Comments on "On the Utility and Disutility of JEBAR"

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The subject paper by Cane et al. (1998) deals with the JEBAR term (Sarkisyan and Ivanov 1971) in the linear, vertically integrated vorticity equation.

$$J(\psi,\ f/H) = \mathrm{curl}(\boldsymbol{\tau}/H) + \mathrm{JEBAR}; \qquad (1a)$$

$$\mathrm{JEBAR} \equiv J\left[\int_{-H}^{0} gz(\rho/\rho_o)\,dz,\ 1/H\right], \qquad (1b)$$

another form of which is

$$\beta\psi_x = \mathrm{curl}\boldsymbol{\tau} + \mathrm{BT}; \qquad (2a)$$

$$\mathrm{BT} \equiv J(p_b,\ H), \qquad (2b)$$

where J is the Jacobian operator, ψ is the transport streamfunction, f and H are the Coriolis parameter and bottom topography, $\beta \equiv \partial f/\partial y$, $\boldsymbol{\tau}$ is the kinematic surface wind stress vector, and p_b is the bottom kinematic pressure. Equation (2) is the more physically appealing form containing the intuitive bottom torque term BT, which, however, renders it unsuitable for direct solution since p_b contains the unknown surface elevation. Equations (1a) and (2a) reduce to the Sverdrup balance equation for a flat bottom, whence JEBAR = BT = 0.

Cane et al. state: "As a rule, we expect the flow to try to behave like Taylor columns, arranging itself to go around hills and valleys, avoiding vortex tube shrinking or stretching. Thus, we expect isolines of p_b and H to be nearly parallel. If so, we expect the Sverdrup relation [Eq. (2) with BT = 0] holds." Cane et al. cite the solution to Eq. (1) by Mellor et al. (1982) and Greatbatch et al. (1991) and the implication is that their solutions may be greatly in error since they differ so much from the solution of the Sverdrup relation; therefore, I am moved to disagree.

To illustrate and evaluate the possibility of error due to density and topographical measurement errors in solving Eq. (1), Cane et al. invoked a reduced-gravity model for which the Sverdrup solution is correct. However, this is a strange choice in the present context since JEBAR is small relative to, say, the Atlantc Ocean and BT is nil; the model can only be forced by surface wind stress, and, unlike the real ocean, the surface density (or buoyancy) flux is nil. In this case, the small JEBAR term is a function of the wind stress curl as is the explicit wind stress curl term in (1a).

Mellor et al. solved Eq. (1) [actually an analytically equivalent but numerically more robust form of (1), which was then numerically integrated along f/H characteristics] using observed climatogical winds (Hellerman and Rosenstein 1983) and density fields (Levitus 1982). I estimate (Mellor et al. and Cane et al. provide only enough information for an estimate) that the Mellor et al. JEBAR is one to two orders of magnitude larger than that produced by the reduced gravity model of Cane et al.

But the real test is observations for which it is useful to repeat two graphs excerpted for the North Atlantic from Mellor et al. Figure 1 is the wind-driven Sverdrup solution, (2a) with BT = 0, and is qualitatively similar

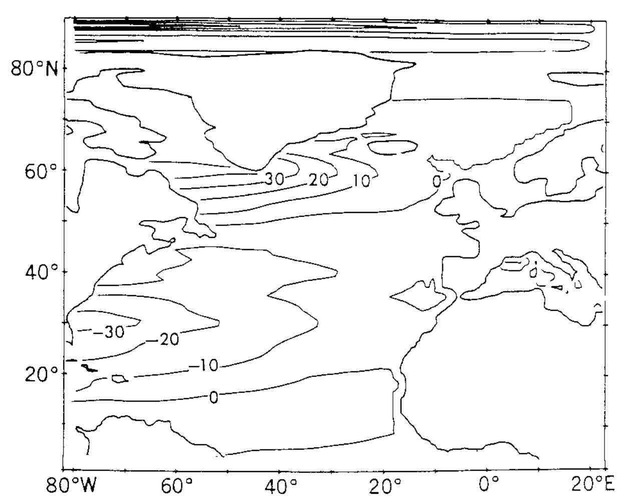

FIG. 1. A solution to the Sverdrup balance equation, that depends only on surface wind stress. Excerpted from Fig. 10 of Mellor et al. (1982).

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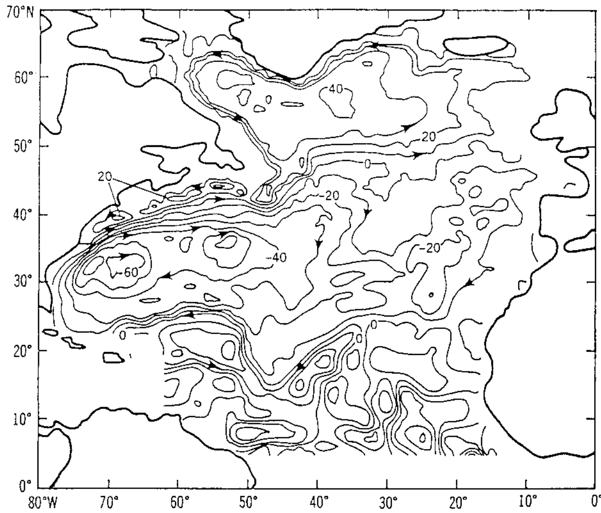

FIG. 2. A solution of the linear vertically integrated vorticity equation that depends on surface wind stress, the ocean density field, and bottom topography. Excerpted from Fig. 11 of Mellor et al. (1982).

to the Cane et al. idealized solution (see also Leetma and Bunker 1978). Figure 2 is the Eq. (1a,b) solution (see Mellor et al. for a discussion of errors and smoothing for data and solutions; smoothing of the calculations north of 30°N was insignificant), which also accounts for density variations and bottom topograpy through the JEBAR term. Which solution is closer to reality? Observations (Knauss 1969; Richardson 1985; Hogg et al. 1986; Johns et al. 1995; see these papers for additional references) and, as proxy circulation data, any satellite image strongly favor the Eq. (1) solution. The magnitude of the Gulf Stream transport has long ago been known to greatly exceed the Sverdrup transport and the recirculation gyre north of the Gulf Stream in Fig. 2, a feature verified by Richardson, Hogg et al., Johns et al., and others is now known to be important to Gulf Stream separation and incorporates the deep western boundary current. Ezer and Mellor (1994) obtained a solution with a primitive equation model which was similar to the Figure (2) solution.

The Sverdrup solution is part of every oceanographer's education, but only the first part; it is surely not realistic. Of the two, analytically equivalent forms, one might conclude that (1a,b) is useful if one wishes to obtain diagnostic solutions to the general circulation problem whereas (2a,b) may be useful as an aid in understanding known solutions.

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