MANUAL OF THE E-PHYSICS

(Mellor - Yamada turbulence closure model,
Monin - Obukhov surface layer processes,
and land/sea conditions)

by

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Preface

Figure 1 illustrates the general concept of the so-called "physics" in a general circulation model (GCM). The solar radiation is the whole source of energy given to the atmosphere. It first penetrates into the earth's surface. The energy is then released to the atmosphere in the form of evaporation for a large portion and in the form of sensible heat transfer for the rest of the portion. The moisture thus produced condenses, releasing a great deal of latent heat. This subject, however, will be discussed separately as the F-physics. Fig. 1

Near the earth's surface, there is a boundary layer, which is the transitional zone from solid or liquid surfaces to gaseous media. The thickness of the layer is 0 ~ 1.5 km over land and 200 ~ 700 m over ocean. This layer is referred to as the planetary boundary layer (PBL), which is formed and controlled mostly by surface heating. In fact, the depth over land varies considerably with the local time (figure 2). Wind speed in the PBL decreases with approaching the earth's surface, simply because of the geometric constraint. If one emphasizes the aspect of vertical profile of wind speed, considering the effect of the earth's rotation, the layer can be considered as the Ekman layer, but the thermal effect dominates the PBL process in reality. Fig. 2

The PBL is full of turbulent energy, because the turbulence is produced due to the buoyancy associated with the surface heating and partially due to the mechanical energy associated with the vertical wind shear. As a result of turbulent mixing, the heat and moisture are well mixed in the entire PBL. Observations reveal that potential temperature and the specific humidity distribute in almost constant with height. For this reason the PBL is also referred to as the mixed layer.

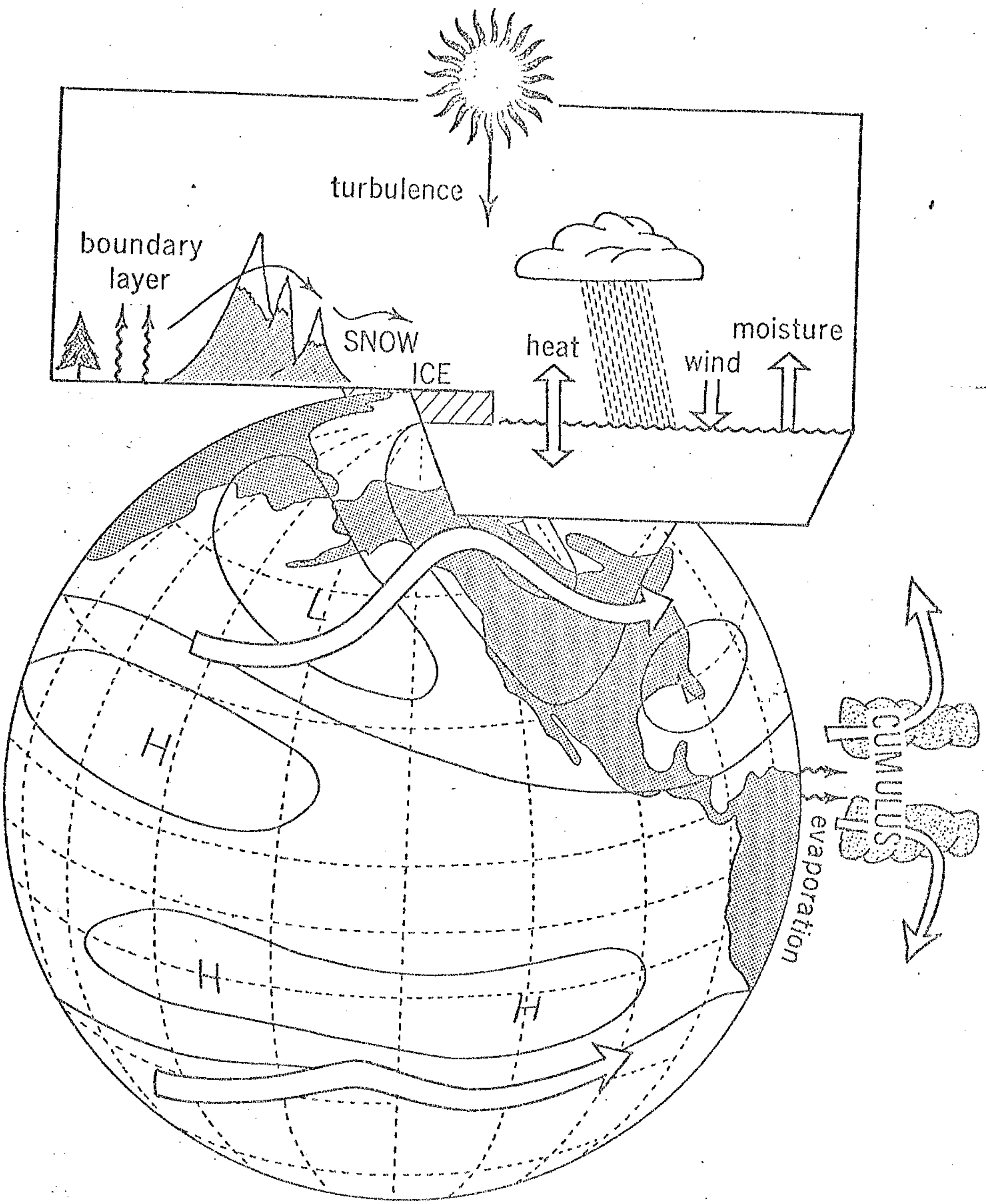

Fig. 1 Schematic picture of a general circulation model (GCM) and its physics. The physics is indicated in the upper box and the cumulus clouds at the equatorial zone in the lower globe.

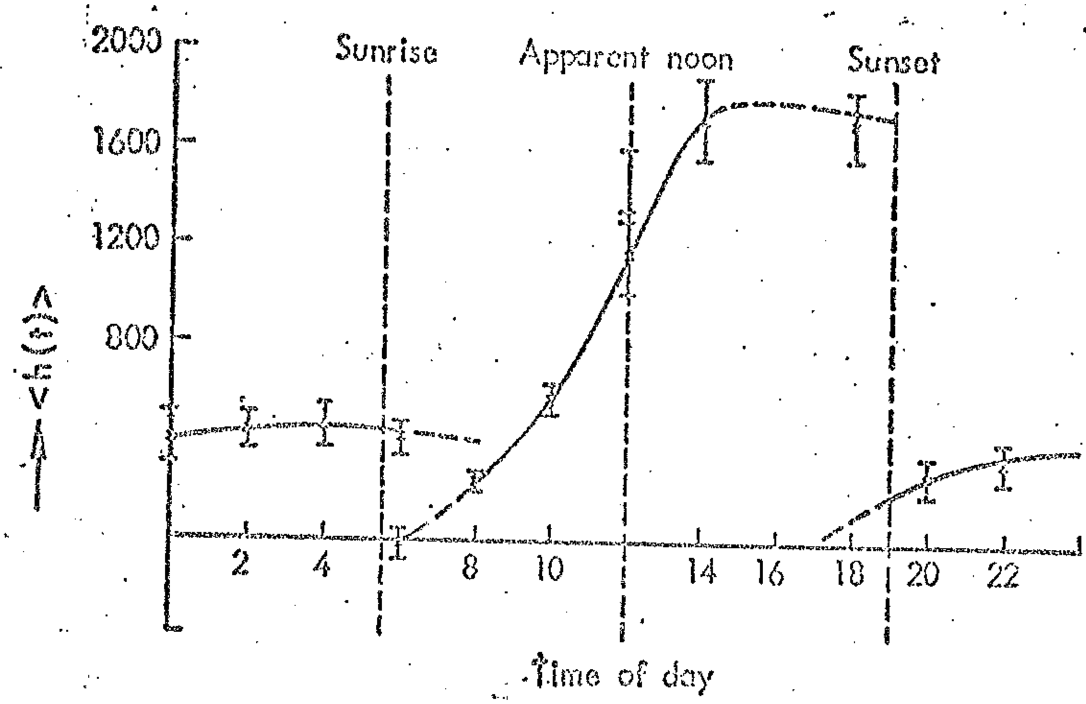

Fig. 2 The mean boundary-layer thickness, <h(t)>, deduced for the O'Neill data and plotted with standard errors in mean solar time (after Carson, 1973).

Inside the PBL (figure 3) the diurnal variability is pronounced; a nocturnal jet is generated; the moisture and sensible heat are vigorously advected in the horizontal; the snow is deposited; and the hydrological process including the vegetation takes place. An understanding of these events and phenomena is important for determining the so-called external (abnormal) forcings that influence substantially the atmospheric state. Fig. 3

The turbulence exists not only in the PBL but also in the free atmosphere. At high altitude, say at 100 mb, a considerable intensity of turbulence is often found, which is known as the clear air turbulence. It is likely that dissipation of kinetic energy takes place at the jet stream level in addition to the major dissipation in the PBL.

The E-physics of the GFDL GCM treats the turbulent processes in the PBL as well as in the free atmosphere, if the grid resolution is sufficiently fine. The E-physics includes the turbulence closure scheme, the treatment of surface boundary layer, the heat balance at the earth's surface, and the soil heat conduction. This model is, therefore, capable of reproducing the nocturnal jet, the overall diurnal variability of the PBL, the clear-air turbulence, mimicing snow deposits, and also a certain degree of land surface hydrology, through which the content of soil moisture is determined.

It is our view that the treatment of the subgrid-scale processes included in the E-physics is essential in calculating proper features of the meteorological and hydrological conditions at or next to the earth's surface and the diffusive character of momentum, heat and moisture in the PBL as well as in the free atmosphere. These conditions in turn exert a substantial impact on determining the intensity of westerly jet, its meandering, and accordingly the teleconnection characteristics of general circulation (see figure 1).

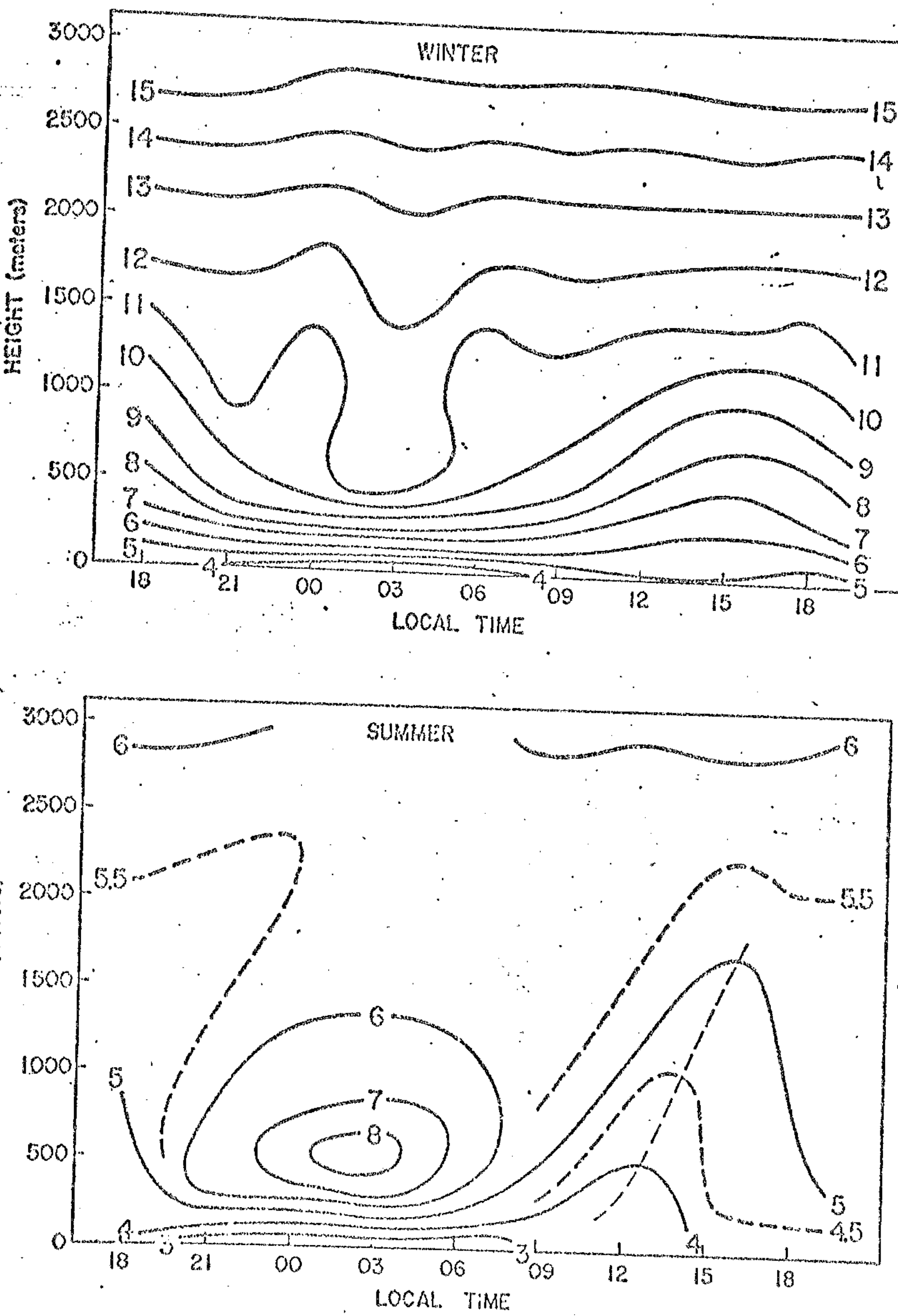

Fig. 3 Diurnal variation of the wind speed as a function of height. Values are based on data from Jackson, Mississippi; Shreveport, Louisiana; and Montgomery, Alabama (after Hoxit, 1973).

1. Introduction

There are three major items in the E-physics, i.e., the turbulence closure scheme, the Monin-Obukhov similarity scheme, and the land/sea surface processes.

The historical development of the turbulence closure theory is not our concern here. The particular model described in this manual is the second-order closure scheme of Mellor-Yamada's (1974) at the hierarchy level 2.5, in which the equation of turbulent kinetic energy is time-dependent, but the equation of the second-moment of thermal fluctuation is stationary. There are six or seven parameters which should be specified based on empirical evidences. Actually they have been determined based on neutral stratification in laboratory experiments. It is one of the major unique features that these parameters are commonly used for any media, such as the ocean or the Jupiter atmosphere without any consideration. Then our strategy is that, once these parameters are decided, they should not be changed arbitrarily, and other ad hoc treatments are not applied. The PBL processes are adequately represented by this scheme. The sharp transition of the thermal profile at the top of the PBL could be reproduced, though the simulated jump is not so conspicuous as in the model of hierarchy level 3. One of the merits in the level 2.5 model is that the turbulence is spatially diffused both in the vertical and the horizontal from the turbulence generating area to the stable area. As a result, the moisture in the PBL, for example, is adequately distributed above the PBL (say up to 3 km), whereas in the level 2 model, the diffusion is confined in the unstable region (Richardson number is less than 0.21) and therefore, limited to the PBL except for eviction through clouds.

The level 2.5 model would be capable of simulating the nocturnal jet (see section 8), the clear-air turbulence, if the grid resolution is adequate, and the turbulent transport process in the surface layer.

However, in order to simulate the fluxes close to the earth's surface, a fine vertical resolution is required for a numerical model. For reasons of economy, a bulk method is employed, i.e., processes based on the Monin-Obukhov similarity theory. They are (almost) consistent in physics to the turbulence closure level 2.5 scheme.

The third aspect of E-physics is the surface processes, which consist of the retention of moisture in soil, the runoff, the snow melt and snow deposit, the evapotranspiration, the soil heat conduction, the heat balance at the surface, the surface albedo, and sea ice heat conduction.

2. Grid-box Mean

It is inevitable that a GCM uses a finite grid length instead of a continuous coordinate in specifying the fluid dynamical equations. Associated with this space descritization, variables and equations are divided into the grid scale averages and their deviations. The effects arising from non-linear coupling of these deviations are the sub-grid scale (SGS) processes.

In practice, however, the mathematical treatment of the grid-box average is too complicated and awkward to handle. So, the ensemble mean and its deviation are normally taken, and the resulting formulas are then switched to the spatial mean and its deviations, ignoring additional terms (Leonard's term, for example; see Leonard, 1973; Clarke et al, 1979).

$$u = \overline{u} + u'$$

$$v = \overline{v} + v'$$

The bar is the ensemble mean, and the prime is its departure. In this manual, the second-moment $\overline{u'v'}$ is hereafter written as $\overline{uv}$ for simplicity, by dropping the prime notations. The bar notation for the first moment is often dropped too. As is readily conceived, the character of the SGS processes is very similar to that of turbulence, thus enabling us to utilize the wealth of knowledge on turbulence accumulated in the past.

2.1 Basic equations

The dependent variables of grid-resolvable scale in the E-physics version of GCM are: u, v, θ , q and b^2, where q is the mixing ratio of water vapor and $1/2b^2$ is the turbulent kinetic energy, $1/2\ b^2 = 1/2(\overline{u^2}+\overline{v^2}+\overline{w^2})$. The equations are normally written on the pressure coordinate (or the

σ -coordinate). However, the SGS terms will be expressed on z-coordinate.

$$\frac{du}{dt} - fv + \frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$= \frac{1}{\rho}\left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}\right) \quad (2.1)$$

$$\frac{dv}{dt} + fu + \frac{1}{\rho}\frac{\partial p}{\partial y}$$

$$= \frac{1}{\rho}\left(\frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}\right) \quad (2.2)$$

$$\frac{d\theta}{dt} = \frac{1}{\rho}\left(\frac{\partial \tau_{\theta x}}{\partial x} + \frac{\partial \tau_{\theta y}}{\partial y} + \frac{\partial \tau_{\theta z}}{\partial z}\right)$$

$$+ Q_R + \gamma(c - e) \quad (2.3)$$

$$\frac{dq}{dt} = \frac{1}{\rho}\left(\frac{\partial \tau_{qx}}{\partial x} + \frac{\partial \tau_{qy}}{\partial y} + \frac{\partial \tau_{qz}}{\partial z}\right)$$

$$-(c - e) \quad (2.4)$$

$$\frac{d\,b^2/2}{dt} = \frac{\partial}{\partial z}\left(\frac{5}{3}\, l\, b\, \frac{\partial b^2/2}{\partial z}\right)$$

$$- \overline{w\, V}\,\frac{\partial V}{\partial z} + \frac{g}{\theta_0}\,\overline{w\,\theta_v} - \frac{b^3}{\Lambda} \quad (2.5)$$

where $\gamma \equiv \frac{L \cdot \theta_o}{C_p \cdot T_{oo}}$, θ_o is the horizontal mean of θ, T_{oo} is the reference temperature, L is the latent heat, c and e are the rates of condensation and evaporation, ℓ and Λ are the turbulent scale length, g is the acceleration of gravity, and θ_v is the virtual potential temperature.

τ_{xx}, τ_{xy}.....τ_{yz} are the Reynolds stress terms, i.e.,

$$\begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \end{pmatrix} = \begin{pmatrix} -\rho\overline{uu} & -\rho\overline{uv} & -\rho\overline{uw} \\ -\rho\overline{vu} & -\rho\overline{vv} & -\rho\overline{vw} \end{pmatrix}$$

and the covariances of θ or q and momentum are:

$$\begin{pmatrix} \tau_{\theta x} & \tau_{\theta y} & \tau_{\theta z} \\ \tau_{qx} & \tau_{qy} & \tau_{qz} \end{pmatrix} = \begin{pmatrix} -\rho\overline{\theta u} & -\rho\overline{\theta v} & -\rho\overline{\theta w} \\ -\rho\overline{qu} & -\rho\overline{qv} & -\rho\overline{qw} \end{pmatrix}$$

2.2 Lateral diffusion

The formulation of eddy viscosity is made using an assumption on the rate of strain of fluid analogous to an elastic media (Smagorinsky, 1963). The shear stress is assumed to be linearly proportional to the rate of deformation. Let tensor τ_{ij} represent τ_{xx}, τ_{xy}.......Then

$$\tau_{ij} = K_M \cdot D_{ij} \tag{2.8}$$

where K_M is a parameter, and D_{ij} is the rate of deformation. So for the 2-dimensional case

$$[D_{ij}] = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{yx} & D_{yy} \end{bmatrix} = \begin{pmatrix} 2\dfrac{\partial u}{\partial x} - \nabla\cdot\mathbb{V} & \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} \\ \dfrac{\partial v}{\partial x} + \dfrac{\partial u}{\partial y} & 2\dfrac{\partial v}{\partial y} - \nabla\cdot\mathbb{V} \end{pmatrix}$$

$$= \begin{pmatrix} \dfrac{\partial u}{\partial x} - \dfrac{\partial v}{\partial y} & \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} \\ \dfrac{\partial v}{\partial x} + \dfrac{\partial u}{\partial y} & \dfrac{\partial v}{\partial y} - \dfrac{\partial u}{\partial x} \end{pmatrix} = \begin{pmatrix} D_T & D_S \\ D_S & -D_T \end{pmatrix} \qquad (2.9)$$

Later we will use the notations of D_T and D_S.

Similarly

$$\frac{1}{\rho}\begin{pmatrix} \tau_{\theta x} & \tau_{\theta y} \\ \tau_{q x} & \tau_{q y} \end{pmatrix} = K_M \begin{pmatrix} \dfrac{\partial \theta}{\partial x} & \dfrac{\partial \theta}{\partial y} \\ \dfrac{\partial q}{\partial x} & \dfrac{\partial q}{\partial y} \end{pmatrix} \qquad (2.10)$$

non-linear viscosity

Smagorinsky (1963) derived the formula of K_M to be consistent to the Kolmogoroff theory on the turbulence scale, i.e., E (wave energy) $\propto L^{5/3}$, L being the length scale. The formula is,

$$K_M = \text{const.}\ \Delta^2 |D| \qquad (2.11)$$

where Δ is the grid size of a GCM, and

$$D^2 = D_{xx}^2 + D_{xy}^2 + D_{yx}^2 + D_{yy}^2$$
$$= 2\left(D_S^2 + D_T^2\right)$$

As will be discussed later

$$K_H = \frac{5}{2} K_M \qquad (2.11')$$

The value for the constant in (2.11) will be explained in setion 3.4.

If a function of $|D|$ is a variable in space, K_M and K_H are non-linear. This situation is similar to that in the PBL; the vertical eddy viscosity coefficient is proportional to $\left|\frac{\partial V}{\partial z}\right|$. In other words, if the latter is true (dependent on the local value of $\left|\frac{\partial V}{\partial z}\right|$), K_M and K_H should be non-linear.

Smagorinsky used the form

$$\boxed{\begin{aligned} K_M &= \left(\frac{C\cdot\Delta}{\sqrt{2}}\right)^2 \cdot |D| \\ &= \alpha \cdot |D| \end{aligned}} \qquad (2.12)$$

where c = 0.14 (see later).

Then the equations are:

$$\boxed{\frac{du}{dt} = \text{------} + \frac{\partial}{\partial x}\left(\alpha\,|D|\,D_T\right) + \frac{\partial}{\partial y}\left(\alpha\,|D|\cdot D_S\right) \qquad (2.13)}$$

$$\frac{dv}{dt} = \text{-------}$$

$$+ \frac{\partial}{\partial x}(\alpha |D| \cdot D_S) - \frac{\partial}{\partial y}(\alpha |D| \cdot D_T) \quad (2.14)$$

$$\frac{d\theta}{dt} = \text{---} + \frac{\partial}{\partial x}\left(\alpha' |D| \frac{\partial \theta}{\partial x}\right) + \frac{\partial}{\partial y}\left(\alpha' |D| \frac{\partial \theta}{\partial y}\right)$$

$$\alpha' = \frac{5}{2}\alpha \quad (2.15)$$

Note that, as will be discussed in section 3.4, these non-linear viscosity can be derived from the turbulence closure theory (Lilly, 1967).

On the other hand, if $|D|$ is constant in space, $K_M=K$ is constant. The equations are written simply as

$$\frac{du}{dt} = \text{------} + K \nabla^2 u \quad (2.16)$$

$$\frac{dv}{dt} = \text{-------} + K \nabla^2 v \quad (2.16')$$

$$\frac{d\theta}{dt} = \text{------} + \frac{5}{2} K \nabla^2 \theta$$

(2.16'')

where

$$K = \beta \cdot \Delta^{4/3} \quad (2.17)$$

and the constant β is

$$\beta = \frac{c^2}{2} \frac{\langle |D| \rangle}{\Delta^{3/2}} \quad (2.18)$$

$<|D|>$ being the ensemble mean of $|D|$ in the grid of Δ.

Recently the del-four diffusion, ∇^4 has been used; this is linear and yet the tendency of the non-linear viscosity is presumably included. So

$$\boxed{\begin{aligned} \frac{du}{dt} &= \text{------} \quad + \mu \nabla^4 u && (2.19) \\ \frac{dv}{dt} &= \text{------} \quad + \mu \nabla^4 v && (2.19') \\ \frac{d\theta}{dt} &= \text{------} \quad + \frac{5}{2} \mu \nabla^4 \theta && (2.19'') \end{aligned}}$$

where

$$\mu = \beta' \Delta^{10/3} \tag{2.20}$$

and

$$\beta' = \frac{c^2}{2} \frac{<|D|>}{\Delta^{4/3}} \tag{2.21}$$

The Kolmogoroff theory concerns the three-dimensional turblence. In the case of GCM, however, the range in which the grid discritization falls is the two-dimensional turbulence inertia domain, in which enstrophy cascade is constant with wavenumber. This implies that E (wave energy) $\propto L$ instead of $L^{5/3}$. In this connection, Leith (1969) postulated that

$$K_M = \text{const.}\ \Delta^3 \, |\nabla \zeta| \, ,$$

where ζ is the relative vorticity.

However, there are only few papers that report successful tests with this formulation (Yamagishi, 1980, const=1.25).

3. Turbulence Closure Model

3.1 Philosophy of second-order closure model

There are several principles for the architecture of an adequate GCM, particularly for the parameterization of SGS processes. The parameterization should meet certain standards for the accuracy, the simplicity, the versatility, the robustness, the computer adaptability, the historical consistency, and perhaps the elegance. A mechanistic model is not always suited for this principle, because this type of model is developed for an explanation of mechanism. The mechanistic model tends to lack in the generality; the mixed layer model is one of the examples.

The turbulence closure model has a long historical background, has mathematical preciseness, the simplicity, and has the versatility. Mellor and Yamada (1974) have clarified the hierarchial levels of the second-order closure model by making a scale analysis. The particular scheme employed in this manual is closed by Rotta's energy redistribution hypothesis and the Kolmogoroff's isotropic turbulence dissipation hypothesis.

The level 2 model is sufficiently simple, because the scheme consists of the exact same forms of eddy viscosity, which are functions of Richardson number. As was mentioned earlier, however, the diffusion operates only in the unstable region, and outside of it no diffusion is allowed to exist. As a result, the diffusivity varies abruptly in space; this is unrealistic and unfavorable. It is often the case in the GCM output that a model atmosphere is too dry above the PBL. The level 3 model consists of a number of non-steady equations of turbulent variances and covariances, which can correct the above deficiency. But this model requires a considerable computational burden, treating more equations of the grid-resolvable variables.

A compromise is to introduce a level 2.5 model, which includes the non-steady equation, Only for $\partial b^2/\partial t$, but all other equations exclude time-derivatives and diffusion terms. Consequently the scheme is considerably simplified; the turbulence transfers are all expressed by algebraic equations. The equation of $\partial \theta^2/\partial t$ is steady. As a result, a jump of θ_v at the top of PBL is not sharp, and the downward heat flux $\overline{w\theta}$ at the top of PBL is not strong. These drawbacks are the penalty one has to pay due to the simplification.

3.2 The equations of level 2.5 model

The equations of Mellor (1973) and Deardorff (1973a,b) are very similar except that, the characteristic parameters for the closure assumptions are different. Namely the turbulence scale length, ℓ , is used in Mellor (subsequently referred to M), whereas the grid size, Δ, is used in Deardorff (subsequently referred to D).

For example, the term of the pressure-velocity gradient correlations which are called "energy redistribution terms" are parameterized by Rotta's hypothesis but in different ways.

In the following, using the tensor notation u_i, u_j, u_k, two versions are shown, i.e.; first by M and second by D.

$$\overline{\frac{p}{\rho}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)} =$$

$$\begin{cases} -\dfrac{b}{3\ell_1}\left(\overline{u_i u_j}-\dfrac{1}{3}\delta_{ij}b^2\right)+C\cdot b^2\left(\dfrac{\partial u_i}{\partial x_j}+\dfrac{\partial u_j}{\partial x_i}\right), \\ -\dfrac{C_m\cdot b}{2^{1/2}\Delta}\left(\overline{u_i u_j}-\dfrac{1}{3}\delta_{ij}b^2\right)+\dfrac{1}{5}b^2\left(\dfrac{\partial u_i}{\partial x_j}+\dfrac{\partial u_j}{\partial x_i}\right) \end{cases} \qquad (3.1)$$

where p is the pressure, ℓ, is a scale length, c and c_m are constants. The viscous dissipation is expressed, based on Kolmogoroff's hypothesis, i.e.,

$$\nu \overline{\frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k}}$$

$$= \begin{cases} \dfrac{b^3}{\Lambda_1} , \\ C_E \dfrac{b^3}{2^{3/2} \Delta} \end{cases} \tag{3.2}$$

where ν is the molecular viscosity, Λ_1 is a scale length, and C_E is a constnt. The triple correlation is written as

$$\overline{u_i u_j u_k} =$$

$$\begin{cases} -\dfrac{5}{3} b \lambda_1 \left(\dfrac{\partial \overline{u_i u_j}}{\partial x_k} + \dfrac{\partial \overline{u_i u_k}}{\partial x_j} + \dfrac{\partial \overline{u_j u_k}}{\partial x_i} \right), \\ -\dfrac{5}{3} \dfrac{C_{3m}}{2^{1/2}} \Delta \cdot b \left(\quad '' \quad '' \quad '' \quad \right) \end{cases} \tag{3.3}$$

where λ_1 is a scale length and C_{3m} is a constant. Thus the equation of turbulent kinetic energy is written by M and D, respectively, as

$$\frac{d\, b^2/2}{dt} =$$

$$\begin{cases} \dfrac{\partial}{\partial z} \left(\ell\, b\, \dfrac{5}{3} \dfrac{\partial b^2/2}{\partial z} \right) \\ - \overline{vw} \dfrac{\partial V}{\partial z} + \dfrac{g}{\theta_0} \overline{w \theta_v} - \dfrac{b^3}{\Lambda_1} , \end{cases} \tag{3.4}$$

$$\left\{ \frac{\partial}{\partial z}\left(\frac{5}{3} \frac{C_{3m}}{2^{1/2}} \Delta_1 b \frac{\partial b^2/2}{\partial z} \right) - \frac{1}{2} A_{kl} D_{kl} + \frac{g}{\theta_0} \overline{w\theta_v} - \frac{C_E}{2^{3/2}} \frac{b^3}{\Delta} \right. \tag{3.4'}$$

where

$$A_{ij} = \overline{u_i u_j} - \frac{1}{3} \delta_{ij} b^2$$
$$D_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \tag{3.5}$$

The terms on the right hand side of eqs. (4) represent the diffusion, the mechanical work due to wind shear, the work done by buoyancy, and the dissipation.

The vertical boundary condition for eq. (3.4) or (3.4') is

$$b^2 = B_1^{2/3} V_*^2 \qquad at \quad z = z_0 \tag{3.6}$$

Zo and V_* being the roughness length and the friction velocity defined later in eq. (4.8), B_1=15, which will be explained in eq. (3.14).

Other equations of the second-moment of turbulence are assumed to be stationary and they are written by M and D, respectively, below.

$$\frac{\partial \overline{u_i u_j}}{\partial t} :$$

$$\left\{ \begin{aligned} 0 = {} & \frac{b}{3l_1}\left(\overline{u_i u_j} - \frac{1}{3}\delta_{ij} b^2\right) \\ & + C_1 \cdot b^2 \left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right) - \frac{2}{3}\frac{b^2}{\Lambda_1}\delta_{ij} \\ & - \overline{u_i u_k}\frac{\partial u_j}{\partial x_k} - \overline{u_j u_k}\frac{\partial u_i}{\partial x_k} - 2\frac{g}{\Theta_0}\overline{w\theta_v}, \\ 0 = {} & \frac{2}{15} b^2 D_{ij} - \frac{1}{3}\delta_{ij} \cdot A_{kl} D_{kl} + \frac{C_m}{2^{1/2}}\frac{b}{\Delta} A_{ij} \\ & + A_{ik}\frac{\partial u_j}{\partial x_k} + A_{jk}\frac{\partial u_i}{\partial x_k} \\ & - \frac{g}{\Theta_0}\left(\delta_{i3}\overline{u_j\theta} + \delta_{j3}\overline{u_i\theta} - \frac{2}{3}\delta_{ij}\overline{w\theta_v}\right) \end{aligned} \right.$$

(3.7)

(3.7')

$$\frac{\partial \overline{u_i \theta}}{\partial t} :$$

$$\left\{ \begin{aligned} 0 = {} & -\overline{u_i u_j}\frac{\partial \Theta}{\partial x_j} - \overline{u_j \theta}\frac{\partial u_i}{\partial x_j} \\ & - \frac{b}{3l_2}\overline{u_i\theta} - \frac{g}{\Theta_0}\overline{\theta\theta_v}, \\ 0 = {} & -A_{ij}\frac{\partial \Theta}{\partial x_j} - \overline{u_j\theta}\frac{\partial u_i}{\partial x_j} - \frac{1}{3} b^2 \frac{\partial \Theta}{\partial x_i} \\ & - C_s \frac{b}{2^{1/2}\Delta}\overline{u_i\theta} + \frac{2}{3}\delta_{i3}\frac{g}{\Theta_0}\overline{\theta^2} \end{aligned} \right.$$

(3.8)

(3.8')

$$\frac{\partial \theta^2}{\partial t}:$$

$$\begin{cases} 0 = -\overline{u_k\theta}\,\dfrac{\partial\theta}{\partial x_k} - \dfrac{q}{\Lambda_2}\,\overline{\theta^2} & (3.9) \\[2ex] 0 = -\overline{u_k\theta}\,\dfrac{\partial\theta}{\partial x_k} - C_\theta\,\dfrac{q}{2^{3/2}\Delta}\,\overline{\theta^2} & (3.9') \end{cases}$$

3.3 Eddy viscosity formulas

The manipulation of eq. (3.7), (3.8) and (3.9) for various i, j, k leads to algebraic equations. The results are summarized by the two equations as follows.

$$-\left(\overline{uw},\ \overline{vw}\right) = K_M\left(\frac{\partial u}{\partial z},\ \frac{\partial v}{\partial z}\right) \qquad (3.10)$$

$$-\left(\overline{\theta w},\ \overline{q w}\right) = K_H\left(\frac{\partial \theta}{\partial z},\ \frac{\partial q}{\partial z}\right) \qquad (3.11)$$

where

$$\begin{aligned} K_M = {} & l_1\Big[(1-3c)\,q^5 + 3l_2\Big\{(\Lambda_2 - 3l_2)\,q^3 \\ & \qquad - 3(4l_2 + \Lambda_2)\,c\,q^3\Big\}\frac{g}{\Theta_0}\frac{\partial\theta}{\partial z}\Big] \div \\ & \Big[q^4 + 6l_1^2 q^2\left|\frac{\partial V}{\partial z}\right|^2 + 3l_1 l_2\frac{g}{\Theta_0}\frac{\partial\theta}{\partial z}\times \\ & \quad \times\Big\{6l_1(\Lambda_1 - 3l_2)\left|\frac{\partial V}{\partial z}\right|^2 + \left(7 + \frac{\Lambda_2}{l_1}\right)q^2 \\ & \qquad + 9l_2(4l_1 + \Lambda_2)\frac{g}{\Theta_0}\frac{\partial\theta}{\partial z}\Big\}\Big] \end{aligned} \qquad (3.12)$$

$$K_H = l_2 \left[b^3 - 6 l_1 K_M \left|\frac{\partial V}{\partial z}\right|^2 \right] \div \left[b^2 + 3 l_2 (4 l_1 + \Lambda_2) \frac{g}{\theta_0} \frac{\partial \theta}{\partial z} \right] \tag{3.13}$$

The numerical values of the constants are given basd on laboratory experiments (Deardorff, 1973; Mellor, 1973) as follows

(M)

$$l_1 = A_1 \cdot l \ , \quad l_2 = A_2 \, l$$
$$\Lambda_1 = B_1 \cdot l \ , \quad \Lambda_2 = B_2 \, l$$
$$\lambda_1 = \lambda_2 = \lambda_3 = 0.23 \, l$$
$$(A_1, A_2, B_1, B_2, C) = (0.78, 0.78, 15, 8, 0.056)$$

(D)

(3.14)

$$(C_m, C_s, C_E, C_\theta) = (4.13, 4.13, 0.70, 0.58)$$
$$(C_{3m}, C_{3\theta}) = (0.2, 0.2) \tag{3.15}$$

$$\frac{1}{\theta_0} = 0.00367 \tag{3.16}$$

Mellor and Yamada (1982) reviewed the more recent data and gave:

$$(A_1, A_2, B_1, B_2, C) = (0.92, 0.74, 16.6, 10.1, 0.08)$$

(3.17)

(See also Yamada and Mellor, 1979).

3.4 Turbulent length scale

The master length scale, ℓ, has to be specified. Two versions are in current use.

Version I

This version is based on Blackadar's (1962) method. The formula combines two properties, i.e., $\ell \sim k_0 Z$ in the lowest level (k_0, Karman constant) and ℓ approaching to ℓ_0 at $z \to \infty$. Thus

$$\boxed{\ell = \frac{k_0 \cdot z}{1 + \frac{k_0 z}{\ell_0}} \qquad (3.22)}$$

where

$$\ell_0 = 0.10 \times \frac{\int_0^z b\, z\, \rho\, dz}{\int_0^z b\, \rho\, dz} \qquad (3.23)$$

This version has been used in this manual (see Miyakoda and Sirutis, 1977).

Version II

Mellor and Herring (1973) and Mellor and Yamada (1977) proposed an equation which is a variant of Rotta's (1951) equation, i.e.,

$$\frac{\partial (b^2 \ell)}{\partial t} = \frac{\partial}{\partial z}\left(b\ell \quad S_\ell \frac{\partial}{\partial z}(b^2 \ell) \right)$$

$$- \ell\, E_1 \left[-\overline{uw}\frac{\partial u}{\partial z} - \overline{vw}\frac{\partial v}{\partial z} + \frac{g}{\theta_0}\overline{w\theta_v} \right]$$

$$- \frac{b^3}{B_1}\left\{ 1 + E_2 \left(\frac{\ell}{k_0 z}\right)^2 \right\} \qquad (3.24)$$

S_ℓ is a stability function related to λ_1 in the equation of b^2, and E_1 and E_2 are empirical constants. Z is the distance from the earth's surface. Mellor and Yamada (1977) uses: (E_1, E_2, S_ℓ)=(1.8, 1.33, 0.2).

Within the whole framework of the turbulence closure model, the determination of this turbulent length scale is a weakest area. However, it is true that the final results are rather insensitive to the choice of ℓ. Readers refer to Mellor and Yamada (1982).

Yokoyama et al. (1979) collected various observational values of the turbulence scale (Table 1). Figure 3.1 shows these values (dashed lines and solid lines). At Z=10^3m, the length scale ℓ falls in 100$\sim$400 m. Yokoyama et al. compared these data with the values of a formula Fig. 3.1

$$\ell = z\left(1 + \beta\frac{z}{h}\right)^{-1}, \tag{3.25}$$

where h is the depth of PBL. The formula's values of β=0, 2 and 4 are plotted by the solid lines. It appears, β=2 $\sim$ 4 fit the observations.

Figures 3.2 $\sim$ 3.5 are the results of a GCM based on E-physics. All variables are for 00GMT, 15 March, 1965, which is the 10th day in the predictions.

Figure 3.2 is the latitude-height distribution of the zonally averaged turbulent kinetic energy. It is pronounced that two or three maxima are included in the vertical. One is the lowest layer, and the second is below the jet stream level in the middle latitudes and is at the tropopause level in the equatorial region. It would be worthy to note that turbulent energy Fig. 3.2 3.3 3.4 3.5

Table 1. Key to the scales of turbulence given in Fig. 1 and additional information.

No.	Authors	Additional information
3	Lettau (1950)	Neutral condition
4	Clarke (1970)	Clarke's class III (near neutral)
5	Yokoyama (1971)	$\lvert z/L \rvert \leq 1$ (near neutral)
6	Gamo and Yokoyama (1971)	Strong wind, by tethered balloon
7	Gamo et al.	Forced convection (KAW-MAR-1972)
8	Busch and Panofsky (1968)	Unstable
9	Kukharets and Tsvang (1969)	Summer, developed convection
10	Clarke (1970)	Clarke's class I (deep convection)
11	Clarke (1970)	Clarke's class II (shallow convection)
12	Yokoyama (1971)	$z/L < -1$ (unstable)
13	Warner (1972)	Convective situations (above the sea)
14	Gamo et al.	Free convection (KAW-MAR-1972)

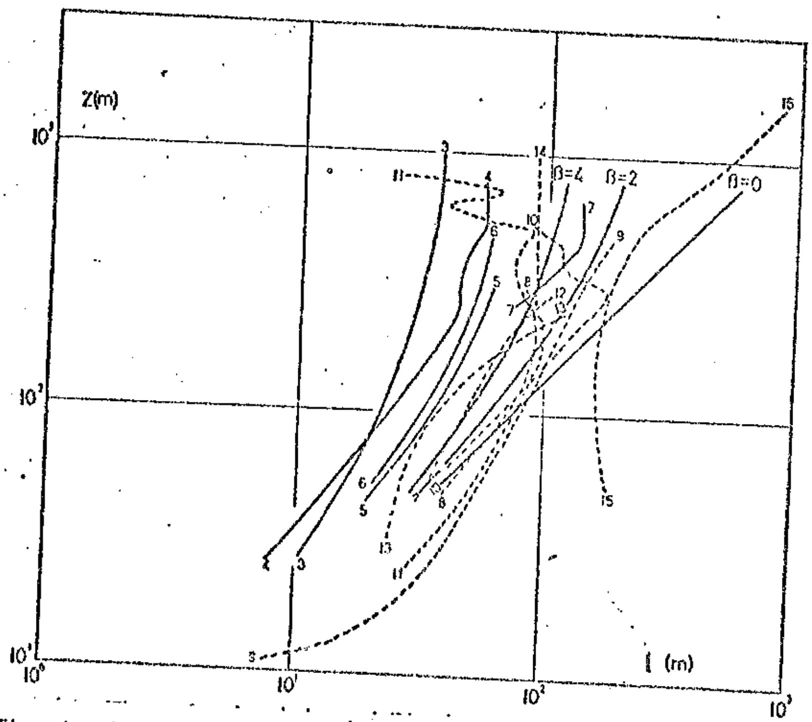

Fig. 3.1 The turbulence scale ℓ. Numbers indicate the papers cited in Table 1. Solid lines are for neutral condition, and dashed lines for unstable conditions, and the solid lines of $\beta = 0$, 2 and 4 are based on the formula (3.32) (after Yokoyama et al. 1979).

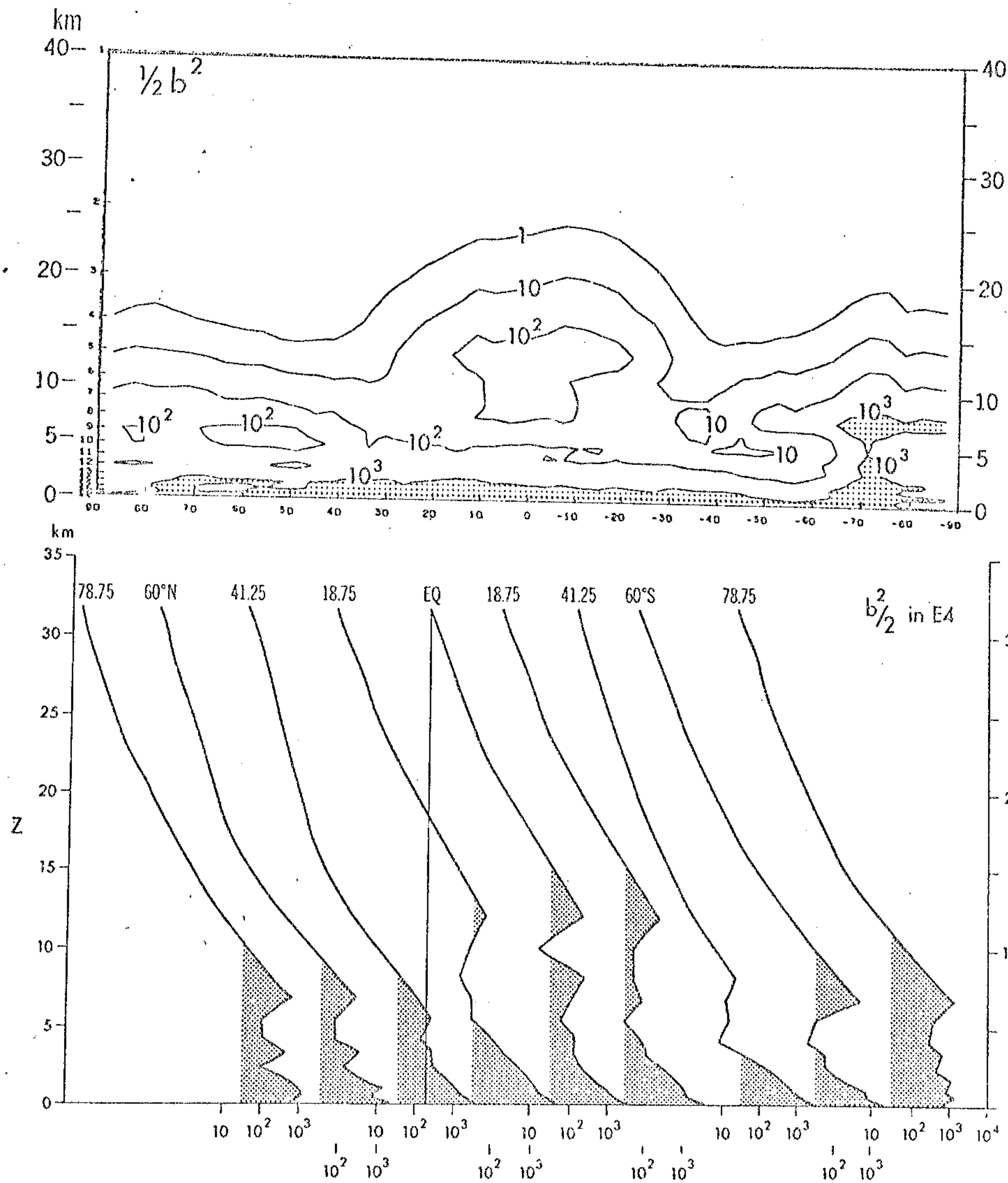

Fig. 3.2 Turbulent kinetic energy, 1/2 b^2, in the latitude-height section (upper) and the vertical distributions of 1/2 b^2 at selected latitudes (lower) in units of cm^2s^{-2}. In the upper panel, the region in which 1/2 $b^2>10^3$ is shaded, and in the lower panel, the layers in which 1/2 $b^2>2 \times 10^2$ are shaded.

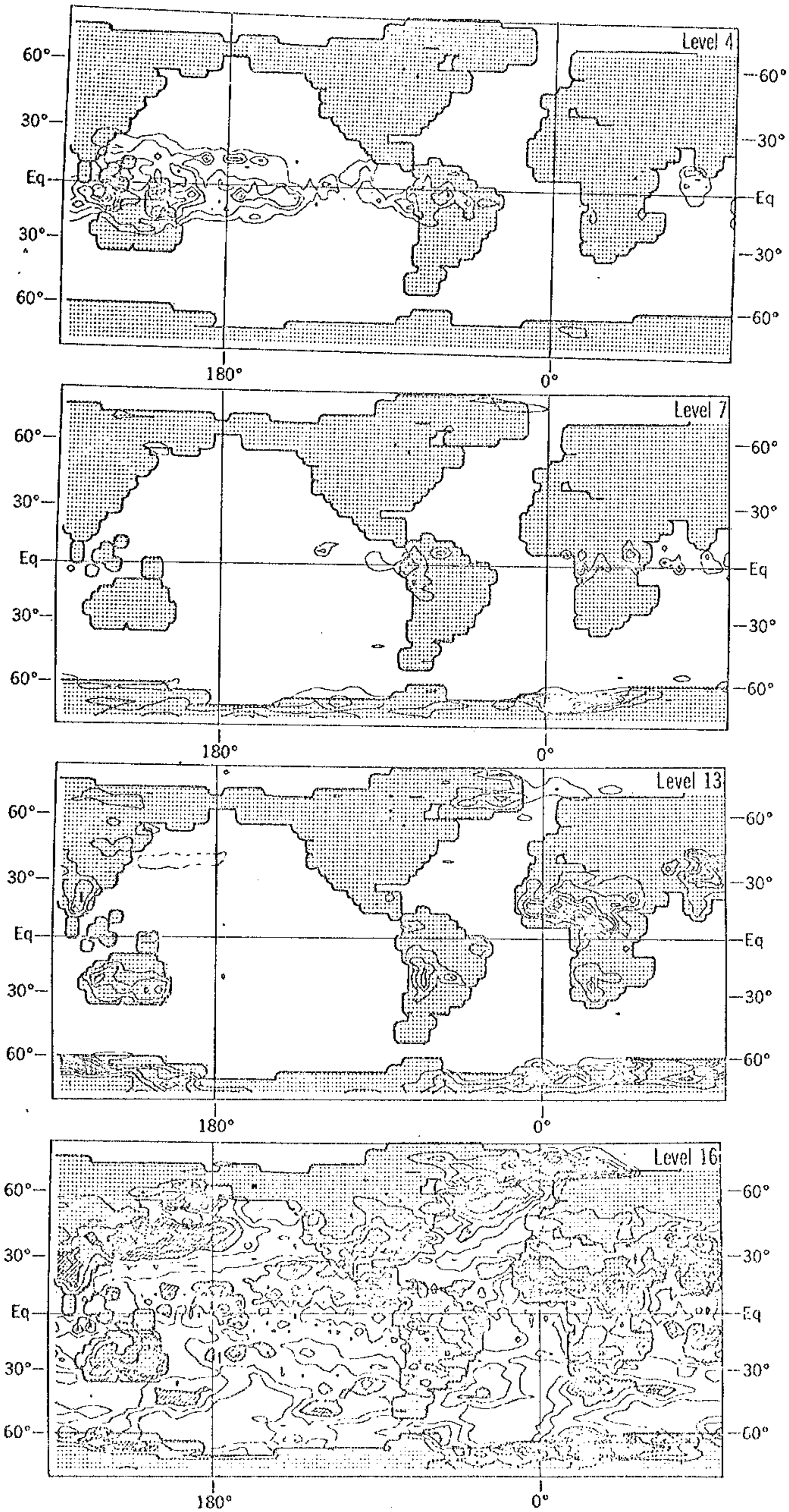

Fig. 3.3 Horizontal distribution of 1/2 b^2 at model's level 4 (99 mb), level 7 (297 mb), level 13 (777 mb) and level 16 (948 mb).

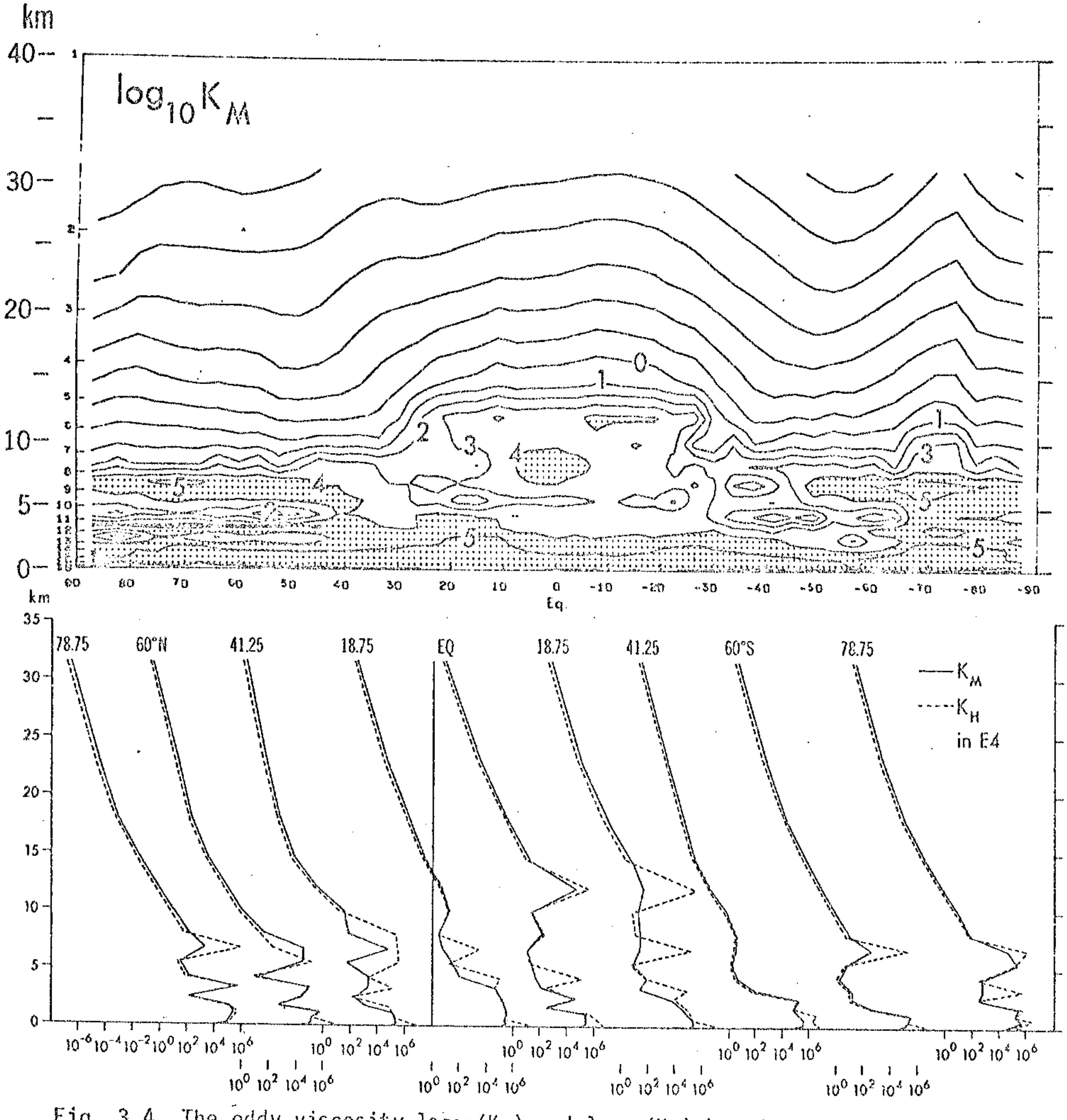

Fig. 3.4 The eddy viscosity $\log_{10}(K_M)$ and $\log_{10}(K_H)$ based on (3.12) and (3.13), respectively, which are revealed in the run of E-physics model. The upper panel is the latitude-height distribution of $\log_{10}(K_M)$. The contour interval is 10 in $\log_{10}(K_H)$, where K_M is in units of c.g.s. The area in which K_M is larger than 10^4 are shaded. The lower panel is the vertical profiles of $\log_{10}(K_M)$ and $\log_{10}(K_H)$ at selected latitudes (after Miyakoda and Sirutis, 1977).

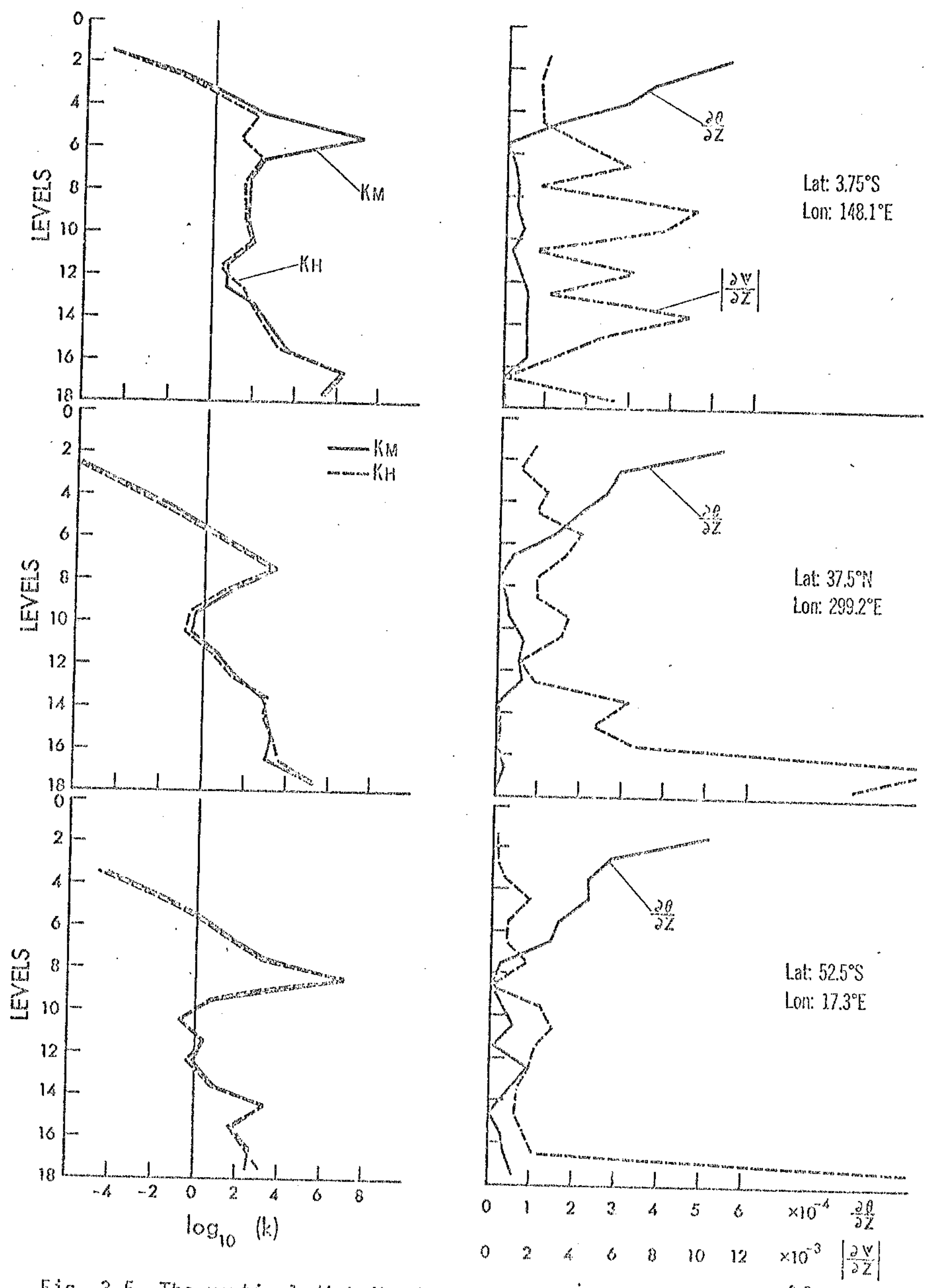

Fig. 3.5 The vertical distributions of K_M and K_H (left) and of $\frac{\partial\theta}{\partial z}$ and $\left|\frac{\partial V}{\partial z}\right|$ (right) at three geographical locations in units of c.g.s.

is found even above PBL. Figure 3.3. is the horizontal distribution at various levels. Level 16 corresponds to the upper portion of PBL. Turbulent energies are larger over continents, western boundary of the oceans, i.e., over Kuroshio or Gulf Stream, ITCZ and the regions of stratocumulus-topped PBL. Turbulence at level 13 is dominantly large over continents, particularly the Sahara desert, Himalaya, and continents of the Southern Hemisphere (summer). At level 4, turbulence is large over the cumulous convection area. The Indonesian Archipelago and Amazon basin are outstanding.

Figure 3.4 is the eddy viscosities, K_M and K_H, which are based on eqs. (3.12) and (3.13). It is interesting to note that K_H is larger than K_M in the troposphere (coinciding with eq. (2.11')). In order to see which factors contribute most to the large values of K_M and K_H, the comparative display is made with the thermal stratification in terms of $\partial\theta/\partial z$ and the vertical shear $|\partial V/\partial z|$ in figure 3.5. In general, when $\partial\theta/\partial z \to 0$, K_M and K_H become large except at the top panel. However, the association of turbulent kinetic energy with $\partial\theta/\partial z$ and $|\partial V/\partial z|$ is not simple, indicating that the effect of the level 2.5 model over that of level 2 model is appreciable.

3.5 Comment on the lateral viscosity

The formulation of non-linear lateral viscosity (2.12) is derived from the turbulence closure model of hierarchy level 1 or even simpler (Lilly, 1967; Deardorff, 1971, 1973a,b).

$\frac{\partial b^2}{\partial t}$ (from (3.4'));

$$0 = -\frac{1}{2} A_{ij} D_{ij} + \cancel{\frac{g}{\theta_0} w\theta} - \frac{C_E}{2^{3/2}} \frac{b^3}{\Delta} \qquad (3.26)$$

$$\frac{\partial A_{ij}}{\partial t} \left(\text{from } (3.7') \right):$$

$$0 = -\frac{2}{15} b^2 D_{ij} - \frac{C_m}{2^{1/2}} \frac{b}{\Delta} A_{ij} \tag{3.27}$$

$$\frac{\partial \overline{u_i \theta}}{\partial t} \left(\text{from } (3.8') \right):$$

$$0 = -\frac{1}{3} b^2 \frac{\partial \theta}{\partial x_i} - C_s \frac{b}{2^{1/2}\Delta} \overline{u_i \theta} + \cancel{\frac{2}{3} \delta_{ij} \frac{g}{\theta_0} \overline{\theta^2}} \tag{3.28}$$

The problem of the lateral viscosity is not related to the buoyancy, so the term with slash is not used. Combination of the terms of mechanical conversion and dissipation in (3.26), and (3.27) leads to

$$b^2 = \frac{4}{15} \frac{\Delta^2}{C_E \cdot C_m} D^2 \tag{3.29}$$

Next insertion of (3.29) into (3.27) gives

$$A_{ij} = -\left\{ \frac{\frac{4}{15}\left(\frac{2}{15}\right)^{1/2}}{C_E^{1/2}\, C_m^{3/2}} \Delta^2 \cdot D \right\} D_{ij} \tag{3.30}$$

Likewise, insertion of (3.29) into (3.28) gives

$$\overline{u_i \theta} = -\left\{ \frac{\frac{2}{3}\left(\frac{2}{15}\right)^{1/2}}{C_s \cdot C_E^{1/2}\, C_m^{1/2}} \cdot \Delta^2 \cdot D \right\} \frac{\partial \theta}{\partial x_i} \tag{3.31}$$

C in (2.12) is, therefore, written by

$$\boxed{C = \left(\frac{4}{15}\right)^{3/4}\left(C_E \cdot C_m^{\,3}\right)^{-1/4} \qquad (3.32)}$$

and $C_S = C_m$.

Thus

$$A_{ij} = -\left\{\frac{(C\cdot\Delta)^2}{\sqrt{2}} D\right\} D \tag{3.33}$$

$$\overline{u_i\theta} = -\frac{5}{2}\left\{\frac{(C\cdot\Delta)^2}{\sqrt{2}} D\right\}\frac{\partial\theta}{\partial x_i} \tag{3.34}$$

Assuming, as in (3.15), that C_m=4.13, C_E=0.70, C_θ=0.58, C is given by

$$C = 0.14\,, \tag{3.35}$$

where 5/2 in (3.34) was derived by Schemm and Lipps (1976), though this point is controversial.

4. Surface Layer

Near the earth's surface, the molecular viscosity plays a major role in vertical transfer of heat, moisture and momentum. The turbulence develops only above a certain level, which is defined as the roughness height, Z_0. Once the turbulence starts, it is efficient in transferring heat, etc.

Yet, because of the proximity of heat source, the temperature difference can remain quite large. (For example, over sand desert, $25° \sim 50°C$ temperature difference is often found within $1 \sim 2$ meters), and the thermal instability can continue to exist in a shallow layer (Figure 4.1).

The overall situation in this layer is that the vertical turbulent transfer is very strong and that there is no other balancing effect. For this reason, the vertical fluxes can be approximated to be constant with height.

This layer is technically called as "the constant-flux layer," which normally extends from the level of Z_0 to about 20 meter height. Because of the assumption of constant-flux, simple formulas for eddy transfers can be obtained.

4.1 Monin-Obukhov theory

As mentioned earlier, because of computational economy, the model's layer between the surface and the lowest level is treated jointly as the surface layer, in which the molecular as well as the turbulent fluxes are combined, and the bulk transfer scheme based on the similarity theory is used instead of the turbulent closure scheme. The shortcoming of this arrangement is that only the vertical transfer is considered and other effects such as lateral advection, baroclinicity and the effect of pressure gradient in space are all ignored.

First let us re-denote

$$\tau_{xz} \longrightarrow \tau_x$$

$$\tau_{yz} \longrightarrow \tau_y$$

$$\tau_{\theta z} \longrightarrow -H/c_p$$

$$\tau_{qz} \longrightarrow -E$$

Thus the equations (2.1) - (2.4) are written

$$\frac{du}{dt} + \cdots\cdots = \frac{1}{\rho}\frac{\partial \tau_x}{\partial z} \tag{4.1}$$

$$\frac{dv}{dt} + \cdots\cdots = \frac{1}{\rho}\frac{\partial \tau_y}{\partial z} \tag{4.2}$$

$$\frac{d\theta}{dt} + \cdots\cdots = -\frac{1}{\rho}\frac{\partial H/c_p}{\partial z} \tag{4.3}$$

$$\frac{dq}{dt} + \cdots\cdots = -\frac{1}{\rho}\frac{\partial E}{\partial z} \tag{4.4}$$

where H, and E>0 if transfers are directed upward, and τ_x and τ_y>0 if downward. τ_x, τ_y, H and E at the surface layer are denoted by the suffix zero, i.e., τ_{ox}, τ_{oy}, H_0 and E_0.

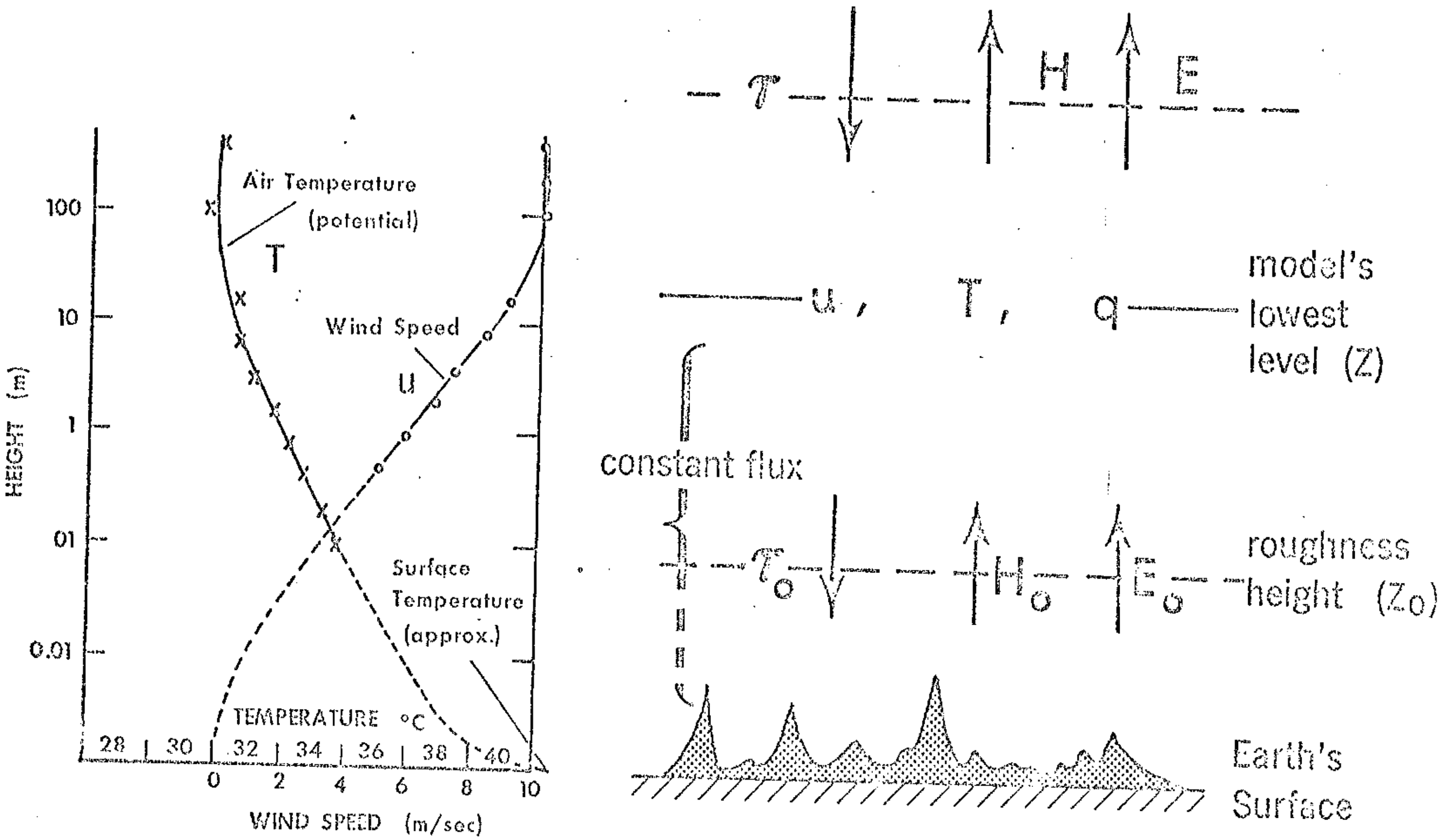

Fig. 4.1 The constant-flux layer. The vertical distributions of wind speed and potential temperature (left) (after Deacon), and the model's levels, wind stress, sensible and latent heat fluxes (right).

On the dimensional base, one can define a parameter, V_*, i.e., the friction velocity, which is constant in Z.

$$\tau_o / \rho = V_*^2 \tag{4.5}$$

where