conclusions

The state of the art of representing aerosol effects on droplet nucleation in models has advanced considerably in the last two decades. Several physically-based methods produce estimates of maximum supersaturation and the number concentrations of droplets nucleated in good agreement (to within 30%) with detailed numerical integrations of the nucleation process under a wide variety of conditions. However, consistent differences are found for all schemes, and larger differences are found for the Ming scheme. The ARG scheme estimates lower $S_{\text{max}}$ and activation than Nenes for low aerosol concentrations and estimates more activation than Nenes for high aerosol concentrations, and hence is more sensitive to increases in aerosol concentration. The Shipway scheme agrees with the Nenes scheme except under polluted conditions, when the Shipway scheme overestimates supersaturation and activation. The Ming scheme overestimates $S_{\text{max}}$ and the number activated for low aerosol concentrations such as for the marine aerosol but under-estimates $S_{\text{max}}$ and activation for increasingly high aerosol concentrations such as the urban aerosol. These differences are attributed to differences between the detailed numerical models that were used to develop the schemes. In particular, the ARG scheme was tuned using simulations by the same numerical model used in this comparison, while the kinetic effects in the Ming scheme were empirically determined using simulations by a numerical model that agrees much better with the Ming parameterization than with the numerical simulations reported here. This suggests the need to compare simulations by numerical models to determine the conditions in which the simulations of droplet nucleation agree and disagree, and to determine which models need revision to reduce differences between the simulations. Kreidenweis et al. [2003] explore this issue to some extent, but a more complete investigation that separates numerics from physics and chemistry is clearly needed.

Two of the nucleation parameterizations have been applied to the same global model. The 10% difference in droplet number produced by the two schemes can be explained on the basis of the offline tests presented here. Because the second indirect effect amplifies the first indirect effect (which scales as the cube root of droplet number and hence would produce only a 3% difference), the 10% droplet number difference produces a 10% difference (0.2 W m$^{-2}$) in the estimated effect of anthropogenic aerosol on short-wave cloud forcing (a measure of the aerosol first and second indirect effect).

Further extensions of this work include applications to treat the global influence of organic surfactants on clouds and the influence of aerosol on shallow and deep cumulus clouds. Thus, although in some respects the work on this subject is mature, there are plenty of opportunities to improve on the progress reported here.

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Appendix A: Symbols

The Kelvin coefficient is defined as

\[ A \equiv \frac{4M_w \sigma}{RT \rho_w} \]  

(A1)

where \( M_w \) is the molecular weight of water, \( \sigma \) is the surface tension (assumed to be that of water unless surfactants are present), \( R \) the ideal gas constant for water vapor, \( T \) temperature, and \( \rho_w \) the density of liquid water.

The hygroscopicity parameter for a particle composed of a mixture of soluble and insoluble components can be expressed [Hänel, 1976; Pruppacher and Klett, 1997; Abdul-Razzak and Ghan, 2000; Petters and Kreidenweis, 2007] as the volume mean of the hygroscopicity of each component,

\[ k = \frac{q_j k_j / \rho_j}{\sum_j q_j / \rho_j} \]  

(A2)

where

\[ k_j = \frac{\rho_j M_v v_j \varepsilon_j}{\rho_w M_j} \]  

(A3)

is the hygroscopicity of component \( j \) comprising the particle, \( M_j \) is the molecular weight of the solute component \( j \), \( \rho_j \) the density of component \( j \), \( q_j \) the mass fraction of component \( j \), \( v_j \) the number of ions the solute component dissociates into, and \( \varepsilon_j \) is the soluble fraction of component \( j \) cloud and environment, J. Atmos. Sci., 30(2), 256–261, doi: 10.1175/1520-0469(1973)030<0256:TSPOCE>2.0.CO;2.


in the dry particle. For an internal mixture of soluble and insoluble components, all particles are assumed to have the same composition, so (A2) describes the hygroscopicity of the mixture of components, with \( q_j \) the mass mixing ratio of component \( j \). Petters and Kreidenweis [2007] show that the volume mixing rule accurately determines the hygroscopicity of an internal mixture of components with very different hygroscopicities.

The growth coefficient is defined [Pruppacher and Klett, 1997; Seinfeld and Pandis, 1998]

\[
G \equiv \left( \frac{\rho_w RT}{e_i D_r, M_w} + \frac{L p_w}{K_v T} \left( \frac{L M_w}{RT} - 1 \right) \right)^{-1}
\]  

(A4)

where \( e_i \) is the saturation vapor pressure of water, \( L \) is the latent heat of vaporization of water, and the modified vapor diffusivity and thermal conductivity depend on droplet size [Fukuta and Walter, 1970; Fitzgerald, 1974]:

\[
D_v' \equiv \frac{D_v}{r + \Delta_v} + \frac{D_v'}{\Delta_v} \left( \frac{2 \pi M_w}{RT} \right)^{1/2}
\]  

(A5)

\[
\frac{r}{r + \Delta_v} + \frac{K_v}{\Delta_v} \left( \frac{2 \pi M_w}{RT} \right)^{1/2}
\]

where \( D_v \) and \( K_v \) are the diffusivity of water vapor and the thermal conductivity of air [Pruppacher and Klett, 1997; Seinfeld and Pandis, 1998], \( \Delta_v \) and \( \Delta_T \) are the vapor and temperature jump lengths [Fuchs, 1959; Fitzgerald, 1974; Pruppacher and Klett, 1997], \( \alpha_c \) is the condensation coefficient [Mozurkewich, 1986], and \( \alpha_T \) is the thermal accommodation coefficient.

In addition,

\[
\alpha = \frac{g M_w L}{c_p RT^2}
\]  

(A7)

where \( g \) is acceleration due to gravity, \( p \) is atmospheric pressure, \( M_a \) is the molecular weight of dry air, and \( c_p \) is the specific heat of air.