JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 86, NO. C3, PAGES 1903–1916, MARCH 20, 1981

Response of Equatorial Oceans to Periodic Forcing

S. G. H. PHILANDER AND R. C. PACANOWSKI

Geophysical Fluid Dynamics Laboratory/NOAA, Princeton University, Princeton, New Jersey 08540

Oscillating wind with a period P induce variability with the following characteristics in the upper few hundred meters of the equatorial zone (5°N to 5°S) of the ocean. (1) $P < 10$ days: these winds fluctuate too rapidly to generate strong currents and excite primarily waves. (2) 10 days $< P <$ 50 days: At these periods the winds generate intense equatorial jets in the upper 50 m, but at greater depths the variability has a small amplitude. Nonlinear eastward jets are more intense, are narrower, and are deeper than the corresponding westward jets so that winds with a zero mean value give rise to a mean eastward surface current. If the wind is always westward, then its fluctuating component intensifies the eastward equatorial undercurrent maintained by the mean winds. The surface flow is eastward and convergent when winds that are always westward go through a weak phase. (3) 50 days $< P <$ 150 days: An eastward pressure force exists sufficiently long to generate an intense eastward equatorial undercurrent. Variability has a large amplitude in the surface layers and in the thermocline. Eastward phase propagation associated with Kelvin waves is prominent in the upper ocean because the nonlinear currents impede the Rossby waves. (4) $P >$ 150 days: The amplitude of variability is almost independent of frequency. An equilibrium response which is in phase with the forcing and which corresponds to a succession of steady states is approached asymtotically. These time scales are for a basin 5000 km wide. If the width of the basin exceeds 5000 km, then the 150 daytime scale increases. In the deep ocean below the thermocline, motion corresponds to propagating waves generated by the divergence of the nonlinear currents in the upper ocean.

1. INTRODUCTION

Variability in the equatorial oceans is primarily atmospherically forced. (Instabilities of the mean currents seem to be important at a period of 1 month only and do not contribute significantly to variability at other periods [*Philander,* 1980]). Up to now, studies of motion induced by the atmosphere have focused on the oceanic response to sudden changes in the intensity of the winds. Figure 1 for example shows the evolution of equilibrium conditions along the equator after the sudden onset of westward winds in a nonlinear numerical model. From studies such as this one it emerges that the following time scales are important in the adjustment of the upper ocean (above a depth of a few hundred meters) in the equatorial zone (5°N to 5°S).

One week. The sudden onset of zonal winds at first causes the surface layers of the ocean to accelerate in the direction of the wind. Motion is most intense near the equator where a jet with a half width of 200 km forms after 1 week [*Yoshida,* 1959]. This length scale is the radius of deformation λ; the time scale is the inertial period at a distance λ from the equator.

One month. The next phase in the adjustment of the ocean is associated with wave fronts, excited initially at the coasts, that propagate across the basin. In the tropics the vertical structure of the waves principally responsible for the adjustment of the upper ocean corresponds to a mode trapped in and above the strong shallow thermocline because of internal reflection there [*Philander and Pacanowski,* 1980]. This mode has an equivalent depth of approximately 25 cm. Near the equator, the Kelvin wave front with this vertical structure propagates most rapidly across the basin and establishes, within 1 month, basin-wide zonal density gradients. The associated pressure forces generate an undercurrent, with a direction opposite to that of the wind, in the equatorial thermocline. (Figure 1 shows the generation of an eastward equatorial undercurrent in the wake of a Kelvin wave front.)

150 days. The adjustment of the equatorial zone is essentially complete after the eastward propagation of the Kelvin wave and the westward propagation of the gravest equatorially trapped Rossby wave front across the basin—after about 150 days in the case of a 5000 km wide basin [*Cane,* 1979].

The results described above suggest the following response to oscillatory winds with a period P. We shall associate a period P with a time scale $P/2$ approximately. The dispersion diagram from which we infer which waves can be excited at different periods is the one that corresponds to an equivalent depth $h \sim 25$ cm. This is the equivalent depth of the earlier mentioned thermocline-trapped mode primarily responsible for the adjustment of the upper ocean.

1. $P <$ 10 days. The fluctuations are too rapid for equatorial jets to be generated. The divergence of the wind-driven motion in the surface layers excites waves, including equatorially trapped inertia-gravity waves.

2. 10 days $< P <$ 50 days. In this frequency range the winds generate intense equatorial jets in the surface layers of the ocean. Superimposed on the jets are propagating waves, primarily Kelvin waves.

3. 50 days $< P <$ 300 days. At these periods, basin-wide pressure forces could persist for a sufficient length of time to generate intense undercurrents. Hence, variability will have a large amplitude, at least to the depth of the thermocline. Westward phase propagation may be evident because Rossby waves are excited (in addition to Kelvin waves).

4. $P <$ 300 days. If the period of the forcing exceeds the adjustment time of the (5000 km wide) ocean, then the response is an equilibrium one. The wind changes so gradually that the ocean is always in an adjusted state and at each moment is in equilibrium with the wind at that moment. There is no explicit wave propagation, and the response is in phase with the winds. An example of such an equilibrium response is the seasonal change in the slope of the equatorial thermocline in the western Atlantic Ocean; it varies in phase with the seasonally changing wind stress. [*Katz et al.,* 1977].

This discussion of motion induced by periodic forcing is

Paper number 80C1234.

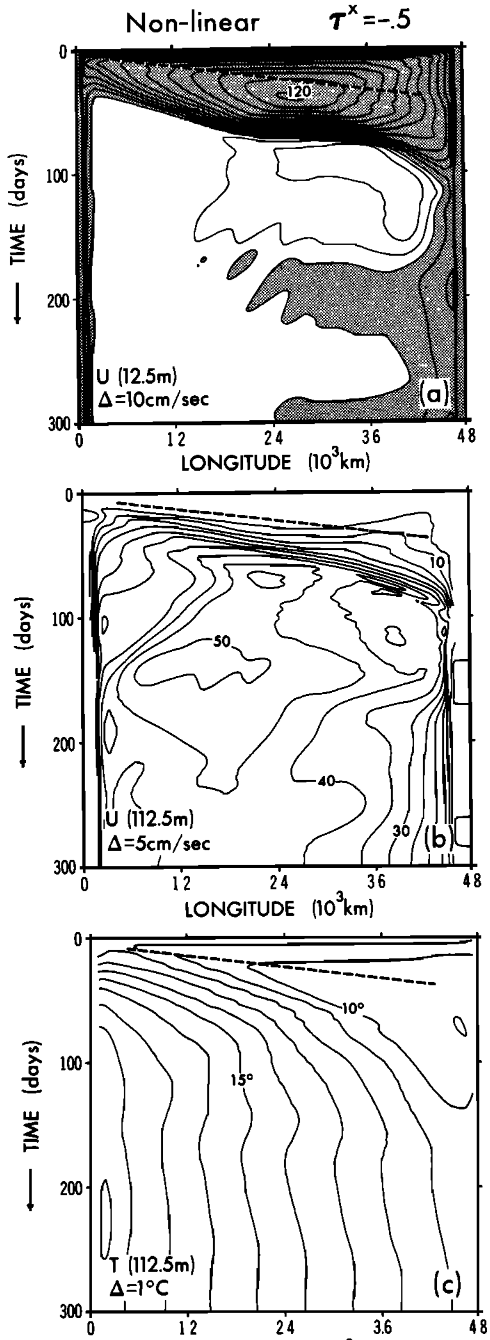

Fig. 1. Evolution of the motion along the equator in response to the sudden onset of westward winds of intensity 0.5 dynes/cm^2 in the numerical model described in section 3. The dashed line corresponds to the equatorially trapped Kelvin wave front, with an equivalent depth of 25 cm, excited initially at the western coast. Motion is westward in shaded areas.

based on results for a sudden intensification of the wind, but we have disregarded an important difference between the response to an abrupt intensification of the wind and the response to oscillatory winds. Whereas periodic forcing will excite continuous wave trains, the sudden onset of winds will generate discontinuous wave fronts with a longitude (x) and time (t) dependence of the form $f(x - ct)$ where f is a step function. Winds that intensify suddenly excite primarily the Kelvin and gravest equatorially trapped Rossby wave fronts. These propagate so rapidly that they are hardly affected by the mean currents [*Philander*, 1979*b*; *McPhadden and Knox*, 1979]. In the case of periodic winds, on the other hand, the importance of the gravest equatorially trapped Rossby wave depends on the period of the forcing: At a period less than 30 days, no Rossby waves are excited; at periods between 1 and 2 months, the gravest equatorially trapped Rossby wave is the only one that can be excited, and as the period increases, more and more Rossby modes are possible. With increasing period, the importance of the gravest equatorially trapped mode decreases while that of the higher latitudinal modes increases. The latter waves travel so slowly that mean currents are likely to affect them seriously. This possibility casts doubt on our earlier description of the response to forcing at periods longer than 50 days when Rossby waves are important.

In this paper we describe the response of a nonlinear, stratified numerical model to periodic forcing. One of our principal results is that mean currents do indeed impede Rossby wave trains significantly. This result is best appreciated by contrasting the solutions given by the nonlinear stratified model, with the solutions given by the shallow water equations which are incapable of simulating mean currents. Section 2 of this paper therefore concerns the shallow water equations. Section 3 is a description of the nonlinear stratified model. Sections 4 and 5 describe the response to oscillatory winds with a zero mean value and a mean value of -0.5 dynes/cm^2, respectively. The effect of spatially varying winds is explored in section 6.

Currents in the upper ocean are either directly wind driven or are due to large-scale pressure gradients maintained by the wind. The deep ocean, below the thermocline, is not directly affected by the wind but is indirectly affected when the divergence of the nonlinear currents in the upper ocean causes the downward radiation of waves into the deep ocean. The deep equatorial currents are discussed in section 7. Section 8 summarizes the principal results of this paper.

2. SOLUTIONS FOR THE SHALLOW WATER EQUATIONS

Consider the response of a model described by the linear shallow water equations when the oscillatory zonal wind stress τ^x acts as a body force:

$$\tau^x = \tau_0 \sin (2\pi t/P) \quad (1)$$

The (equivalent) depth of the model is taken to be 25 cm which is appropriate for the modes primarily responsible for the adjustment of the upper ocean. We obtained solutions numerically, for a basin 5000 km wide for a number of different values of P in (1). Figure 2 shows solutions for the cases P = 25, 100, 200, and 400 days. (Initially, there is no motion. Integration then proceeds for 10 cycles. Figure 2 shows the tenth cycle. The coefficient of lateral viscosity in this model is 2×10^7 cm^2/s). *Kindle* [1979] describes similar solutions for winds with spatial variations.

The response consists of two parts: directly wind-driven currents (which is the forced part of the solution) plus free waves excited at the coasts to satisfy boundary conditions there. We confine our attention to periods sufficiently long for inertia-gravity waves to be unimportant.

At periods less than 50 days, Kelvin waves excited at the western coast are far more important than Rossby waves excited at the eastern coast. The zonal velocity component can therefore be written

$$u = U_1(y) \sin (2\pi t/P) + U_2(y) \sin (kx - 2\pi t/P) \quad (2)$$

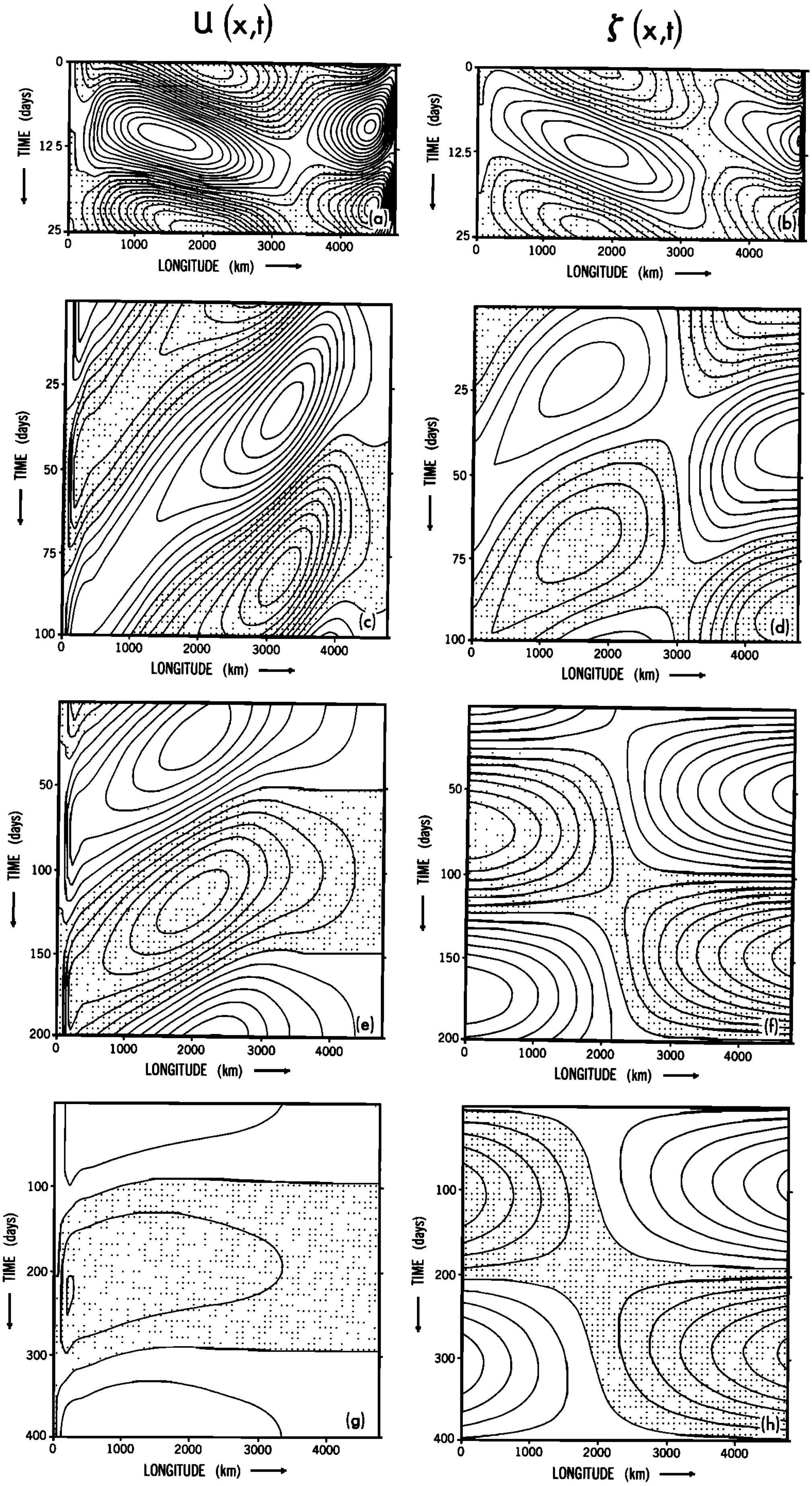

Fig. 2. Zonal velocity component (2*a*, 2*c*, 2*e*, 2*g*) and sea level (2*b*, 2*d*, 2*g*, 2*h*) along the equator as given by the shallow water equations for oscillatory forcing at periods of 25, 100, 200, and 400 days. Shaded regions denote westward flow or sea level below the mean value of 25 cm. Contours are at intervals of $\tau^x/(h^2\beta C)^{1/2}$ for u and $\tau^x/\beta C^2)^{1/2}$ for ζ where $c = (gh)^{1/2}$.

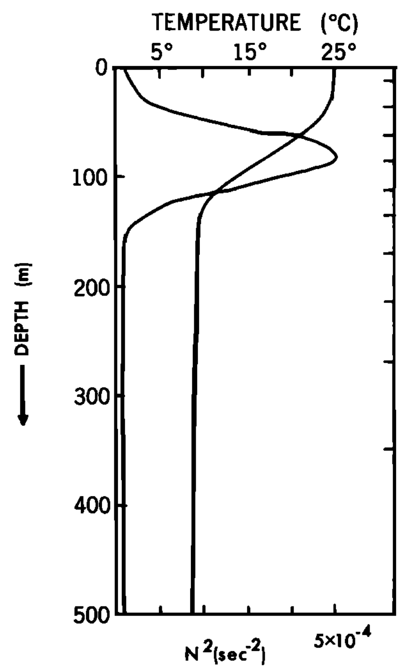

Fig. 3. Initial temperature distribution and associated Brunt-Vaisala frequency in the upper 500 m of the model. The positions of grid points are shown along the ordinate. Below 500 m the temperature decreases linearly to 4°C at a depth of 3000 m.

where U_1 is the intensity of the wind-driven current and U_2 is the amplitude of the Kelvin wave which propagates with speed $2\pi/Pk = (gh)^{1/2} = 160$ cm/s. Near the equator ($y = 0$)

$$u \sim 2U_1 \sin (kx/2) \cos (2\pi t/P - kx/2) \qquad (3)$$

provided U_1 is equal to U_2. According to this solution phase, propagation is at twice the speed of Kelvin waves, and there is a node at a distance equal to the Kelvin wavelength from the western boundary. The results in Figures 2*a* and 2*b* are in accord with this solution: Eastward phase propagation is at about 75 cm/s, and the approximate nodes are at the predicted longitudes.

At periods much longer than 50 days, eastward phase propagation is no longer evident even though Kelvin waves are excited. This happens because the time it takes a Kelvin wave to propagate across the basin is less than 50 days. Hence, on time scales long compared to 50 days, eastward phase propagation is difficult to discern. Instead, westward phase propagation associated with Rossby waves from the eastern coast is evident at periods of 100 and 200 days in Figures 2*c*–2*f*.

At periods greater than 300 days (see Figures 2*g* and 2*h*, for example) explicit wave propagation disappears because the response is an equilibrium one. At these periods, which exceed the adjustment time of the equatorial oceans, the wind stress and zonal pressure force are always in balance close to the equator:

$$g\zeta_x = \tau^x/h \qquad (4)$$

(Here, ζ is the sea-level and g is the gravitational acceleration.) Velocities are negligibly small and are a quarter period out of phase with the wind.

The approach to an equilibrium response as the period P of the forcing increases from 100 to 300 days is intriguing. We find that at a period of 200 days, for example (Figures 2*e* and 2*f*), the eastern part of the basin has an equilibrium response, but the western part does not. With increasing P, the adjusted region expands westward according to the approximate equation

$$\bar{x} = (8P - 0.12\ L) \text{ km} \qquad (5)$$

where $\bar{x}$ measures the size of the region with an equilibrium response: It is the distance between the eastern coast and the point where westward phase propagation starts. In (5), P is measured in days, and L is the width of the basin in kilometers. Equation (5) is valid only on time scales longer than the time T it takes the Kelvin wave to propagate across the basin: $P \gtrsim 2T$. This is consistent with the result that after a sudden

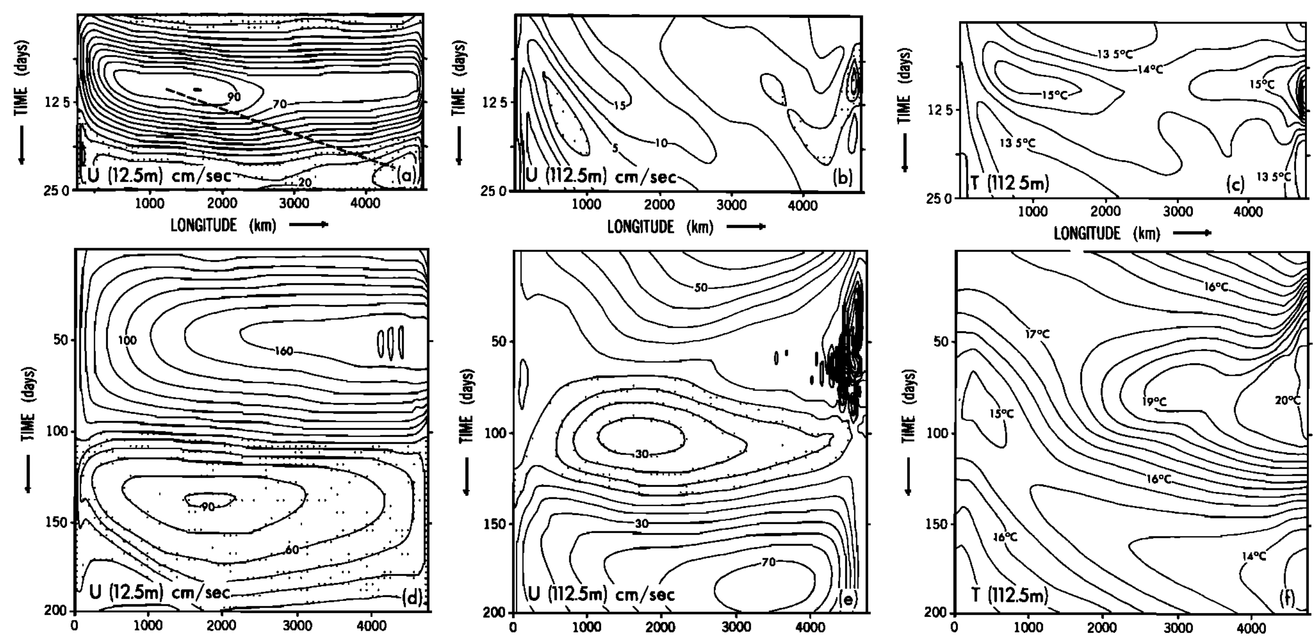

Fig. 4. Motion along the equator during one cycle in response to the winds $\tau^x = 0.5 \sin (2\pi t/P)$ dynes/cm^2 where the value of P is 25 or 200 days. Shaded areas indicate westward motion.

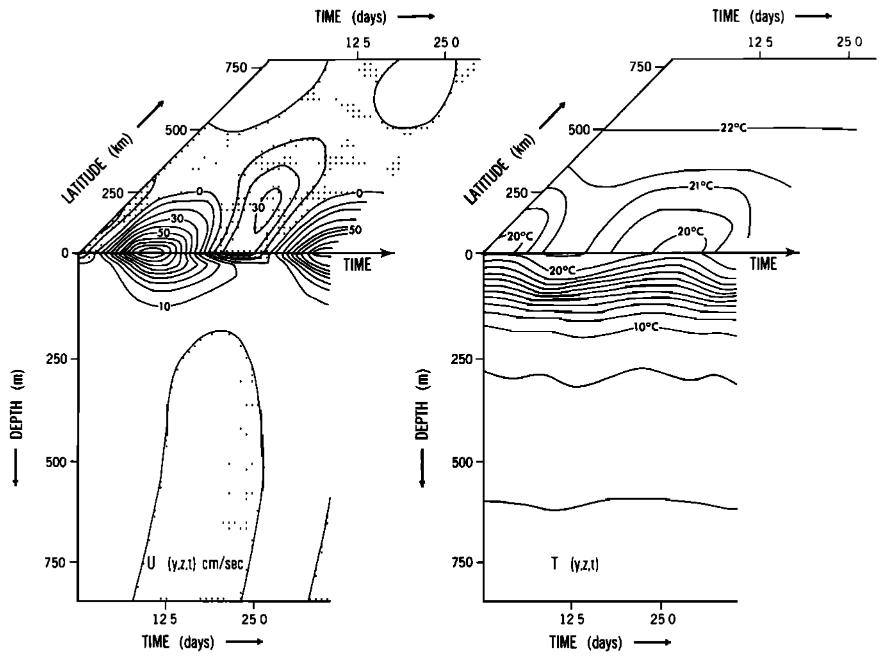

Fig. 5. Zonal velocity component and the temperature field as a function of time, latitude, and depth along the central meridian in response to sinusoidally varying winds with a period of 25 days. Shaded areas indicate westward motion.

intensification of the wind, the ocean is in an adjusted state in the wake of a Rossby wave front which is excited when the initially generated Kelvin wave reflects off the eastern coast [*Cane*, 1979].

This description of the response to periodic forcing is entirely in accord with the qualitative one given in the introduction (1). There we mentioned that mean currents may invalidate these results. This possibility cannot be explored by solving the nonlinear shallow water equations because at low frequencies the expression in (4) is a solution of the nonlinear equations too. In a realistic model, on the other hand, there should be intense surface currents and an equatorial undercurrent. We therefore proceed with a nonlinear stratified model.

3. Model

The equations of motion (the primitive equations) are simplified by making the Boussinesq and hydrostatic approximations and by assuming an equation of state of the form

$$\rho = \rho_0(1 - \alpha T)$$

where ρ is the density, T is the temperature, $\alpha = 0.0002/°C$ is the coefficient of thermal expansion, and $\rho_0 = 1$ gm/cm^3. The coefficients of horizontal (ν_H) and vertical (ν_v) eddy viscosity and the thermal diffusivity (κ) are assumed to have the constant values $\nu_v = 10$ cm^2/s; $\nu_H = 2 \times 10^7$ cm^2/s; $\kappa_v = 1$ cm^2/s; and $K_H = 10^7$ cm^2/s. *Philander and Pacanowski* [1980] discuss the sensitivity of the model to the values of these parameters. The equations are solved numerically by using the method described by *Byran* [1969] who discusses the finite differencing schemes in detail.

The model ocean is a rectangular box with a longitudinal extent of 4800 km, a depth of 3000 m, and a latitudinal extent of 1400 km with the equator along the southern boundary. In a horizontal plane the 35 × 70 grid points are spaced at regular intervals (of 40 and 70 km in the latitudinal and longitudinal directions, respectively). In the vertical, the first 10 grid points are spaced irregularly as shown in Figure 3. The other grid points are at the depths 825, 1250, 1750, 2250, and 2750

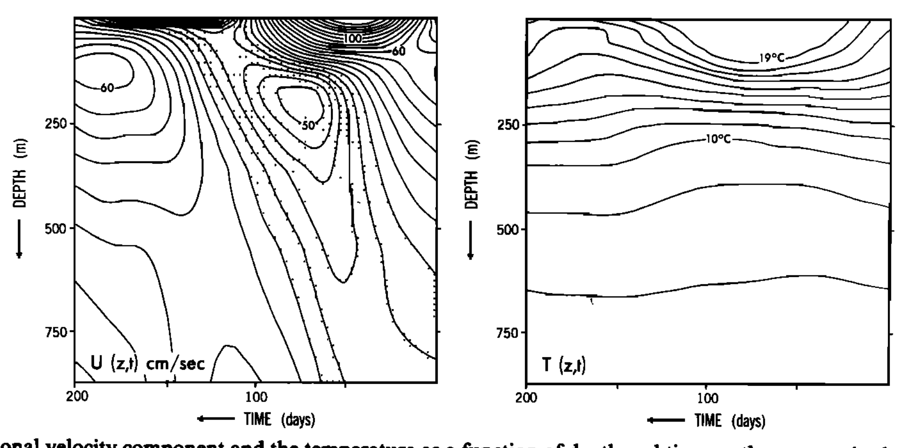

Fig. 6. Zonal velocity component and the temperature as a function of depth and time on the equator in the center of the basin for winds with a period of 200 days and a zero mean value. Shaded areas indicate westward flow.

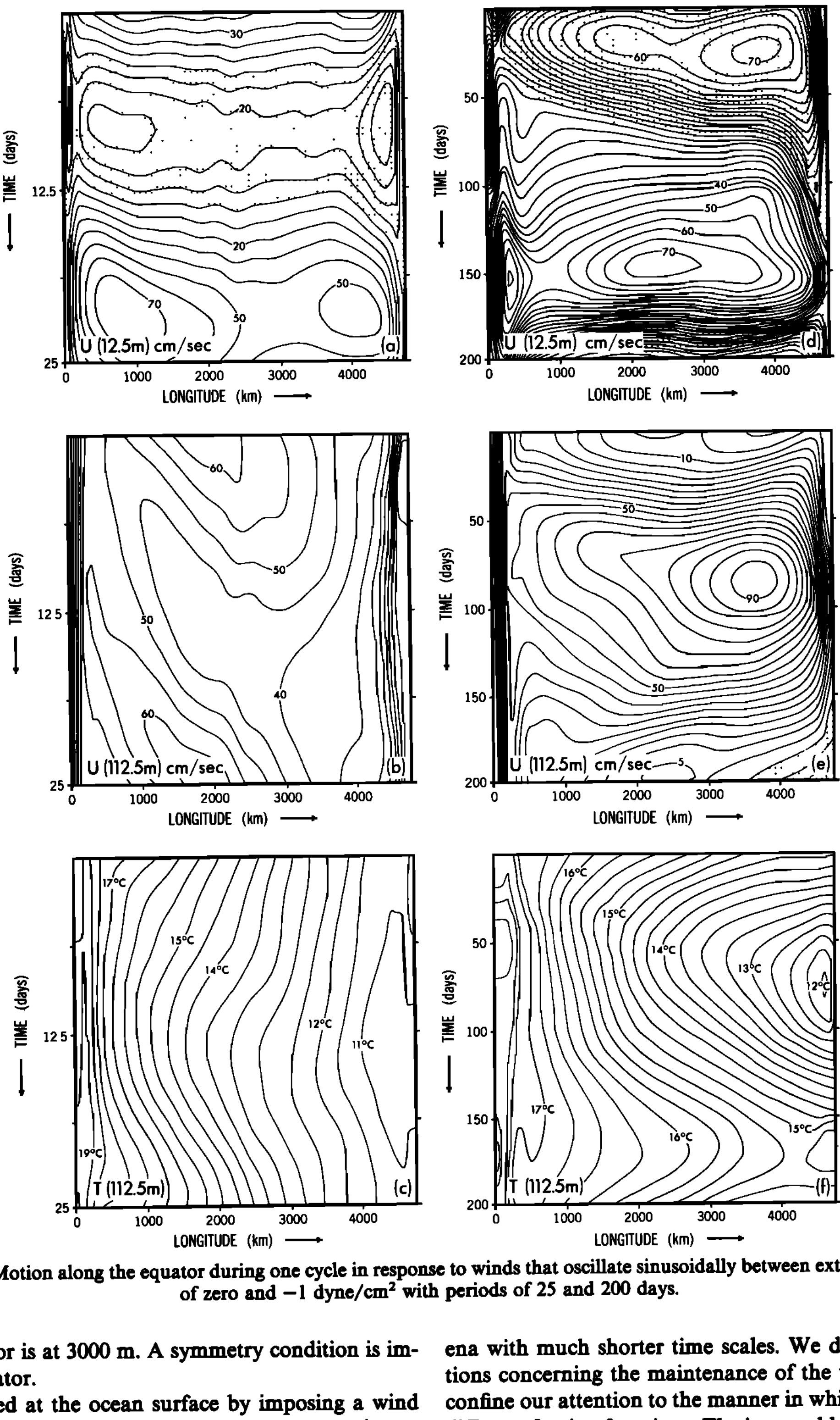

Fig. 7. Motion along the equator during one cycle in response to winds that oscillate sinusoidally between extreme values of zero and −1 dyne/cm² with periods of 25 and 200 days.

m. The ocean floor is at 3000 m. A symmetry condition is imposed at the equator.

Motion is forced at the ocean surface by imposing a wind stress and a heat flux: $\nu U_z = \tau^x$; $\nu V_z = 0$; $w = 0$, $T_z = 10^{-4}(T - 25°C)$. At the vertical walls, each of the velocity components and the heat flux is zero. The ocean floor is stress free and has a temperature of 4°C.

The initial temperature (and Brunt-Vaisala frequency) is shown in Figure 3. Since ours is a diffusive model, the temperature will immediately start to evolve to a linear profile even in the absence of any forcing. This diffusive change, with which no motion is associated, is a very slow one so that the thermocline only disappears on a time scale long compared to a decade. Here we concern ourselves with periodic phenomena with much shorter time scales. We do not address questions concerning the maintenance of the thermocline, but we confine our attention to the manner in which it is deformed by different forcing functions. The imposed heat flux at the ocean surface reduces the diffusive cooling there and causes only a very slight warming of the deep ocean.

Numerical integrations start from a state of no motion and are continued for seven cycles of the periodic wind stress. In all of our calculations, the differences between the end of the sixth and seventh cycles are extremely small when compared to the differences that occur during the cycle. In our various figures we therefore show results from the seventh cycle which can be regarded as the steady state periodic solution.

The sharp shallow tropical thermocline which in effect has

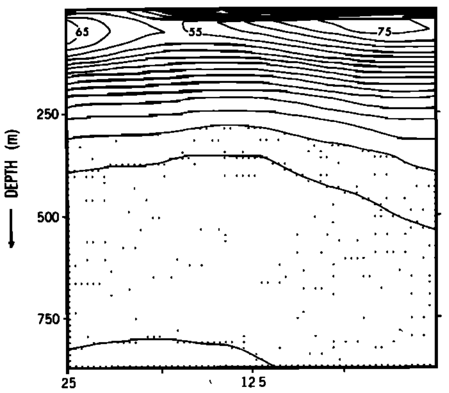

Fig. 8. Zonal velocity component as a function of depth and time on the equator in the center of the basin. The winds have a period of 25 days, a mean value of −0.5 dynes/cm^2, and an amplitude of 0.5 dynes/cm^2.

two discontinuities in N, as shown in Figure 3, will cause certain modes to be trapped in and above the thermocline because of internal reflection there [*Philander and Pacanowski*, 1980]. These modes, the gravest of which has an equivalent depth of about 25 cm, are primarily responsible for the adjustment of the upper ocean to fluctuating winds. In the initial value problems studied by *Philander and Pacanowski* [1980], these waves are clearly discernible because they propagate as nondispersive wave fronts. (This happens even when the initially density field has considerable large-scale horizontal variations.) In the case of oscillatory winds, these modes propagate as wave trains and are superimposed on fluctuating currents. Evidence of their presence is therefore much more indirect than in the examples where they propagate as nondispersive wave fronts.

4. Spatially Uniform Winds With a Zero Mean Value

This section describes the response of the nonlinear stratified model to sinusoidally varying winds. We keep the amplitude of the winds fixed at 0.5 dynes/cm^2 and study the response as a function of period.

As the period of the forcing increases, the amplitude of the response increases too. At first—at periods between 10 and 50 days—large velocity fluctuations are confined to the surface layers near the equator where the wind-driven jets are very intense. At periods longer than 50 days, the amplitude of variability in the thermocline also starts to grow because undercurrents are generated. At periods longer than approximately 150 days, the amplitude of the variability of the upper ocean does not increase significantly with increasing period. From a statistical point of view these results say that forcing with a white frequency spectrum will in the upper equatorial ocean generate motion with a spectrum that is red up to periods of about 150 days and is white at longer periods.

Figure 4 shows the difference between the response at a period of 25 days (which is representative of periods between 10 and 50 days) and the response at a period of 200 days (which is representative of periods longer than 50 days). Note than an increase in period is accompanied by an increase in the amplitude of fluctuations and in the depth to which large fluctuates extend. Another intriguing change with increasing period is the phase difference between the wind and surface jets. At periods between 10 and 50 days, the nonlinear eastward jet persists for a considerable time after the onset of westward winds, but at periods longer than 50 days the persistence of the eastward jet decreases. Compare Figures 4*a* with 4*d* for example. The difference can be attributed to the role of the zonal pressure gradients. At high frequencies the winds change direction too rapidly for large pressure gradients to be established, but at periods longer than about 50 days the eastward winds (for example) generate a considerable westward pressure force which hastens the demise of the eastward jet even before the winds start to blow westward.

25 Days

According to the linear results of section 2 the surface layers should, at a period of 25 days, show eastward phase propagation at twice the speed of the Kelvin wave (see (3)). In the nonlinear response, no phase propagation is evident. The dashed line in Figure 4*a* corresponds to the expected linear phase speed so that the only features in common between the linear and nonlinear cases are the locations of the zonal velocity maxima and their separation in time. Below the surface jets, variability has a small amplitude and is associated with linear Kelvin waves excited at the western coast. The eastward phase propagation in Figures 4*b* and 4*c* corresponds to that of Kelvin waves with an equivalent depth of about 25 cm.

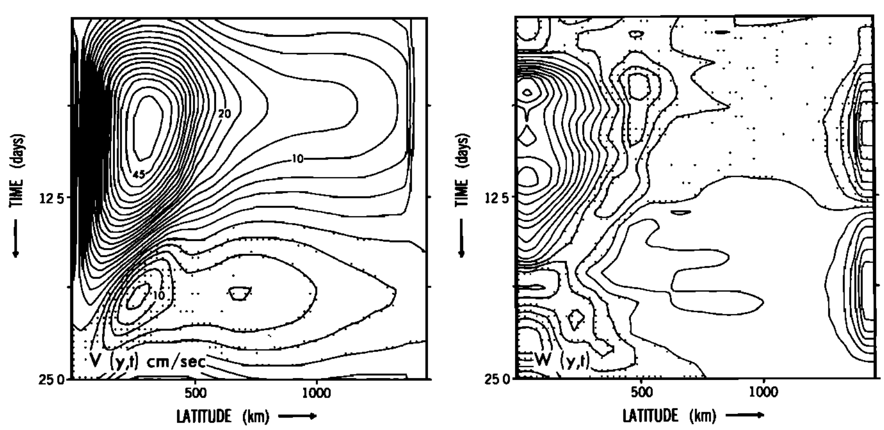

Fig. 9. Meridional velocity component and the vertical velocity component as a function of latitude and time at a depth of 12.5 m along a central meridian in response to winds with a period of 25 days and a mean value of −0.5 dynes/cm^2. Shaded areas indicate equatorward or downward motion. Contour intervals for the vertical velocity component are 0.005 cm/s.

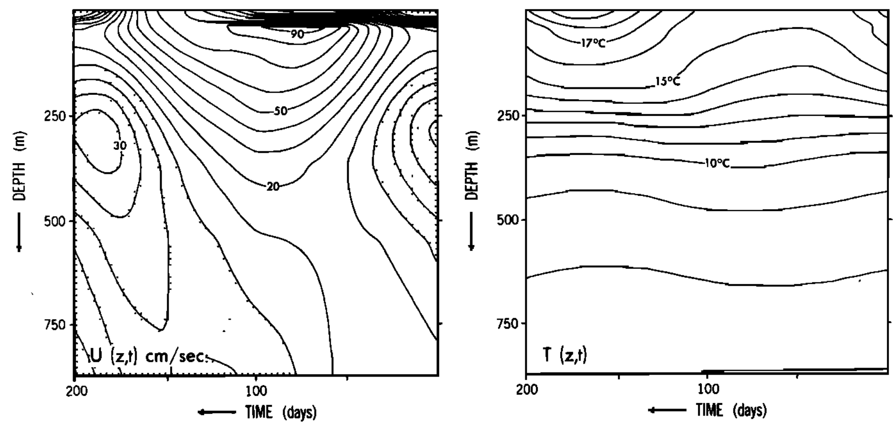

Fig. 10. Zonal velocity component and the temperature at a point on the equator in the center of the basin. The period of the wind is 200 days; its mean value is −0.5 dynes/cm².

The eastward jets in Figures 4*a* and 5 are clearly more intense, narrower, and deeper than the westward jets. The reason for this is the meridional circulation: Eastward winds impart their momentum to the surface layers which converge on the equator where there is downwelling; westward winds induce upwelling which transports water with little momentum into the surface layers where there is divergent poleward motion [*Philander*, 1979*a*]. Because of these nonlinear effects, a mean eastward current is generated even though the winds have a zero mean value.

200 Days

At a period of 200 days, density gradients in the thermocline persist sufficiently long for the associated pressure forces to generate intense undercurrents which reach their maximum speed a few weeks after the winds do (Figure 6).

The linear response is characterized by westward phase propagation at periods longer than 50 days (see Figure 2), but the nonlinear response of the upper ocean shows either no phase propagation or slight eastward propagation (see Figures 4*d*, 4*e*, and 4*f* for example). It appears that the effect of nonlinear currents on the Rossby waves is severe. As a consequence, the approach to an equilibrium response is very different in linear and nonlinear models. In section 2 we found that the eastern side of the basin reaches an equilibrium state first and that this region expands westward as the period increases. In the nonlinear ocean, such longitudinal differences are much less pronounced. With increasing period, the flow remains qualitatively similar to that shown in Figures 4*d*, 4*e*, and 4*f*), but the phase lag between the forcing and the response decreases gradually.

The main result in this section is that nonlinear currents alter the linear response at periods longer than 10 days completely. At periods up to 50 days the surface jets are so intense that the superimposed Kelvin waves are hardly discernible. Below the surface jets in the thermocline, eastward phase propagation is evident. At periods longer than 50 days there are intense undercurrents in the thermocline, and phase propagation continues to be eastward. Rossby waves are apparently unimportant in the response of the upper equatorial ocean to oscillatory winds. (Below the thermocline, where directly wind-driven currents are weak, Rossby waves are prominent at periods longer than 50 days. (See section 7.)

5. SPATIALLY UNIFORM WINDS WITH A WESTWARD MEAN VALUE

The prevailing winds over the equatorial Atlantic and Pacific oceans have a westward component and are very seldom eastward. The results of the previous section, which show that eastward winds induce motion strikingly different from that

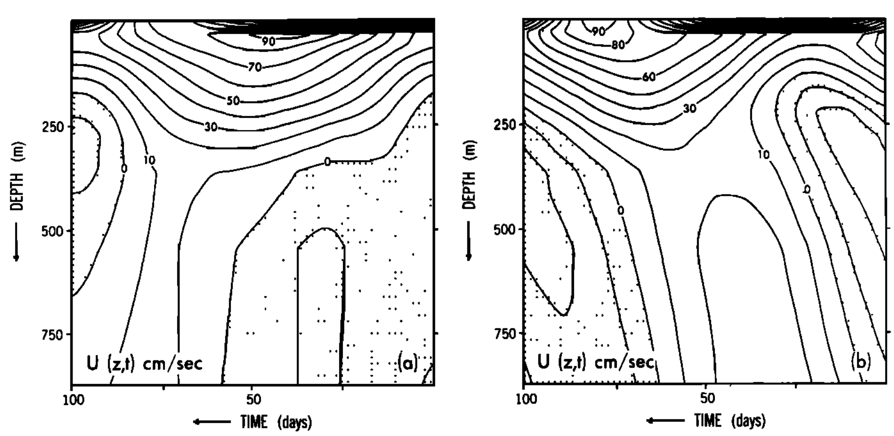

Fig. 11. Zonal velocity component as a function of time and depth at points on the equator (*a*) 1600 km from the western coast and (*b*) 1600 km from the eastern coast. The winds have a period of 100 days and a mean value of −0.5 dynes/cm². There is westward flow in shaded areas.

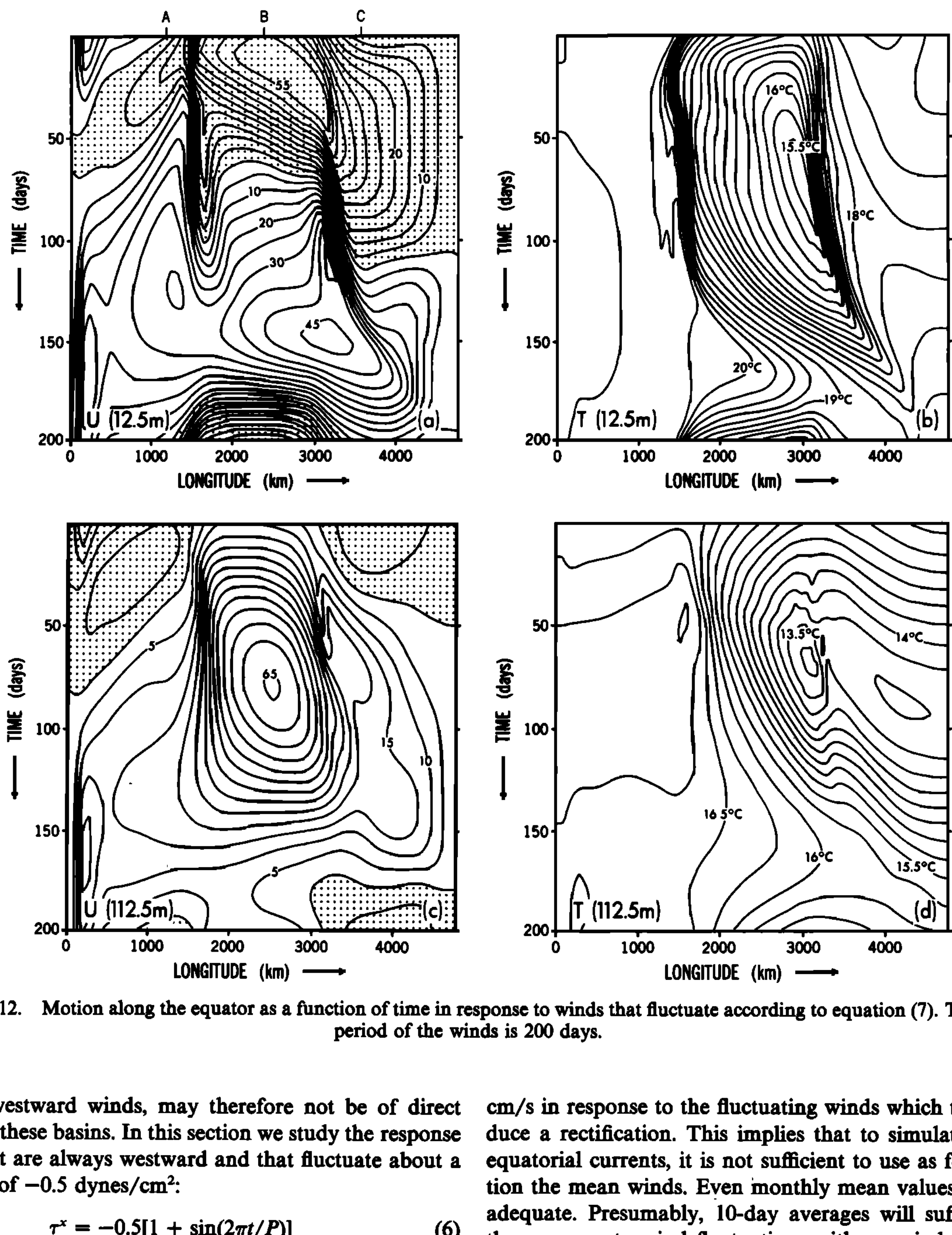

Fig. 12. Motion along the equator as a function of time in response to winds that fluctuate according to equation (7). The period of the winds is 200 days.

caused by westward winds, may therefore not be of direct relevance to these basins. In this section we study the response to winds that are always westward and that fluctuate about a mean value of -0.5 dynes/cm^2:

$$\tau^x = -0.5[1 + \sin(2\pi t/P)] \qquad (6)$$

The winds are most intense after a $\frac{1}{4}$ cycle and are zero after $\frac{3}{4}$ cycle.

If the flow were linear, then it would have a mean component (in response to the mean winds) and a fluctuating component (in response to the fluctuating winds). This is indeed the case at periods up to 10 days. As the period of the wind increases, nonlinear effects become more and more important. At periods between 10 and 50 days we expect nonlinearities to be important in the surface layers but not in the thermocline or at greater depths. It is therefore surprising to find that oscillatory winds with a short period (between 10 and 50 days) cause a nonlinear intensification of the equatorial undercurrent. Steady winds with an intensity of -0.5 dynes/cm^2 drive an undercurrent with a maximum speed of 55 cm/s in the model. Figure 8 shows that the maximum speed of the undercurrent is on the average higher than 55 cm/s in response to the fluctuating winds which therefore induce a rectification. This implies that to simulate the mean equatorial currents, it is not sufficient to use as forcing function the mean winds. Even monthly mean values may be inadequate. Presumably, 10-day averages will suffice because the response to wind fluctuations with a period less than 10 days is essentially linear. The intensity of the fluctuating component of the wind is of course another important parameter. It will be necessary to use observed winds as a forcing function to assess the importance of rectification associated with high frequency fluctuations.

Some of the properties of the surface flow at periods between 10 and 50 days are unexpected. Even though the wind is always westward, it does not drive divergent westward motion all the time. Convergent, eastward motion, with which is associated downwelling, occurs when the winds are weak. (This can be seen in Figures 7 and 9 for the case of winds with a period of 25 days.) This surfacing of the equatorial undercurrent occurs because the wind stress fluctuates on a time scale too short for the pressure force to adjust. The large-scale pressure force is maintained by the mean winds. Hence, when the wind stress is weak, the eastward pressure force is tempo-

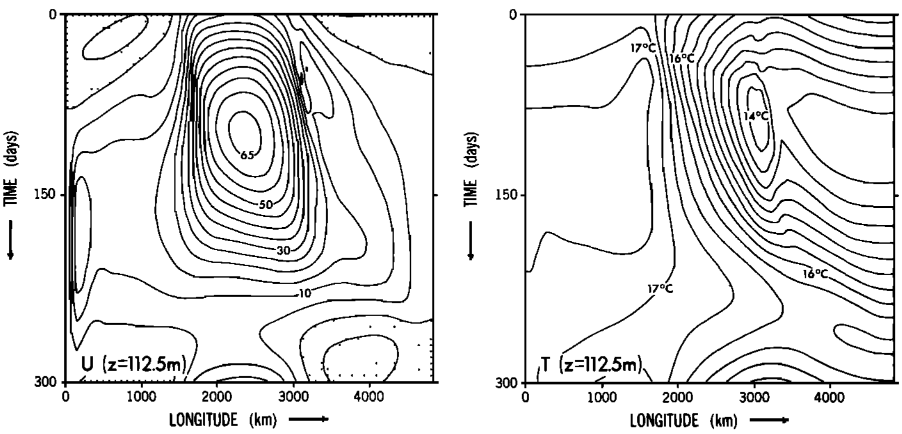

Fig. 13. Same as for Figure 12, but the period of the winds is 300 days.

rarily unbalanced so that it accelerates the surface flow eastward.

The mean winds should not be of particular importance when the period of the forcing is long when compared to the adjustment time of the ocean. In such a case the ocean at each instant is in equilibrium with the winds at that instant and therefore has no 'memory' of the mean winds. Thus when the wind stress is zero, the thermocline is horizontal, and the currents are very weak. During the subsequent gradual intensification of westward winds, linear theory is approximately valid. Both the westward surface flow and a linear eastward undercurrent will then intensify. (In a steady state, linear, stratified model there is westward flow above the eastward undercurrent, because the vertically integrated transport vanishes [*McCreary,* 1980; *Philander and Pacanowski,* 1980]. When nonlinearities become important, the undercurrent continues to intensify, but the westward surface flow weakens because upwelling transfers eastward momentum to the surface layers. The undercurrent attains its maximum speed, and zonal density gradients are at a maximum, when the wind stress is most intense.

The oceanic response at a period of 200 days is surprisingly close to the equilibrium response just described (Figure 7). The phase lag between the maximum of the undercurrent and that of the wind stress is only $\frac{1}{8}$ cycle approximately (Figure 10). The surface flow is now eastward when the westward winds are most intense because of upwelling of eastward momentum. (Compare the shaded parts of Figures 7*a and 7d*). Zonal density gradients are large when the wind is intense and small when the wind is weak. Temperature fluctuations are very small near the western coast and increase in an eastward direction. In the equatorial plane the depth of a given isotherm is almost fixed at the western coast which is a pivotal point for the vertical fluctuations of the isotherm. It follows that heat is not redistributed zonally along the equator during one cycle. Rather, heat is redistributed latitudinally. When the wind is intense, isotherms trough markedly near 4° latitude where heat is temporarily stored. When the winds relax, this heat flows back to the equator.

Whereas the linear theory (section 2) shows pronounced westward phase propagation at periods longer than 50 days, the nonlinear response of the upper ocean is characterized by eastward phase propagation. The mean currents have a severe effect on the Rossby waves but not on the Kelvin wave. Since the Kelvin wave emanates from the western coast, we expect longitudinal differences in the oceanic response. This is indeed so at periods between 50 and 150 days. Figure 11 for example shows the vertical structure of the flow for forcing with a period of 100 days at points in the western and eastern parts of the basin. In the western part (Figure 11*a*) the response resembles the nearly equilibrium response in Figure 10: For example, the undercurrent is most intense shortly after the winds are most intense. In the eastern half of the basin, however, zonal pressure gradients are not as large and do not persist as long as in the western side. Current fluctuations therefore have a different structure in the eastern side. Compare Figure 11*a* to Figure 11*b*. At periods longer than 150 days these longitudinal differences are small. The response at a period of 200 days is typical of this low frequency range.

In summary, fluctuations with a large amplitude are confined to the upper 50 m or so at periods less than 50 days. At periods longer than 50 days, variability of the equatorial undercurrent in the thermocline can be as large as that of surface currents. Longitudinal differences exist at periods between 50 and 150 days with the western part of the basin closer to an equilibrium state than the eastern part, but at periods longer than 150 days, longitudinal differences are small and the approach to an equilibrium response is gradual.

6. EFFECTS OF SPATIALLY VARYING WINDS

Let us suppose that in a closed ocean basin there is forcing over a band of longitudes in the center of the basin only. Currents similar to those described earlier are generated in the forced region where zonal density gradients are large. From this region, waves propagate into the unforced region. The currents and waves owing to a sudden intensification of the winds have been described elsewhere (see, e.g., *Philander and Pacanowski* [1980]); here we consider zonal winds τ^x that fluctuate periodically over a 1600-km meridional band in the middle of the basin:

$$\tau^x = -0.5H(x)(1 + \sin(2\pi t/P)) \qquad (7)$$

where

$$H(X) = 0.5\left[\tanh\left(\frac{x - 1600 \text{ km}}{120 \text{ km}}\right) - \tanh\left(\frac{x - 3200}{120 \text{ km}}\right)\right]$$

Figure 12 shows that at a period of 200 days, motion in the forced region is very similar to motion owing to winds with no spatial structure (see section 5 and Figure 7). West of the forced region where westward phase propagation is evident, motion is due to Rossby waves generated at the boundaries of the forced region plus Kelvin waves generated by reflection at the western coast. Temperature fluctuations west of the forced region are insignificant.

In the eastern part of the basin, motion is due to (nonsinusoidal) Kelvin wave trains excited at the extremes of the forced region plus Rossby waves due to reflections at the eastern coast. Eastward phase propagation is nonetheless evident in the eastern unforced region even at periods of 200 and 300 days (Figure 13). With increasing period, the phase lag between changes in the east and the fluctuating winds over the forced area graudally decreases.

As is in the case of spatially uniform winds (section 5), heat is not redistributed zonally in the equatorial plane during a cycle. When the winds are intense and the heat content everywhere along the equator is small, then heat is stored in the neighborhood of 4° latitude where the isotherms trough. This storage is only at the meridians where the wind blows. When the winds relax, the heat returns to the equator, including the meridians east of the forced region.

There is remarkably little difference between the response at a period of 200 days and the response at a period of 300 days, except for a slight decrease in the phase lag between the forcing and the response. The approach to an equilibrium response is a very gradual one.

7. DEEP EQUATORIAL CURRENTS

The previous sections concerned currents in and above the thermocline. The divergences of these currents cause vertical fluctuations of the thermocline, which in turn cause waves to propagate downward into the deep ocean. Given the wavelength and the period of the fluctuations in the upper ocean, the response of the deep ocean can be calculated readily. *Wunsch* [1977] for example calculates the downward propagating Rossby waves excited by a westward traveling disturbance with a period of 1 year and a wavelength of approximately 3000 km. Let us consider a disturbance with a different spatial structure. Suppose the forcing is independent of longitude (so that the zonal wave number is zero). In that case the amplitude of the response will decay exponentially with depth

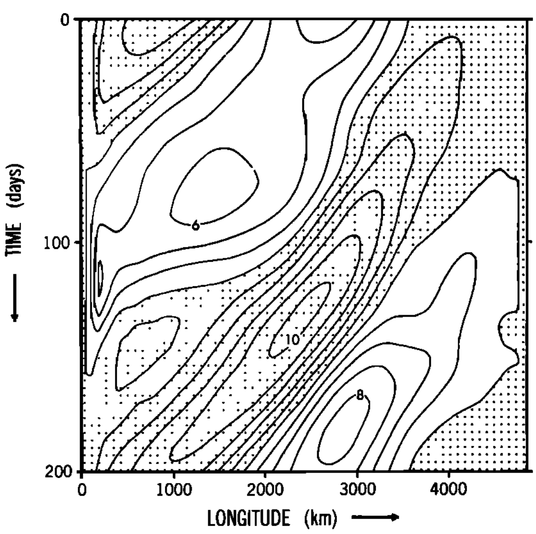

Fig. 14. Zonal velocity component fluctuations in centimenters per second at a depth of 2500 m along the equator in response to spatially uniform surface winds that oscillate sinusoidally with a period of 200 days. The winds have a mean value of -0.5 dynes/cm^2 and an amplitude of 0.5 dynes/cm^2. Motion is westward in shaded regions.

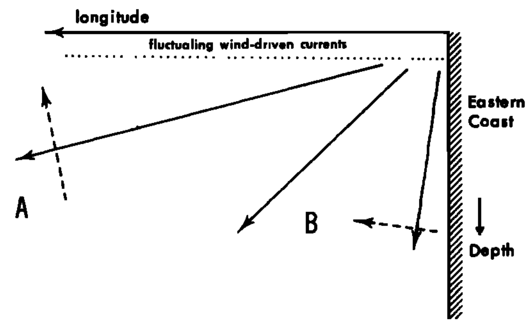

Fig. 15. Schematic diagram of wave rays emanating from the eastern coast near the surface. Dashed lines indicate phase propagation.

below the thermocline because no downward propagating waves are excited (except for inertia-gravity waves which at low frequencies are so strongly trapped about the equator that they are unimportant). However, this prediction is not borne out in our model. We find that the response of the deep ocean to longitudinally uniform winds with a period of 200 days does not decay exponentially with depth but corresponds to westward propagating fluctuations shown in Figure 14. How is this motion generated?

To explain the currents shown in Figure 14, we have to take into account that waves are excited when the wind-driven currents in the upper ocean impinge on the meridional coasts. Most of the waves are trapped in and above the thermocline and are responsible for the adjustment of the upper ocean as was explained earlier. Some of the energy propagates into the deep ocean. At the western coast of the basin the most important waves to be excited are Kelvin waves. The dispersive Rossby waves that can be generated there have such small group velocities that they remain confined to the neighborhood of that coast. From the eastern coast, on the other hand, rapid nondispersive Rossby waves will propagate into the oceanic interior. Figure 15 suggests that the vertical phase speed of these waves should be greater at meridian A than at meridian B. This is indeed the case. Figure 16 shows that at a distance of 1600 km from the eastern coast, a typical vertical phase speed is less than at a distance of 3200 km from that coast. (Compare the dashed lines in Figures 16*a* with those in Figure 16*b*).

The properties of the waves that are excited depend on their periods. Between a period of 1 week and a period of 6 months approximately, the waves are predominantly equatorially trapped [*Philander*, 1979*c*]. At periods longer than 6 months, latitudinally propagating Rossby waves become more and more important. It has been shown by means of ray theory [*Schopf et al.*, 1980] and by means of analytical solutions (M. Cane et al., private communication, 1980) that these low-frequency latitudinally propagating waves excited at coasts can focus on the equator at certain meridians. It is evident from Figure 17 that there is a similar phenomenon in our numerical model: Equatorially trapped waves are prominent at one meridian but not at another.

In summary, waves in the deep ocean can be generated either by vertical fluctuations of the thermocline or by oscillating surface currents that impinge on coasts. Both these generation mechanisms are associated with the divergence of upper ocean currents. In the example discussed here, the winds have no spatial structure, and the waves are generated only at the coasts. If the spatial variability of the wind is taken into account, then the two generation mechanisms are inseparable.

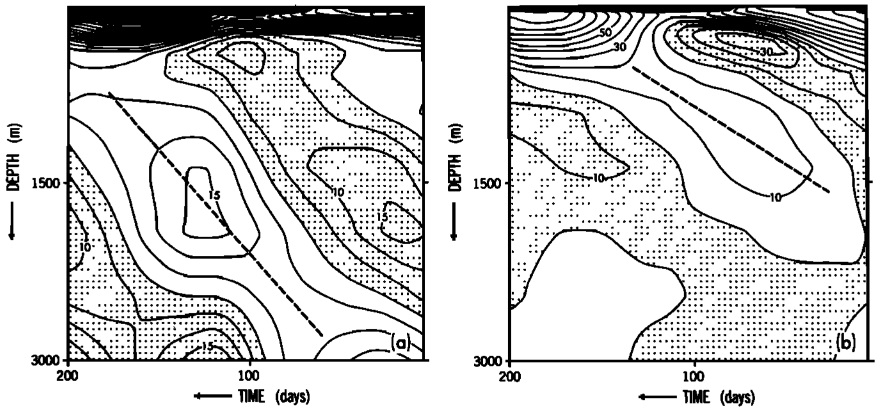

Fig. 16. Vertical structure of the motion in Figure 14 at points on the equator (*a*) 3200 km and (*b*) 1600 km from the eastern coast. The contour interval is 5 cm/s. Motion is westward in shaded areas.

Motion in the deep ocean will again correspond to propagating linear waves, but it will be difficult to relate the variability of the deep currents to that of the surface winds because the response of the intervening upper ocean is highly nonlinear. It is unclear whether the longitudinal variations evident in Figures 16 and 17 will persist when the winds vary spatially in a realistic manner.

8. SUMMARY AND DISCUSSION

The amplitude of variability in the upper equatorial ocean increases as the period of the fluctuating surface winds increases, up to a period of about 150 days. The following frequency ranges characterize the response to zonal winds with a period P. The limits of the different frequency bands are approximate.

$P < 10$ days. These winds fluctuate too rapidly to generate intense currents and excite primarily waves, including equatorially trapped inertia-gravity waves.

10 days $< P <$ 50 days. At these periods, intense equatorial jets are generated in the surface layers (i.e., the upper 50 m) of the ocean; however, at greater depths, variability has a small amplitude. The nonlinear eastward jets are more intense, narrower, and deeper than westward jets so that winds with a zero mean value give rise to a mean eastward current. If the winds are always westward, then their fluctuating component intensifies and eastward equatorial undercurrent maintained in the thermocline by the mean winds. In the surface layers above the undercurrent, there is convergent eastward flow when the winds are weak, even though the winds are always westward.

50 days $< P <$ 150 days. At periods greater than 50 days, the fluctuating zonal pressure force is eastward for a sufficient length of time to generate an intense equatorial undercurrent. Variability therefore has a large amplitude in the surface layers and in the thermocline. The upper ocean currents affect Rossby waves so severely that fluctuations in and above the thermocline show only eastward phase propagation, typical of Kelvin waves. The Kelvin waves, which are now principally responsible for the adjustment of the equatorial zone, emanate from the western coast so that the variability has prominent longitudinal variations: The western side of the basin is closer to an equilibrium response than is the eastern side. At periods longer than 150 days, the longitudinal differences are much less pronounced.

$P >$ 150 days. The amplitude of variability is now almost independent of frequency if the intensity of the wind stress remains unchanged. With increasing period, the phase lag between forcing and response decreases gradually. Eastward phase propagation of the fluctuations persist. In the equilibrium response, which is approached asymptotically as the period increases, currents are weak, and zonal density gradients are small when the wind is weak. A gradual intensification of westward winds causes a linear intensification of the westward surface flow and eastward undercurrent. When nonlinear effects become important, the westward surface flow decelerates and can become eastward, because upwelling transfers eastward momentum of the undercurrent into the surface layers. The undercurrent is most intense, and zonal density gradients are at a maximum when the westward winds are most intense.

Because of the effect of currents on Rossby wave trains, solutions to the shallow water equations (which describe the usual one- or two-level models) bear little resemblance to the motion in nonlinear stratified models.

This discussion is based on the response of a numerical model in which the basin is 4800 km wide. Will the time scales change if the dimensions of the basin change? In our description of the response in different frequency ranges, the time it takes a Kelvin wave to propagate across the basin and to establish large-scale zonal density gradients is of central importance. This time is of the order of 1 month for our basin (we associate it with a period of 50 days). Suppose that the basin were 10,000 km wide and that the winds were spatially uniform. It would take a Kelvin wave 1 month to establish large density gradients in the western part of the basin but 2 months to establish basin-wide gradients. The intermediate frequency range that extends from 50 to 150 days for a 5000 km basin will extend from 50 days to a period longer than 150 days for a larger basin. For a basin 10,000 km wide a reasonable estimate for this frequency range is 50–300 days. At these periods, fluctuations at the depth of the undercurrent are large and have pronounced longitudinal variations. At periods longer than 300 days, the approach to an equilibrium state is almost longitudinally uniform in a 10,000 km basin.

If the winds were to vary spatially—if they were confined to a band of meridians for example—then the forced region would respond essentially as if it were a closed basin. Time scales for example are determined by the zonal extent of the forced region. There are significant current (but insignificant temperature) fluctuations west of the forced region where westward phase propagation is prominent. East of the forced region, both temperature and current fluctuations are large

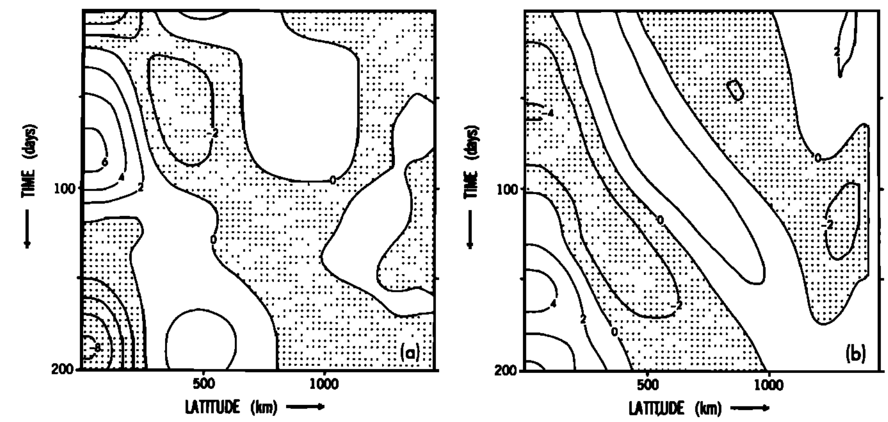

Fig. 17. Latitudinal structure of the zonal flow in Figure 16 along meridians (*a*) 3200 km and (*b*) 1600 km from the eastern coast at a depth of 2500 m. The contour interval is 2 cm/s. Motion is westward in shaded areas.

and are associated with eastward phase propagation. See Figures 12 and 13.

Observations of the seasonal cycle in the Atlantic Ocean provide a test for the results presented here. At a period of 1 year, we expect a response close to an equilibrium one. Zonal density gradients, for example, should vary nearly in phase with the seasonally varying zonal winds over the western equatorial Atlantic Ocean. This is precisely what *Katz et al.* [1977] find. When the winds are weak, the thermocline is almost horizontal; when the winds are intense, upwelling is strong and zonal density gradients along the equator are large. In the model the pivotal point about which isotherms oscillate in the equatorial plane is the western extreme of the forced region (see Figure 12). Hence, heat storage along the equator is at a minimum when the wind is intense. This result is at variance with observations which show that heat storage is at a maximum when the wind is intense [*Merle,* 1980]. It appears that in the cean (but not in the model) there is a mechanism that counters the wind-induced upwelling and that suppresses the thermocline in the western Atlantic when the wind is intense. One mechanism that will act in this manner is a mixing process which depends on the intensity of the wind. In the model we assume tht the coefficients of eddy viscosity and diffusivity are constants. This could be the reason why the model does not simulate the absolute depth of an istherm correctly, although it does simulate the slope of an isotherm realistically.

Seasonal variability in the equatorial Pacific Ocean is unfortunately too poorly known to provide a further test for the model. The reason for this is the considerable interannual variability in that ocean: According to *Wyrtki* [1975, Figure 4], the surface winds over the equatorial zone had practically no periodic seasonal cycle between 1954 and 1961, but between 1961 and 1968 the annual cycle was very strong at meridians west of 180°W. Because the oceanic response in nonlinear, a study of the seasonal cycle must focus on those periods during which the annual cycle has a large amplitude (1961 to 1968, for example). *Meyers* [1979] did not isolate such periods so that the monthly mean conditions he calculates on the basis of all the available historical data may not describe an observable seasonal cycle. *Tsuchiya* [1979] recently analyzed seven hydrographic sections, but that data set is insufficient for a description of the seasonal cycle.

Simultaneous time series of oceanographic variables and surface winds are required to check our results concerning the oceanic response in different frequency ranges. Of particular value will be a study of the seasonal cycle, especially in the Atlantic Ocean where it is regular and has a large amplitude. A data set that describes this seasonal cycle will permit rigorous tests for different parameterizations of mixing processes which seem to affect the oceanic response critically.

Velocity fluctuations in the deep equatorial oceans are observed to have two distinctive features: They have smaller vertical scales and, at periods longer than a day approximately, have higher energy levels than fluctuations poleward of a few degrees latitude [see *Luyten and Swallow,* 1976] The results of section 7 suggest that these deep currents correspond to linear vertically propagating waves generated by the divergence of the nonlinear wind-driven currents in the upper ocean. The equatorial zone is distinct because equatorially trapped waves are possible at all frequencies, but extraequatorial waves are not possible in a frequency band immediately below the inertial frequency. If this explanation is correct, then the deep equatorial currents should be distinctive only in a finite frequency band which probably extends from a period of about 1 week to a period of approximately 6 months [*Philander,* 1979*c*]. At lower (or higher) frequencies, latitudinally propagating Rossby (or inertia-gravity) waves, which are not confined to the immediate vicinity of the equator, become prominent so that the equator is no longer distinctive. In section 7 we show that at a period of 200 days the latitudinally propagating waves are so important that the equatorial zone is distinctive only at certain meridians where focusing occurs. At this stage there are no measurements to determine to what extent these results are correct (but see *Eriksen* [1980]). The possibility that the velocity fluctuations in the deep equatorial oceans correspond not merely to waves, but to mean currents generated by waves—see *Holton* [1975] for a discussion of this topic—can therefore not be ruled out.

Acknowledgment. B. Williams and P. Tunison provided invaluable assistance with the preparation of this paper.

REFERENCES

Bryan, K., A numerical method for the study of the world ocean, *J. Comp. Phys., 4,* 347–376, 1969.

Cane, M., The response of an equatorial ocean to simple wind-stress patterns, *J. Mar. Res., 37,* 233–252, 1979.

Eriksen, C. C., Evidence for a continuous spectrum of equatorial waves in the Indian Ocean, *J. Geophys. Res., 85,* 3285–3303, 1980.

Holton, J., The dynamic meteorology of the stratosphere and mesosphere, *Meteorol. Monogr., 15*(37), 216, 1975.

Katz, E. et al., Zonal pressure gradient along the equatorial Atlantic, *J. Mar. Res., 35,* 293–307, 1977.

Kindle, J. C., Equatorial Pacific Ocean variability, Ph.D. dissertation, Florida State University, Tallahassee, 1979.

Luyten, J., and J. Swallow Equatorial undercurrents, *Deep Sea Res., 23,* 1005–1007, 1976.

McCreary, J., A linear stratified ocean model of the equatorial undercurrent, submitted to Philos. Trans. R. Soc. London, Ser. A., 1980.

McPhadden, M. J., and R. A. Knox, Equatorial Kelvin and inertia-gravity waves in zonal shear flow, *J. Phys. Oceanogr., 9,* 263–277, 1979.

Merle, J., Seasonal heat budget in the Equatorial Atlantic Ocean, *J. Phys. Oceanogr., 10,* 464–469, 1980.

Meyers, G., Annual variation in the slope of the 14°C isotherm along the equator in the Pacific Ocean, *J. Phys. Oceanogr., 9,* 885–891, 1979.

Philander, S. G. H., Nonlinear equatorial and coastal jets, *J. Phys. Oceanogr., 9,* 739–747, 1979*a*.

Philander, S. G. H., Equatorial waves in the presence of the equatorial undercurrent, *J. Phys. Oceanogr., 9,* 254–262, 1979*b*.

Philander, S. G. H., Variability of the tropical oceans, *Dyn. Atmos. Oceans, 3,* 191–208, 1979*c*.

Philander, S. G. H., The equatorial undercurrent revisited, *Ann. Rev. Earth Planet. Sci., 8,* 191–204, 1980.

Philander, S. G. H., and R. C. Pacanowski, The generation of equatorial currents, *J. Geophys Res. 85,* 1123–1136, 1980.

Schopf, P., D. Anderson, and R. Smith, Beta-dispersion of low frequency Rossby waves, *Dyn. Atm. Oceans,* in press, 1980.

Tsuchiya, M., Seasonal variation of the equatorial zonal geopotential gradient in the eastern Pacific Ocean, *J. Mar. Res., 37,* 399–407, 1979.

Wunsch, C., Response of an equatorial ocean to a periodic monsoon, *J. Phys. Oceanogr., 7,* 497–511, 1977.

Wyrtki, K., El Niño—The dynamic response of the equatorial Pacific Ocean to atmospheric forcing, *J. Phys. Oceanogr., 5,* 572–584, 1975.

Yoshida, K., A theory of the Cromwell Current and equatorial upwelling, *J. Oceanogr. Soc. Jpn., 15,* 154–170, 1959.

(Received January 14, 1980;
revised June 18, 1980;
accepted August 18, 1980.)