On the Coherence between Progressive and Retrogressive Waves and a Partition of Space-Time Power Spectra into Standing and Traveling Parts

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ABSTRACT

A formula is derived to express the PR coherence between the progressive and retrogressive components in terms of the CS coherence between the cosine and sine space-Fourier coefficients. By the use of the PR coherence the space-time power spectra are partitioned into standing and traveling wave parts.

If the PR coherence is zero, the progressive and retrogressive components do not interfere with each other to form standing wave oscillations with nodes. In this case the CS coherence also becomes zero, if and only if these components have equal amplitudes.

1. Introduction

Hayashi (1971, 1973, 1977) developed a technique of computing space-time cross spectra through the time cross spectra of the space-Fourier coefficients. By the space-time spectral analysis transient waves are resolved into progressive and retrogressive waves and their wave characteristics, structure and energetics can be studied in terms of space-time power spectra, cospectra, phase difference and coherence. This method has been extensively applied to a wave analysis of the output data of a GFDL general circulation model (Hayashi, 1974; Hayashi and Golder, 1977) as well as observational data (Gruber, 1974; Zangvil, 1975 a, b; Hartmann, 1976, Sato, 1977).

The space-time power spectrum is equivalent to the wavenumber-frequency power spectrum defined by Kao (1968), except that the former is defined for a frequency band, while the latter is defined for a single discrete frequency. The advantage of the present technique is that it allows the time spectra to be computed by any methods such as the lag correlation method, the direct Fourier transform method and the maximum entropy methods. This technique is also a generalization of the quadrature-spectrum method by Deland (1964, 1972a) who found that the positive (or negative) value of the quadrature spectrum between the cosine and the sine space-Fourier coefficients gives a measure of the variance of retrogressive (or progressive) waves. Hayashi (1971) proved that this quadrature spectrum is equal to the difference between the progressive and retrogressive components of the space-time power spectra.

Hayashi (1973) further interpreted that if this quadrature spectrum is zero, the progressive and retrogressive waves have equal amplitudes and form standing-wave oscillations with nodes and antinodes. However, Pratt (1976) pointed out that progressive and retrogressive waves do not necessarily produce nodes, if the coherence between the cosine and sine (CS) space-Fourier coefficients is low.

If the CS coherence is zero, the quadrature spectrum is also zero and the space-time power spectral formula gives equal amplitude for progressive and retrogressive components. However, this should not be interpreted to mean that the space-time spectral method cannot distinguish between progressive and retrogressive components when the CS coherence is zero. Instead of using the CS coherence, the space-time power spectra should be interpreted in terms of the coherence between progressive and retrogressive (PR) components. For example, the PR coherence is zero for waves traveling back and forth, since the progressive and retrogressive components do not exist simultaneously. In the present paper it will be proven that when the PR coherence is zero, the CS coherence is also zero, only if these components actually have equal amplitudes. By the use of the PR coherence the space-time power spectra will be partitioned into standing and traveling wave components.

2. Review of space-time power spectra

We briefly review the relevant parts of the previous papers (Hayashi, 1971, 1973) in this section.

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The transient waves are generally composed of progressive (−) and retrogressive (+) waves as

$$W(t,x)=\sum_k \sum_{\pm\omega} R_{k,\omega}\cos(kx+\omega t+\phi_{k,\omega}), \quad (2.1)$$

where k is the wavenumber which is discrete for waves surrounding the earth and ω is a discrete representation of an actually continuous frequency.

The space-time power spectrum of these wave components is defined as

$$P_{k,\pm\omega}(W)=\sum_{\Delta\omega} \tfrac{1}{2}R^2_{k,\pm\omega}, \quad (2.2)$$

where the summation over $\Delta\omega$ means an integration over a frequency band. This space-time power spectrum may be computed by the formula in a real form[2]

$$4P_{k,\pm\omega}(W)=P_\omega(C_k)+P_\omega(S_k)\pm 2Q_\omega(C_k,S_k), \quad (2.3)$$

where C_k, S_k are the cosine and sine space-Fourier coefficients and P_ω, Q_ω are the time-power spectrum and quadrature spectrum, respectively.

It follows from (2.3) that the progressive and retrogressive wave components have equal amplitudes, if $Q_\omega(C_k,S_k)=0$.

The CS coherence square is given by

$$\mathrm{coh}_\omega^2(C_k,S_k)=\frac{K_\omega^2(C_k,S_k)+Q_\omega^2(C_k,S_k)}{P_\omega(C_k)P_\omega(S_k)}, \quad (2.4)$$

where K_ω is the time-cospectrum.

If the CS coherence is zero, the quadrature spectrum is also zero and the progressive and retrogressive components have equal amplitudes. However, these components do not necessarily form standing-wave oscillations.

3. Coherence between progressive and retrogressive waves

In this section we shall derive a formula (3.12) to compute the PR coherence (3.11) in terms of the CS coherence (2.4).

The transient waves can be resolved into progressive and retrogressive wave components with wavenumbers ($\pm k$) as

$$W(t,x)=\sum_k [W_k(t,x)+W_{-k}(t,x)], \quad (3.1)$$

where

$$W_{\pm k}(t,x)=\sum_\omega R_{\omega,\pm k}\cos(\omega t\pm kx\pm\phi_{\omega,\pm k}). \quad (3.2)$$

The above expression is equivalent to (2.1) with the relations

$$R_{\omega,\pm k}=R_{k,\pm\omega}, \quad \phi_{\omega,\pm k}=\phi_{k,\pm\omega}. \quad (3.3)$$

The time-power spectrum of these components at x is given by

$$P_\omega(W_{\pm k})=\tfrac{1}{2}\sum_{\Delta\omega} R^2_{\omega,\pm k}=P_{k,\pm\omega}(W). \quad (3.4)$$

It should be noted that this power spectrum does not depend on x for a single wavenumber and coincides with the space-time power spectrum defined by (2.2).

The time cospectrum and quadrature spectrum between the progressive and the retrogressive waves at x are given by

$$K_\omega(W_k,W_{-k}) =\tfrac{1}{2}\sum_{\Delta\omega} R_{\omega,k}R_{\omega,-k}\cos(2kx+\phi_{\omega,k}+\phi_{\omega,-k}), \quad (3.5)$$

$$Q_\omega(W_k,W_{-k}) =-\tfrac{1}{2}\sum_{\Delta\omega} R_{\omega,k}R_{\omega,-k}\sin(2kx+\phi_{\omega,k}+\phi_{\omega,-k}), \quad (3.6)$$

where the argument is the phase difference between the progressive and retrogressive components. The cospectrum (3.5) may be rewritten as

$$2K_\omega(W_k,W_{-k})=\cos(2kx)\sum_{\Delta\omega} R_{\omega,k}R_{\omega,-k}\cos(\phi_{\omega,k}+\phi_{\omega,-k}) -\sin(2kx)\sum_{\Delta\omega} R_{\omega,k}R_{\omega,-k}\sin(\phi_{\omega,k}+\phi_{\omega,-k}). \quad (3.7)$$

Using the relations (3-5) and (4-3) given in Hayashi (1971), the above cospectrum is reduced to

$$2K_\omega(W_k,W_{-k})=\tfrac{1}{2}[P_\omega(C_k)-P_\omega(S_k)]\cos(2kx) +K_\omega(C_k,S_k)\sin(2kx). \quad (3.8)$$

Similarly the quadrature spectrum (3.6) is rewritten as

$$2Q_\omega(W_k,W_{-k})=-\tfrac{1}{2}[P_\omega(C_k)-P_\omega(S_k)]\sin(2kx) +K_\omega(C_k,S_k)\cos(2kx). \quad (3.9)$$

The PR phase difference and PR coherence square over a frequency band $\Delta\omega$ are defined, respectively, by

$$\mathrm{Ph}_\omega(W_k,W_{-k}) =\tan^{-1}[Q_\omega(W_k,W_{-k})/K_\omega(W_k,W_{-k})], \quad (3.10)$$

$$\mathrm{coh}_\omega^2(W_k,W_{-k}) =\frac{K_\omega^2(W_k,W_{-k})+Q_\omega^2(W_k,W_{-k})}{P_\omega(W_k)P_\omega(W_{-k})}. \quad (3.11)$$

Inserting (3.8), (3.9) and (2.3) into (3.11) and using (2.4), we finally obtain a formula relating the PR coherence to the CS coherence as

$$4[1-\mathrm{coh}_\omega^2(W_k,W_{-k})]P_{k,\omega}(W)P_{k,-\omega}(W) =[1-\mathrm{coh}_\omega^2(C_k,S_k)]P_\omega(C_k)P_\omega(S_k). \quad (3.12)$$

It should be noted that the PR coherence does not depend on x for a single wavenumber. It follows

[2] The space-time power spectrum can also be computed by applying the maximum entropy power spectral analysis to the complex space-Fourier coefficients as $4P_{k,\pm\omega}(W)=P_{\pm\omega}(C_k-iS_k)$, after Hayashi (1977).

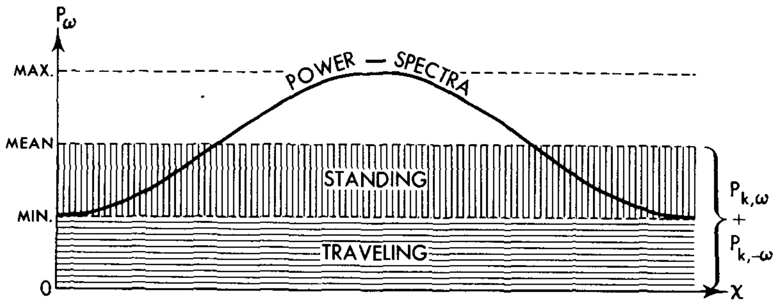

FIG. 1. The time-power spectrum P_ω (solid curve) of disturbances composed of progressive and retrogressive waves with the same wavenumber and frequencies as a function of space (x). The space-mean of this power spectrum is equal to the sum of the space-time power spectra ($P_{k,\omega}+P_{k,-\omega}$). The traveling part is given by the minimum of the time-power spectrum. The standing part is given by subtracting the minimum power spectrum from the space-mean power spectrum.

from this formula that if and only if the CS coherence is 1.0, the PR coherence is 1.0.

On the other hand, if the PR coherence is zero, we have from (3.12) and (2.3)

$$Q_\omega^2(C_k,S_k)=P_\omega(C_k)P_\omega(S_k)\,\mathrm{coh}_\omega^2(C_k,S_k)+\tfrac{1}{4}[P_\omega(C_k)-P_\omega(S_k)]^2. \quad (3.13)$$

It follows from (3.13) that if

$$Q_\omega(C_k,S_k)=0, \quad \text{then} \quad \mathrm{coh}_\omega(C_k,S_k)=0. \quad (3.14)$$

It also follows from (2.4) that if

$$\mathrm{coh}_\omega(C_k,S_k)=0, \quad \text{then} \quad Q_\omega(C_k,S_k)=0. \quad (3.15)$$

This means that when the PR coherence is zero, the CS coherence is also zero, if and only if the progressive and retrogressive components have equal amplitudes. Thus it is not inconsistent that the space-time spectral formula (2.3) gives equal amplitude for these components when the CS coherence is zero.

4. Partition of space-time power spectra

In this section we shall derive formulas (4.9) and (4.10) to compute the standing and traveling parts of the space-time power spectra. This partition is based on the space variation of the following time-power spectrum (see, also, Fig. 1).

The time-power spectrum of disturbances at x composed of progressive and retrogressive waves with a single wavenumber k over a frequency band $\Delta\omega$ is given by the identity

$$P_\omega(W_k+W_{-k})=K_\omega(W_k+W_{-k},\ W_k+W_{-k})=2K_\omega(W_k,W_{-k})+P_\omega(W_k)+P_\omega(W_{-k}). \quad (4.1)$$

The above cospectrum may be written in terms of the phase difference (3.10) and coherence (3.11) as

$$K_\omega(W_k,W_{-k})=P_\omega^{\frac{1}{2}}(W_k)P_\omega^{\frac{1}{2}}(W_{-k})\times\mathrm{coh}_\omega(W_k,W_{-k})\cos[\mathrm{Ph}_\omega(W_k,W_{-k})]. \quad (4.2)$$

This relation is further rewritten by using (3.10), (3.8) and (3.9) as

$$K_\omega(W_k,W_{-k})=P_\omega^{\frac{1}{2}}(W_k)P_\omega^{\frac{1}{2}}(W_{-k})\times\mathrm{coh}_\omega(W_k,W_{-k})\cos(2kx-\alpha), \quad (4.3)$$

where

$$\alpha=\tan^{-1}\left[\frac{2K_\omega(C_k,S_k)}{P_\omega(C_k)-P_\omega(S_k)}\right].$$

Inserting (4.3) and (3.4) into (4.1) we have

$$P_\omega(W_k+W_{-k})=2P_{k,\omega}^{\frac{1}{2}}(W)P_{k,-\omega}^{\frac{1}{2}}(W)\,\mathrm{coh}_\omega(W_k,W_{-k})\times\cos(2kx-\alpha)+P_{k,\omega}(W)+P_{k,-\omega}(W). \quad (4.4)$$

The above power spectrum (4.4) attains its maximum at

$$x=\frac{\alpha+2m\pi}{2k}, \quad (4.5)$$

where $m=0, \pm1, \pm2 \ldots$. However, if the PR coherence is zero, there is no space variation in this power spectrum.

The time-power spectrum (4.4) averaged over x is given by

$$\overline{P_\omega(W_k+W_{-k})}^x=P_{k,\omega}(W)+P_{k,-\omega}(W). \quad (4.6)$$

The minimum value of the time-power spectrum along x is given by

$$\mathrm{Min}=[P_{k,\omega}^{\frac{1}{2}}(W)-P_{k,-\omega}^{\frac{1}{2}}(W)]^2+2P_{k,\omega}^{\frac{1}{2}}(W)P_{k,-\omega}^{\frac{1}{2}}(W)[1-\mathrm{coh}_\omega(W_k,W_{-k})]. \quad (4.7)$$

It follows from (4.7) that if

$$P_{k,\omega}(W)=P_{k,-\omega}(W) \quad \text{and} \quad \mathrm{coh}_\omega(W_k,W_{-k})=1,$$

then

$$\mathrm{Min}=0. \quad (4.8)$$

This means that the minimum value vanishes if the waves are standing waves composed of progressive and retrogressive waves which are of equal amplitude and coherent with each other. Therefore, it will be reasonable to interpret this value as due to waves traveling in opposite directions or due to random noise.

The "standing part" is defined as a space-mean of the space-varying part of the time-power spectra (see Fig. 1). This is given by subtracting the minimum variance (4.7) from the space-mean variance (4.6) as

$$\mathrm{ST}_{k,\omega}(W)=2P^{\frac{1}{2}}_{k,\omega}(W)P^{\frac{1}{2}}_{k,-\omega}(W)\,\mathrm{coh}_\omega(W_k,W_{-k}). \quad (4.9)$$

This standing part may be interpreted as a coherent part of the space-time variance. The right-hand side of (4.9) is rewritten by using (3.11), (3.8) and (3.9) as

$$\mathrm{ST}_{k,\omega}(W) =\{\tfrac{1}{4}[P_\omega(C_k)-P_\omega(S_k)]^2+K_\omega{}^2(C_k,S_k)\}^{\frac{1}{2}}. \quad (4.9)'$$

This coincides with the "standing variance" defined by Pratt (1976) who uses a different approach.

The "traveling" part is defined as the minimum value (4.7) which is rewritten as

$$\mathrm{TR}_{k,\omega}(W)=P_{k,\omega}(W)+P_{k,-\omega}(W)-\mathrm{ST}_{k,\omega}(W). \quad (4.10)$$

It should be mentioned that there is no unique partition of the space-time power spectra into standing and traveling wave parts because of their non-orthogonality and the arbitrariness of the position of the node as discussed by Deland (1972b), Tsay (1974) and Pratt (1976). Nevertheless, the present definition gives reasonable results at least for pure traveling or pure standing waves as follows:

For pure traveling waves

$$\left.\begin{aligned}\mathrm{ST}_{k,\omega}(W)&=0\\ \mathrm{TR}_{k,\omega}(W)&=P_{k,\omega}(W)+P_{k,-\omega}(W)\end{aligned}\right\}. \quad (4.11)$$

For pure standing waves,

$$\left.\begin{aligned}\mathrm{ST}_{k,\omega}(W)&=P_{k,\omega}(W)+P_{k,-\omega}(W)\\ \mathrm{TR}_{k,\omega}(W)&=0\end{aligned}\right\}. \quad (4.12)$$

5. Further partition of space-time power spectra

In Section 4 the space-time power spectrum is partitioned into standing (4.9) and traveling parts (4.10). In this section we shall further divide the traveling part into "pure" progressive and retrogressive parts (5.9).

Let us divide the progressive and retrogressive components ($W_{\pm k}$) defined by (3.2) into the standing part $\hat{W}_{\pm k}$ and the "pure" progressive and retrogressive parts ($W^*_{\pm k}$) as

$$W_{\pm k}(t,x)=\hat{W}_{\pm k}(t,x)+W^*_{\pm k}(t,x). \quad (5.1)$$

In order to be able to determine the power spectra of the above parts uniquely, the following assumptions are invoked:

1) The disturbances with a particular wavenumber and a frequency band consist of only one standing wave, one pure progressive wave and one pure retrogressive wave which are *completely incoherent* with one another.

2) The standing wave consists of progressive and retrogressive components which are of equal magnitude and are *completely coherent* with each other.

The above assumptions are formulated as

$$\mathrm{coh}_\omega(\hat{W}_{\pm k},W^*_{\pm k})=0, \quad (5.2)$$

$$\mathrm{coh}_\omega(\hat{W}_{\pm k},W^*_{\mp k})=0, \quad (5.3)$$

$$\mathrm{coh}_\omega(W^*_k,W^*_{-k})=0, \quad (5.4)$$

$$\mathrm{coh}_\omega(\hat{W}_k,\hat{W}_{-k})=1, \quad (5.5)$$

$$P_\omega(\hat{W}_k)=P_\omega(\hat{W}_{-k}). \quad (5.6)$$

The power spectrum of (5.1) is reduced to

$$\begin{aligned}P_\omega(W_{\pm k})&=P_\omega(\hat{W}_{\pm k})+P_\omega(W^*_{\pm k})+2K_\omega(\hat{W}_{\pm k},W^*_{\pm k})\\ &=P_\omega(\hat{W}_{\pm k})+P_\omega(W^*_{\pm k}),\end{aligned} \quad (5.7)$$

where the assumption (5.2) has been used. On the other hand, (4.9) is reduced to

$$\begin{aligned}\tfrac{1}{4}\,\mathrm{ST}^2_{k,\omega}(W)&=K_\omega{}^2(W_k,W_{-k})+Q_\omega{}^2(W_k,W_{-k})\\ &=K_\omega{}^2(\hat{W}_k,\hat{W}_{-k})+Q_\omega{}^2(\hat{W}_k,\hat{W}_{-k})\\ &=P_\omega(\hat{W}_k)P_\omega(\hat{W}_{-k})\\ &=P_\omega{}^2(\hat{W}_k)=P_\omega{}^2(\hat{W}_{-k}),\end{aligned} \quad (5.8)$$

where (3.11) and the asumptions (5.3)–(5.6) have been used successively. Inserting (5.8) into (5.7) we have the desired formula for computing the "pure" progressive and retrogressive parts of the space-time power spectra as

$$P_\omega(W^*_{\pm k})=P_{k,\pm\omega}(W)-\tfrac{1}{2}\,\mathrm{ST}_{k,\omega}(W). \quad (5.9)$$

In order that $P_\omega(W^*_{\pm k})$ be positive, the following condition must be met:

$$\mathrm{coh}_\omega{}^2(W_k,W_{-k})\leqslant\frac{P_{k,\pm\omega}(W)}{P_{k,\mp\omega}(W)}, \quad (5.10)$$

where (5.9) and (4.9) have been used.

If the condition (5.10) is not satisfied, assumptions 1) and 2) are not valid and the above separation is not meaningful. It is important to remember that if the length of the time record is too short or the frequency band is too narrow, the coherence is overestimated and condition (5.10) cannot be satisfied [for the statistical significance of coherence, see Goodman (1957) and Julian (1975)].

6. Example

As an example, we shall apply the present method to mid-latitude ultralong waves appearing in a GFDL general circulation model. For a detailed analysis of these waves, see Hayashi and Golder (1977).

The following spectra defined in the previous sections are computed for the geopotential height at 60°N

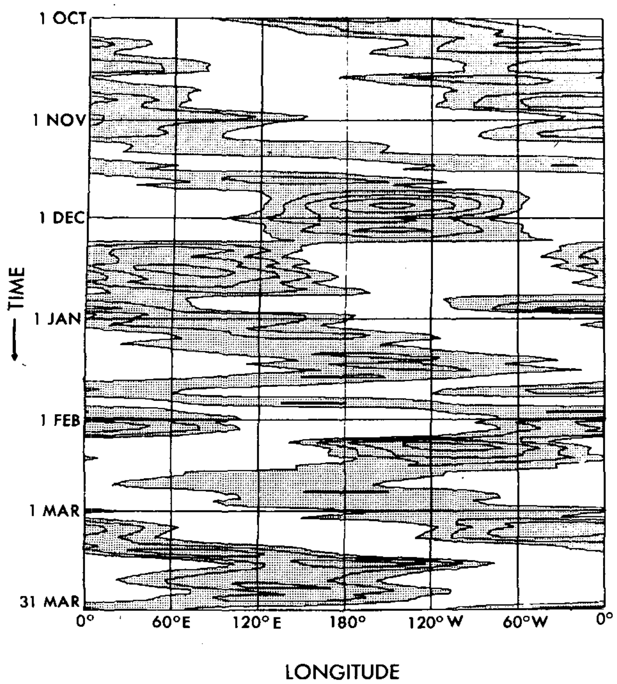

FIG. 2. The longitude-time section of the wavenumber 1 component of the geopotential height at 60°N at 190 mb level of the GFDL model. The time-mean is subtracted out. Contours are drawn only in the negative area (shaded).

from October through March. These spectra are associated with wavenumber 1, periods of 30 days and frequency intervals of 1/60 day^{-1}.

Space-time power spectra (2.2):

$$P_{k,\omega}(W)=23.0$$
$$P_{k,-\omega}(W)=30.0$$

CS coherence (2.4):

$$\mathrm{coh}_\omega(C_k,S_k)=0.32$$

PR coherence (3.11):

$$\mathrm{coh}_\omega(W_k,W_{-k})=0.31$$

Standing part (4.9):

$$\mathrm{ST}_{k,\omega}(W)=16.0$$

Pure traveling parts (5.9):

$$P_\omega(W_k^*)=15.0$$
$$P_\omega(W_{-k}^*)=22.0$$

The above spectral analysis suggests that in addition to standing wave oscillations, there exist both eastward and westward moving waves which are incoherent with each other. This interpretation can be visualized in a longitude-time section (Fig. 2) of wavenumber 1. It is seen that the incoherent waves are identified as traveling waves moving back and forth. These incoherent waves may be interpreted as the vacillation of synoptic patterns.

7. Conclusions

By deriving the formula (3.12) which expresses the PR coherence between the progressive and retrogressive components in terms of the CS coherence between the space-Fourier coefficients, the following conclusions have been obtained:

1) If the PR coherence is zero, progressive and retrogressive components do not interfere with each other to form standing wave oscillations.

2) When the PR coherence is zero, the CS coherence is also zero, if and only if these components have equal amplitudes.

3) The PR coherence determines the partition of the space-time power spectra into the standing part given by (4.9) or (4.9)' and the traveling part (4.10). The position of the antinodes is given by (4.5).

4) The traveling part may further be divided into "pure" progressive and retrogressive parts by (5.9), provided that the condition (5.10) is satisfied.

It is important in estimating the above coherences that the record length and the frequency interval are sufficiently large.

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