A Method of Estimating Space-Time Spectra from Polar-Orbiting Satellite Data

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ABSTRACT

Space-time spectral formulas are modified to estimate wavenumber-frequency spectra correctly from space-time series data sampled at the same local time but at different hours of a day by a polar-orbiting satellite.

It is shown that a significant error occurs in the wavenumber-frequency spectra of the space-time series for wave periods less than 10 days. This error can be eliminated without time interpolation by taking a space-Fourier transform with respect to the frequency-shifted wavenumbers measured at the same local time.

1. Introduction

Hartmann (1976) discussed possible errors in wavenumber-frequency spectra estimated from space-time series data sampled at the same local time but at different hours of a day by a polar-orbiting satellite (see Figs. 1 and 2 for its orbits).[1] As illustrated by Fig. 3a, the sampling along the latitude circle is made at the same local time. He pointed out that there is a shift in the zonal wavenumber measured at the same local time as illustrated by Fig. 3b. This shift is given by the wave frequency and is less than 0.1 for periods greater than 10 days and can be considered negligible.

However, for an eastward moving planetary wave of wavenumber 1 and a period of 4 days, as observed in the polar stratosphere (Venne and Stanford, 1979), the wavenumber shift is 0.25 and is not negligible. For possible 2-day period oscillations, as observed in the tropical stratosphere (Cadet and Teitelbaum, 1979; Coy, 1979), the wavenumber shift is as large as 0.5.

It is expected that this shift in wavenumber causes a significant error in the wavenumber-frequency spectra. It will be demonstrated that this error can be eliminated by taking a space-Fourier transform with respect to the frequency-shifted wavenumbers.

In Section 2 the frequency-shifted wavenumber is introduced. In Section 3 space-time spectral formulas (Hayashi, 1971, 1973, 1977a,b, 1979a,b) are modified to estimate wavenumber-frequency spectra correctly. In Section 4 a test of the modified method is made based on an artificial sinusoidal wave. In Section 5 a summary and remarks are given. Appendix A gives a list of symbols. Appendix B gives formulas for the frequency-shifted Fourier transform and space-time cross spectra. Appendix C gives formulas to calculate the orbital tilt and appendix D reviews the relativistic Doppler effects.

[1] Fig. 2 is reproduced from Nimbus III Catalog, 1969, Vol. 1, Part 1 [available from Goddard Space Flight Center, Greenbelt, MD 20771].

2. Frequency-shifted wavenumber

The standard time t and local time t_l of a satellite at longitude λ are related by

$$t = t_l - \lambda/\Omega, \tag{2.1}$$

where Ω is the rotation of the earth relative to the sun-synchronous satellite (2π radian per solar day) and $-\pi \leq \lambda \leq \pi$.

The space-time Fourier representation of a space-time series is given by

$$w(\lambda,t) = \mathrm{Re} \sum_{m,\omega} \hat{W}_{m,\omega} \exp[i(m\lambda + \omega t)], \tag{2.2}$$

where m is the zonal wavenumber measured at standard time and the angular frequency ω takes both positive (westward) and negative (eastward) values.

The above representation is rewritten in terms of local time (t_l) by inserting (2.1) into (2.2) as

$$w(\lambda,t_l) = \mathrm{Re} \sum_{m,\omega} \hat{W}_{m,\omega} \exp[i(m_\omega\lambda + \omega t_l)], \tag{2.3}$$

where

$$m_\omega = m - \omega/\Omega. \tag{2.4}$$

Hereafter, m_ω will be called the frequency-shifted wavenumber.

Thus at the same local time the wavenumber is

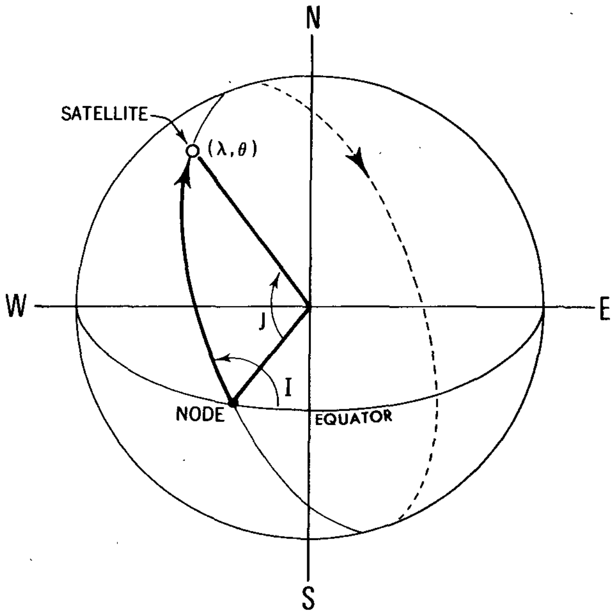

FIG. 1. The orbit of a polar-orbiting sun-synchronous satellite. Twice daily observations are available on the day and night sides. Relative to the rotating earth, the orbits drift westward 360° per solar day. The longitude (λ) and latitude (θ) of the satellite are determined by the angles I and J (see Appendix C).

shifted by $-\omega/\Omega$ and hence a phase discontinuity of $2\pi\omega/\Omega$ occurs at 180° longitude (see Fig. 3b) as pointed out by Hartmann (1976).

This shift in wavenumber is somewhat analogous to the Doppler-shifted frequency[2] measured by a satellite moving westward relative to the earth with the angular velocity Ω, as given by

$$\omega' = \omega - m\Omega, \tag{2.5}$$

by use of a Galilean transformation

$$\lambda = \lambda' - \Omega t. \tag{2.6}$$

Chapman *et al.* (1974) and Rodgers (1976) estimated wavenumber-frequency spectra through the frequency spectra of time series data sampled by a polar-orbiting satellite at different points along the slanted lines in Fig. 3a. With a reasonable guess of a wavenumber, the positive Doppler-shifted frequencies observed by the satellite are converted to positive (westward) or negative (eastward) frequencies observed on the earth. Subsequently, Hirota (1976, 1979) estimated wavenumbers as well as Doppler-shifted frequencies by use of wavenumber-filtered local time-space series. Hirota's method, however, is not free from the wavenumber error and phase discontinuity mentioned above. Moreover, his method cannot make use of twice daily data to resolve waves with periods shorter than 2 days.

In the case of relativistic Doppler shifts, both frequency and wavenumber are shifted (see Appendix D).

3. Modified spectral formulas

a. *Space-Fourier transform*

Wavenumber-frequency power spectra are defined by

$$P_{m,\omega}(w) = \tfrac{1}{2}\langle |\hat{W}_{m,\omega}|^2 \rangle, \tag{3.1}$$

where the angle braces denote ensemble average which can be replaced by a narrow frequency band average, if the time series is ergodic and of sufficient length.

Here, the complex space-time amplitude $\hat{W}_{m,\omega}$ is given by the space-time Fourier transform by virtue of the orthogonality of $\exp(im\lambda + i\omega t)$ as

$$\hat{W}_{m,\omega} = \frac{1}{\pi T}\int_0^{2\pi}\left[\int_0^T w(\lambda,t) \times \exp[-i(m\lambda + \omega t)]dt\right]d\lambda, \tag{3.2a}$$

$$= \frac{1}{\pi T}\int_0^{2\pi}\left[\int_0^T w(\lambda,t_l) \times \exp[-i(m_\omega + \omega t_l)]dt_l\right]d\lambda, \tag{3.2b}$$

where (3.2b) has been transformed from (3.2a) by use of (2.1) and (2.4).

Eq. (3.2b) is rewritten as

$$\hat{W}_{m,\omega} = T^{-1}\int_0^T F_{m,\omega}(t_l)\exp(-i\omega t_l)dt_l, \tag{3.3}$$

where $F_{m,\pm\omega}$ is the "frequency-shifted space-Fourier transform" defined (see Appendix B for its real representation) by

$$F_{m,\omega}(t_l) = \pi^{-1}\int_0^{2\pi} w(\lambda,t_l)\exp(-im_\omega\lambda)d\lambda. \tag{3.4}$$

Eq. (3.3) indicates that the correct space-time Fourier transform is given by taking a time-Fourier transform of $F_{m,\omega}$. This relation is the basis of the present method. In practice, t_l can be replaced by discrete values in the absence of unresolvable high-frequency oscillations.

It should be noted that the frequency-shifted Fourier transform $F_{m,\omega}$ does not reproduce the original space series as

$$w(\lambda,t_l) \neq \mathrm{Re}\sum_m F_{m,\omega}(t_l)\exp(im_\omega\lambda), \tag{3.5}$$

[2] In meteorology the Doppler-shifted frequency often refers to the frequency measured by an observer moving with the basic flow.

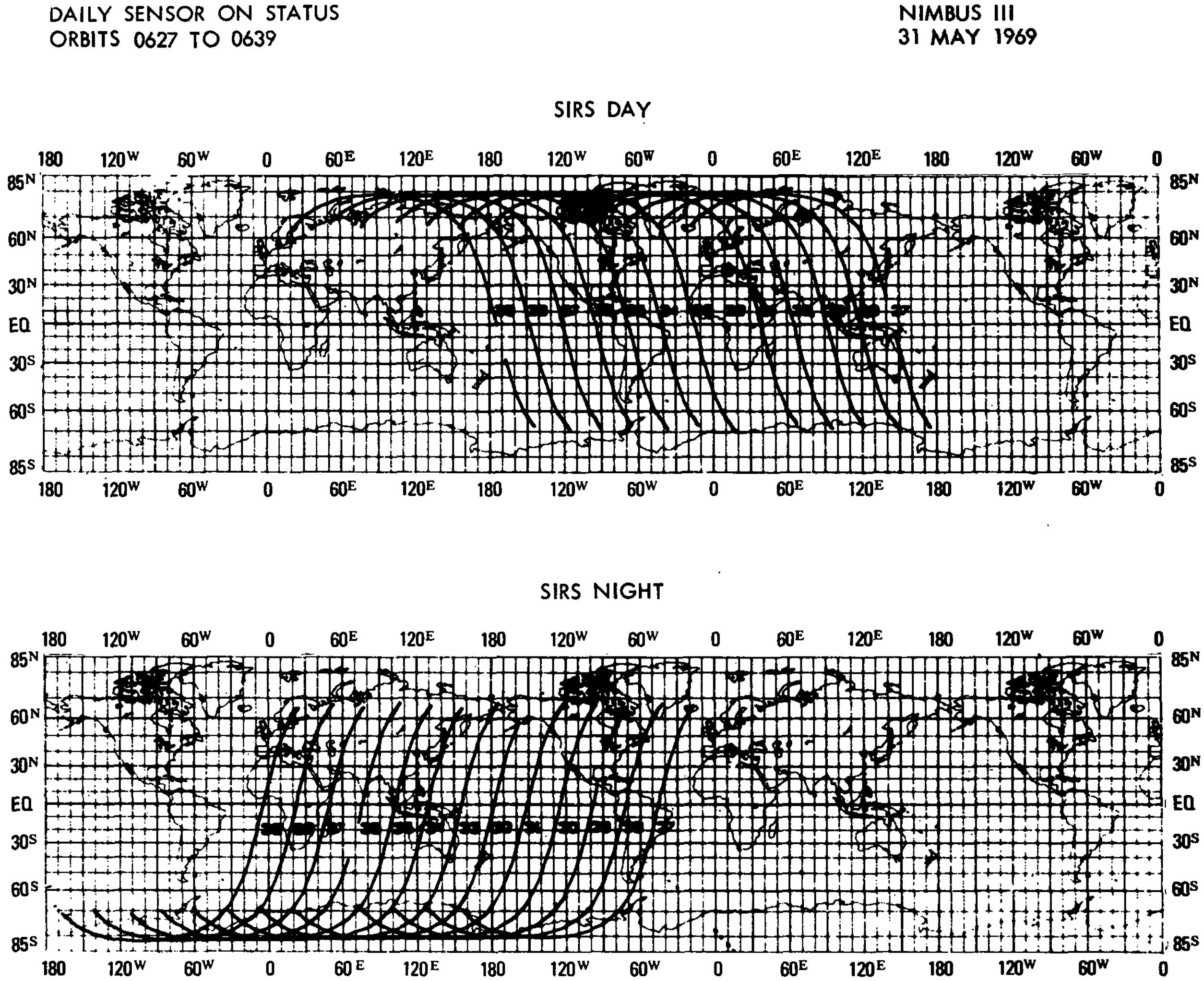

FIG. 2. The orbits of Nimbus 3 (after Nimbus 3 Data Catalog) on 31 GMT May 1969. The day side orbits (upper) at equator is on 31 May in local time, while the night side orbits (lower) are on 31 May in the Western Hemisphere and on 1 June in the Eastern Hemisphere. The longitude is extended to 2.5 cycles.

since $\exp(im_\omega\lambda)$ with non-integer wavenumbers is *not* orthogonal over λ (see Schickedanz and Bowen, 1977). However, it can be proven that $\exp(im_\omega\lambda + i\omega t_l)$ is orthogonal in the (λ,t_l) domain and that (3.2b) also follows directly from (2.3) by virtue of this orthogonality.

b. Space-time spectral formulas

For convenience in computation the above $F_{m,\omega}$ and $\hat{W}_{m,\omega}$ are decomposed into the real and imaginary parts as

$$F_{m,\pm\omega}(t_l) = C_{m,\pm\omega}(t_l) - iS_{m,\pm\omega}(t_l), \quad (3.6)$$

$$2\hat{W}_{m,+\omega} = \hat{C}_{m,\omega} - i\hat{S}_{m,\omega}, \quad (3.7a)$$

$$2\hat{W}_{m,-\omega} = \hat{C}^*_{m,-\omega} - i\hat{S}^*_{m,-\omega}, \quad (3.7b)$$

where the asterisk denotes the complex conjugate.

Here $(\hat{C}_{m,\pm\omega},\hat{S}_{m,\pm\omega})$ is the time Fourier transform of $(C_{m,\pm\omega},S_{m,\pm\omega})$ with respect to $\exp(-i\omega t_l)$ [not $\exp(\mp i\omega t_l)$] defined by

$$(\hat{C}_{m,\pm\omega},\hat{S}_{m,\pm\omega}) = 2T^{-1}\int_0^T (C_{m,\pm\omega},S_{m,\pm\omega})\exp(-i\omega t_l)dt_l. \quad (3.8)$$

Inserting (3.7a) and (3.7b) into (3.1) gives formulas for computing wavenumber-frequency power spectra

$$4P_{m,\pm\omega}(w) = P_{\pm\omega}(C_{m,\pm\omega} - iS_{m,\pm\omega}), \quad (3.9a)$$

$$= P_\omega(C_{m,\pm\omega}) + P_\omega(S_{m,\pm\omega}) \pm 2Q_\omega(C_{m,\pm\omega},S_{m,\pm\omega}). \quad (3.9b)$$

where P_ω and Q_ω are the time power and quadrature spectra respectively.

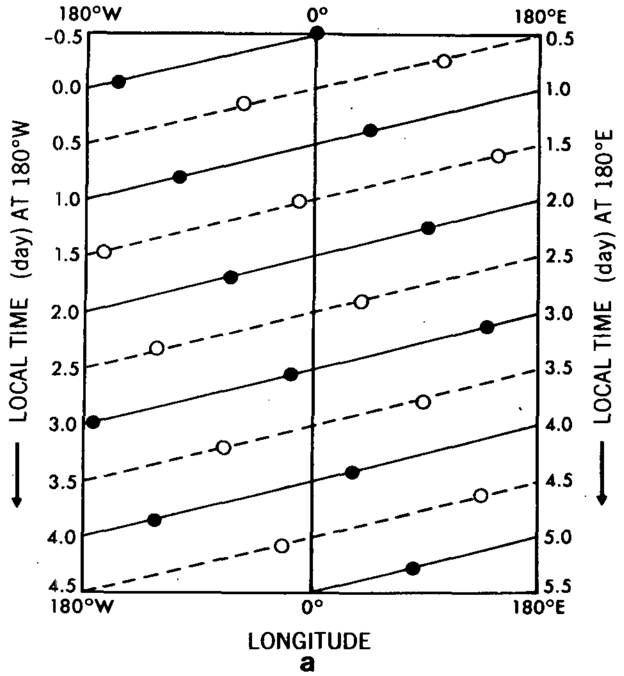

FIG. 3a. Longitude-time section of the position of a satellite with 2.3 orbits per day at midnight (full circle) and noon (open circle) at the equator. Solid lines connect the night side points while dashed lines connect day side points. The numerals indicate local time at 180°W (left) and 180°E (right).

These formulas are the same as those derived by Hayashi (1971, 1977b) except for the use of the frequency-shifted space-Fourier transform. Eq. (3.9b) in real representation is convenient for using the lag correlation method (see Jenkins and Watts, 1968) or the direct Fourier transform method (see Bendat and Piersol, 1971) in estimating the time cross spectra, while (3.9a) in complex representation is suitable for using the maximum entropy method which can be applied to a short time record [see Hayashi (1977) for its application to wavenumber-frequency spectra].

c. Computational scheme

The computational scheme of the modified method is given as follows:

1) Select a wavenumber-frequency range of interest. Sample discrete values of tuning frequencies $\pm f$ for the frequency-shifted Fourier transform with a certain frequency interval Δf which can be wider than that of time spectra but should be sufficiently narrow. If $\Delta f = 0.1$ day^{-1}, for example, the error in power spectra is $\sim$10% (see Section 4).

2) Compute the frequency-shifted Fourier transform by use of the formulas (B1) in Appendix B.

3) Compute space-time spectra by use of the formulas (B3)–(B10). These spectra give the correct estimates over the frequency range $\pm f - \Delta f/2 \sim \pm f + \Delta f$ to a good approximation.

In computing the frequency-shifted Fourier transform, it is important to use day and night data on the same equatorial local date (see Fig. 3b) rather than on the same date in the same standard time (see Fig. 2).

4. Test of the modified method

In this section a test of the modified method described in Section 3 is made based on artificial twice-daily local-time series data. The space-time power spectra are computed by using (3.9b) by the lag correlation method. As an example of waves, a sinusoidal standing wave oscillation is given which consists of both eastward and westward moving components with wavenumber 1 and multiple periods of 10, 4, 2, 1 days.

Fig. 4 shows the ratio of the space-time power spectra (wavenumber 1) estimated from the local time series (2.3) to the peak value of those estimated from a standard time series (2.2). The line spectra have been broadened due to a lag window which has an effect of smoothing the spectral curve by a 1-2-1 running mean. Without any correction (top) the eastward moving components are significantly underestimated. In particular, the power at the eastward 4-day period is underestimated by 40%. The

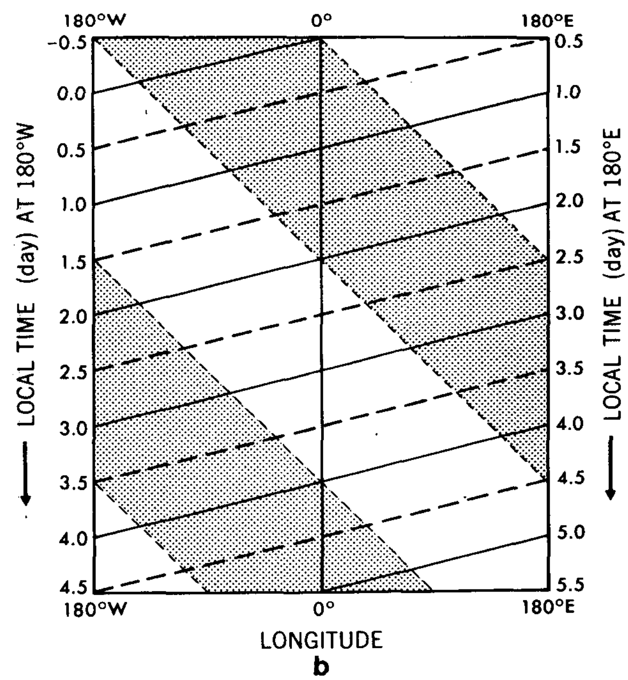

FIG. 3b. Longitude-time section of an eastward moving wave (indicated by shading) with wavenumber 1 and a period of 4 days. The wavenumber 1 is measured as 1.25 along the slanted lines connecting the same local time.

power spectra at the 1-day period leak entirely to their frequency-shifted wavenumbers which are 0 and 2 for the westward and eastward components, respectively. The asymmetry between the eastward and westward components is due to the leakage and interference between these components with non-integer frequency-shifted wavenumbers. These errors also depend on the wavenumber spectral distributions of data (not illustrated). The spectral estimates (middle) are not improved by linearly interpolating the data to the same standard time. This is because a linear interpolation causes a serious distortion in the amplitude and phase of high-frequency oscillations (Shapiro, 1972). By use of the frequency-shifted Fourier transform tuned to the eastward 4-day period, it is shown (bottom) that the power spectrum at this period is perfectly corrected. The other spectral peaks can also be perfectly corrected by tuning the frequency shift to these periods (not illustrated).

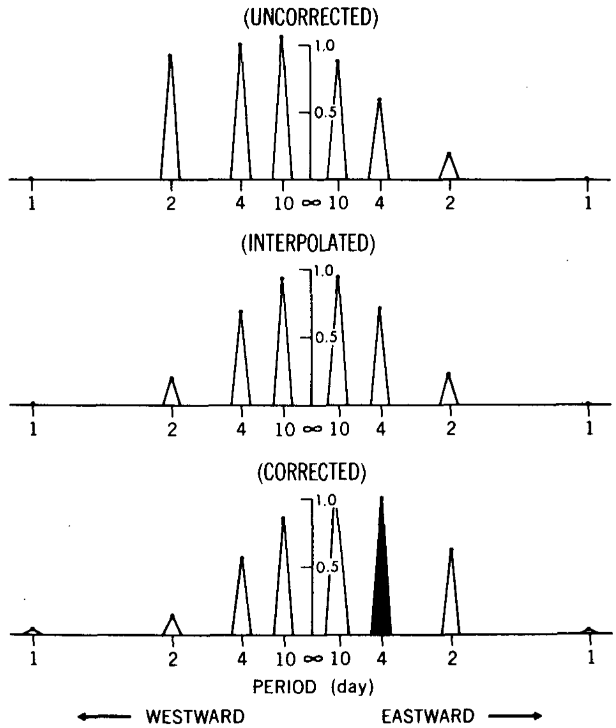

FIG. 4. The ratio of the space-time power spectra (wavenumber 1) estimated from local-time series to the peak value of those estimated from standard time series. A standing wave oscillation is given which consists of both eastward and westward moving components with wavenumber 1 and multiple periods of 10, 4, 2, 1 days. Top: without correction. Middle: with linear interpolation of data between two local noons (or nights). Bottom: with a wavenumber correction tuned to the eastward 4-day period (shaded).

5. Summary and remarks

Space-time spectral formulas are modified to estimate wavenumber-frequency spectra correctly from space-time series data sampled at different hours of a day by a polar-orbiting satellite.

It is shown that a significant error occurs in the wavenumber-frequency spectra of the space-time series for wave periods $\leq$ 10 days. This error is not corrected by linearly interpolating the data to the same standard time nor after the space-time spectra have been computed, since this error depends on the wavenumber spectral distribution.

It is demonstrated that the above error can be eliminated simply by taking a space-Fourier transform with respect to the frequency-shifted wavenumbers. In principle, this method gives the same correction as given by a time-Fourier interpolation which is an ideal interpolation in the absence of unresolvable high-frequency oscillations.

In addition to the above errors, there are the following problems in the use of polar-orbiting satellite data.

(i) The orbits of a sun-synchronous satellite tilt with latitude (see Fig. 1). In the case of Nimbus 3 (Fig. 2), there is a difference of about 22° longitude along the orbit from the equator to 60° latitude (see Appendix C for the calculation of the orbital tilt).

(ii) Orbital points shift (see Fig. 3a) in longitude from day to day (or night) due to a non-integer number of orbits per day.[3] In the case of the Nimbus 3, which makes 13.4 orbits per day, the relative orbit shifts about 11° eastward per 13 orbits.

The problems (i) and (ii) must be avoided by interpolating data to regular longitude-latitude grid points. In the case of planetary-scale waves, a linear interpolation in space does not cause a serious distortion.

In addition, there are the following minor problems:

(iii) Local time changes along the orbit due to its tilt with latitude as discussed in Appendix C. In the case of Nimbus 3 there is a local time difference of ~70 min from the equator to 60° latitude.

(iv) At higher latitudes daytime and nighttime data are not strictly 12 h apart from each other. In the case of Nimbus 3, they are taken about 0100 and 1100 at 60° latitude.

Problems (iii) and (iv) can be ignored or avoided by averaging day and night data.

The present method can be extended to partition the time power spectra of transient waves consisting of multiple wavenumbers into standing and

[3] This shift should not be confused with the eastward drift (360° per year) of the orbit of a sun-synchronous satellite which faces the sun all the year round. This drift is due to the intrinsic tilt of the orbit around an ellipsoidal earth.

traveling parts correctly by modifying the generalized spectral formulas (Hayashi, 1979b). Also, the time modulation of the space amplitude of traveling waves can be estimated correctly by applying a complex demodulation (see Chapman and McGregor, 1978; Julian, 1971) to the frequency-shifted space-Fourier transform.

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APPENDIX A

List of Symbols

θ	latitude
$\lambda(\theta)$	longitude of orbits at θ
$t(\theta)$	Greenwich standard time of a satellite at θ
$t_l(\theta)$	local time of a satellite at θ
T	length of time record
ω	angular frequency
Ω	rotation of the earth relative to a sun-synchronous satellite (2π per solar day)
m	zonal wavenumber
m_ω	frequency-shifted wavenumber
$F_{m,\omega}$	frequency-shifted space-Fourier transform

APPENDIX B

Space-Time Spectral Formulas

The frequency-shifted space-Fourier transform (3.4) is rewritten in real representation as

$$C_{m,\pm\omega}(t_l) = \pi^{-1}\int_0^{2\pi} [w\cos(f\lambda)]\cos(m\lambda)d\lambda \pm \pi^{-1}\int_0^{2\pi} [w\sin(f\lambda)]\sin(m\lambda)d\lambda, \quad \text{(B1a)}$$

$$S_{m,\pm\omega}(t_l) = \mp\pi^{-1}\int_0^{2\pi} [w\sin(f\lambda)]\cos(m\lambda)d\lambda + \pi^{-1}\int_0^{2\pi} [w\cos(f\lambda)]\sin(m\lambda)d\lambda, \quad \text{(B1b)}$$

where

$$f = \omega/\Omega. \quad \text{(B2)}$$

It should be noted that the above transform can be regarded as the space-Fourier transform of w weighted by $\cos(f\lambda)$ and $\sin(f\lambda)$. In practice, the integral should be replaced by a summation over discrete values of λ.

The modified formulas for space-time cross spectra between two variables w and w' are given by

$$4P_{m,\pm\omega}(w) = P_\omega(C_{m,\pm\omega}) + P_\omega(S_{m,\pm\omega}) \pm 2Q_\omega(C_{m,\pm\omega},S_{m,\pm\omega}), \quad \text{(B3)}$$

$$4K_{m,\pm\omega}(w,w') = K_\omega(C_{m,\pm\omega},C'_{m,\pm\omega}) + K_\omega(S_{m,\pm\omega},S'_{m,\pm\omega}) \pm Q_\omega(C_{m,\pm\omega},S'_{m,\pm\omega}) \mp Q_\omega(S_{m,\pm\omega},C'_{m,\pm\omega}), \quad \text{(B4)}$$

$$4Q_{m,\pm\omega}(w,w') = \pm Q_\omega(C_{m,\pm\omega},C'_{m,\pm\omega}) \pm Q_\omega(S_{m,\pm\omega},S'_{m,\pm\omega}) - K_\omega(C_{m,\pm\omega},S'_{m,\pm\omega}) + K_\omega(S_{m,\pm\omega},C'_{m,\pm\omega}), \quad \text{(B5)}$$

$$\mathrm{Ph}_{m,\pm\omega}(w,w') = \tan^{-1}[Q_{m,\pm\omega}(w,w')/K_{m,\pm\omega}(w,w')], \quad \text{(B6)}$$

$$\mathrm{Coh}^2_{m,\pm\omega}(w,w') = \frac{K^2_{m,\pm\omega}(w,w') + Q^2_{m,\pm\omega}(w,w')}{P_{m,\pm\omega}(w)P_{m,\pm\omega}(w')}, \quad \text{(B7)}$$

where P, K, Q, Ph, Coh are the power, co and quadrature spectra, phase difference and coherence, respectively.

The coherence between the eastward and westward component of (Hayashi, 1977a, 1979b) is given by

$$[2P^{1/2}_{m,\omega}(w)P^{1/2}_{m,-\omega}(w)\,\mathrm{Coh}_{m,\omega}(w)]^2 = [K_\omega(C_{m,\omega},C_{m,-\omega}) - K_\omega(S_{m,\omega},S_{m,-\omega}) - Q_\omega(C_{m,\omega},S_{m,-\omega}) - Q_\omega(S_{m,\omega},C_{m,-\omega})]^2 + [Q_\omega(C_{m,\omega},C_{m,-\omega}) - Q_\omega(S_{m,\omega},S_{m,-\omega}) + K_\omega(C_{m,\omega},S_{m,-\omega}) + K_\omega(S_{m,\omega},C_{m,-\omega})]^2. \quad \text{(B8)}$$

The power spectra of standing (w^s) and traveling (w^t) wave components are given by

$$P_{m,\omega}(w^s) = 2P^{1/2}_{m,\omega}(w)P^{1/2}_{m,-\omega}(w)\,\mathrm{Coh}_{m,\omega}(w), \quad \text{(B9)}$$

$$P_{m,\pm\omega}(w^t) = P_{m,\pm\omega}(w) - \tfrac{1}{2}P_{m,\omega}(w^s). \quad \text{(B10)}$$

APPENDIX C

Orbital Tilt

The tilt of "relative" orbits as seen from the rotating earth can be determined as

$$\lambda(\theta) - \lambda(0) = \Delta\lambda(\theta) - \Omega\Delta t(\theta), \quad \text{(C1)}$$

where $\Delta\lambda(\theta)$ is the intrinsic tilt of the "absolute" orbit as seen from a non-rotating earth from latitude 0 to θ as given by (C.3), while $\Delta t(\theta)$ is the standard time interval of the orbit from latitude 0 to θ as given by (C.4). $\Omega\Delta t(\theta)$ is the longitudinal drift of the orbit relative to the rotating earth.

As illustrated by Fig. 5 for Nimbus 3, the tilt of the relative orbit is mainly due to the intrinsic tilt and partially due to the earth's rotation. Both $\Delta\lambda(\theta)$ and $\Delta t(\theta)$ reverse their sign from day to night.

On the other hand, the change of local time along the orbit is given by (2.1) and (C1) as

$$t_l(\theta) - t_l(0) = [\lambda(\theta) - \lambda(0)]/\Omega + \Delta t(\theta), \quad \text{(C2a)}$$

$$= \Delta\lambda(\theta)/\Omega. \quad \text{(C2b)}$$

Thus, in the absence of the intrinsic tilt $\Delta\lambda(\theta)$, the local time does not change along the orbit. This tilt vanishes if day and night orbits (see Fig. 2) are averaged.

The explicit expressions of $\Delta\lambda(\theta)$ and $\Delta t(\theta)$ over a sphere are given essentially based on Kodaira (1972, pp. 161–162)[4] as

$$\cos[\Delta\lambda(\theta)] = \cos J/\cos\theta, \tag{C3}$$

$$\Delta t(\theta) = [J/(2\pi)]T_0, \tag{C4}$$

where

$$\cos J = (\cos^2\theta - \cos^2 I)^{1/2}/\sin I. \tag{C5}$$

Here, I is the inclination angle of the absolute orbit, J the angular distance along the orbit from 0 to θ (see Fig. 1) and T_0 the orbital period. I, T_0, θ are specified.

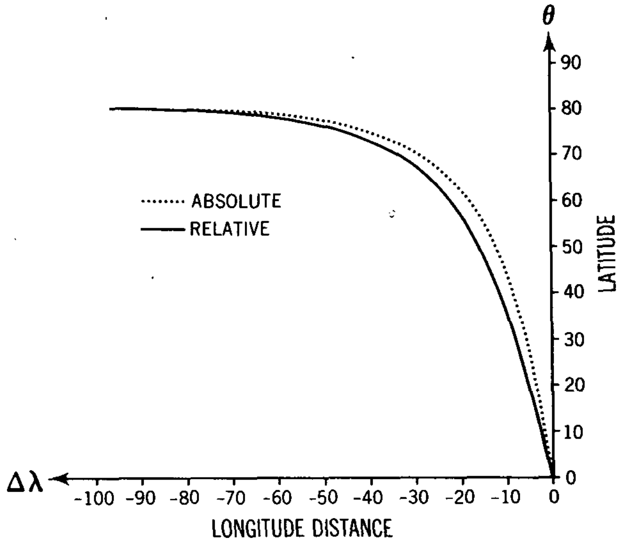

FIG. 5. Latitudinal tilt of relative (solid) orbit as seen from the rotating earth and absolute (dashed) orbit as seen from a non-rotating earth (solid) for the Nimbus 3 with orbital inclination (see Fig. 1) of 99.92° and orbital period of 107 min as calculated by Eq. (C1) with (C3)–(C5).

APPENDIX D

The Doppler Effects

This appendix reviews relativistic and non-relativistic Doppler effects essentially based on Gill (1965) and Born (1965).

The space-time coordinates (x', t') whose origin moves with a velocity U are related to the space-time coordinates (x, t) at rest by Lorentz transformations as

$$t = (t' + Ux'/c^2)/\alpha, \tag{D1}$$

$$x = (x' + Ut')/\alpha, \tag{D2}$$

where

$$\alpha = (1 - U^2/c^2)^{1/2}. \tag{D3}$$

Inserting (D1) and (D2) into a traveling wave of the form

$$\cos(\omega t + kx) = \cos(\omega' t' + k'x'), \tag{D4}$$

the wavenumber (k) and angular frequency (ω) are transformed similar to t' and x', respectively, as

$$k' = (k + U\omega/c^2)/\alpha, \tag{D5}$$

$$\omega' = (\omega + Uk)/\alpha. \tag{D6}$$

The wavenumber shift $U\omega/c^2$ in (D5) occurs because the wavelength of a traveling wave is measured at different points in space at the same time in the moving system but not at the same time in the system at rest. This time difference is due to Einstein's relativity principle of simultaneity.

In special cases, (D5) is reduced to

$$1/k' = \alpha/k, \quad \text{for} \quad \omega = 0, \tag{D7}$$

$$1/k = \alpha/k', \quad \text{for} \quad U = -\omega/k, \tag{D8}$$

$$k' = U\omega/(c^2\alpha), \quad \text{for} \quad k = 0. \tag{D9}$$

Eq. (D7) states that the wavelength of a stationary wave appears shorter to a moving observer than it appears to an observer at rest, whereas (D8) states that the wavelength of a propagating wave appears shorter to an observer at rest than it appears to an observer moving with the wave. Both statements are consistent with the principle of length contraction. Eq. (D9) means that if an observer at rest sees light propagating perpendicular ($k = 0$) to the direction of a moving observer, the moving observer feels that the light comes from a different direction ($k' \neq 0$). This is called "aberration" in astronomy.

On the other hand, the frequency shift Uk in (D6) corresponds to the classical Doppler shift by a Galilean transformation. It should be noted that this Doppler-shifted frequency ($\omega + Uk$) is *increased* by a factor of $1/\alpha$. This may appear contrary to the principle that a clock at rest appears *slow* to a moving observer. However, this is a Lagrangian point of view. From an Eulerian point of view, clocks at rest appear *fast* to a moving observer who sees *different* clocks at rest which happen to be at the same place in the moving system.

If k in (D6) is substituted from (D5), we have

$$\omega' = \omega\alpha + Uk'. \tag{D10}$$

Thus, when an observer is moving and sees a wave propagating perpendicular ($k' = 0$) to his direction, he feels that the frequency is *smaller* than when he is at rest. This is called a "transversal Doppler effect."

In the case of a Galilean transformation ($U/c \approx 0$), (D5) and (D6) are simplified as

$$k' = k, \tag{D11}$$

$$\omega' = \omega + Uk. \tag{D12}$$

[4] Kodaira, N., 1972: The general properties of meteorological satellites. Kishokenkyu Note (in Japanese), No. 111, 159–210. [available from Meteor. Soc. Japan, 1-3-4 Otemachi, Chiyodaku, Tokyo].

Here a shift occurs not in wavenumber but in frequency.

REFERENCES

Bendat, J. S., and A. G. Piersol, 1971: *Random Data: Analysis and Measurement Procedures*. Wiley-Interscience, 407 pp.

Born, M., 1976: *Einstein's Theory of Relativity*. Dover, 372 pp.

Cadet, D., and H. Teitelbaum, 1979: Observational evidence of internal inertia-gravity waves in the tropical stratosphere. *J. Atmos. Sci.*, **36**, 892–907.

Chapman, W. A., M. J. Cross, D. A. Flower, G. E. Peckham and S. D. Smith, 1974: A spectral analysis of global atmospheric temperature fields observed by the Selective Chopper Radiometer on the Nimbus 4 Satellite during the year 1970–1. *Proc. Roy. Soc. London*, **A338**, 57–76.

——, and J. McGregor, 1978: The application of complex demodulation to meteorological satellite data. *Quart. J. Roy. Meteor. Soc.*, **104**, 213–223.

Coy, L., 1979: A possible 2-day oscillation near the tropical stratopause. *J. Atmos. Sci.*, **36**, 1615–1618.

Gill, T. P., 1965: *The Doppler Effect*. Logos Press, Academic Press, 149 pp.

Hartmann, D. L., 1976: The structure of the stratosphere in the Southern Hemisphere during late winter 1973 as observed by satellite. *J. Atmos. Sci.*, **33**, 1141–1154.

Hayashi, Y., 1971: A generalized method of resolving disturbances into progressive and retrogressive waves by space Fourier and time cross-spectral analyses. *J. Meteor. Soc. Japan*, **49**, 125–128.

——, 1973: A method of analyzing transient waves by space-time cross spectra. *J. Appl. Meteor.*, **12**, 404–408.

——, 1977a: On the coherence between progressive and retrogressive waves and a partition of space-time power spectra into standing and traveling parts. *J. Appl. Meteor.*, **16**, 368–373.

——, 1977b: Space-time power spectral analysis using the maximum entropy method. *J. Meteor. Soc. Japan*, **55**, 415–420.

——, 1979a: Space-time spectral analysis of rotary vector series. *J. Atmos. Sci.*, **36**, 757–766.

——, 1979b: A generalized method of resolving transient disturbances into standing and traveling waves by space-time spectral analysis. *J. Atmos. Sci.*, **36**, 1017–1029.

Hirota, I., 1976: Seasonal variation of planetary waves in the stratosphere observed by the Nimbus 5 SCR. *Quart. J. Roy. Meteor. Soc.*, **102**, 757–770.

——, 1979: Kelvin waves in the equatorial middle atmosphere observed by the Nimbus 5 SCR. *J. Atmos. Sci.*, **36**, 217–222.

Jenkins, G. M., and D. G. Watts, 1968: *Spectral Analysis and Its Applications*. Holden-Day, 525 pp.

Julian, P. R., 1971: Some aspects of variance spectra of synoptic scale tropospheric wind components in midlatitudes and in the tropics. *Mon. Wea. Rev.*, **99**, 954–965.

Rodgers, C. D., 1976: Evidence for the five-day wave in the upper stratosphere. *J. Atmos. Sci.*, **33**, 710–711.

Schickedanz, P. T., and E. G. Bowen, 1977: The computation of climatological power spectra. *J. Appl. Meteor.*, **16**, 359–367.

Shapiro, R., 1972: Information loss and compensation in linear interpolation. *J. Comput. Phys.*, **10**, 65–84.

Venne, D. E., and J. L. Stanford, 1979: Observation of a 4-day temperature wave in the polar winter stratosphere. *J. Atmos. Sci.*, **36**, 2016–2019.