Modified Methods of Estimating Space-Time Spectra from Polar-Orbiting Satellite Data

Part I: The Frequency Transform Method

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Abstract

A simplification is made of the nonorthogonal Fourier transform method (Salby, 1982) for estimating space-time spectra from uneven twice-daily data sampled by a polar-orbiting satellite. The modified method transforms frequency only by the use of the Galilean transformation, while Salby's method transforms both frequency and wavenumber. Space-time spectra are obtained from the time-Fourier transform with respect to the Doppler-shifted frequency as viewed from the satellite. The wavenumber-frequency aliasing characteristics are examined and the computer code is exemplified.

1. Introduction

Space-time spectral analysis is a powerful tool for studying the dynamics of large-scale atmospheric waves. By this analysis, waves are decomposed into eastward and westward moving wavenumber-frequency components and their structure and energetics can be examined (see Hayashi, 1982 for a review of the methods and their applications). However, as pointed out by Hartmann (1976), a direct application of the conventional methods (e.g. Hayashi, 1971) of space-time spectral analysis to polar-orbiting satellite data suffers from serious errors in the spectra of waves with periods shorter than 5 days or so. This is because these data are sampled at the same local time but different hours of the day due to the earth's rotation relative to the orbit of the satellite (see Fig. 1).

Instead of applying the conventional space-time spectral analysis, Chapman *et. al.* (1974) computed the power spectra of time series data which are viewed from a westward drifting sun-synchronous satellite as indicated by the dots along the disconnected slanted lines in Fig. 2a. These data are associated with the Doppler-shifted frequency[1)] F which is equal to $f-m$ and is contributed to by various combinations of wavenumber m and frequency f(day^{-1}). If only one pair of f and m is dominant and known, these time spectra can also be regarded as space-time spectra. This method, however, cannot use twice-daily data to estimate the space-time spectra of high frequency waves such as the observed

1) This F itself is ambiguous, since it can be one of the aliased frequencies due to discrete sampling (see Section 2.1 for detail).

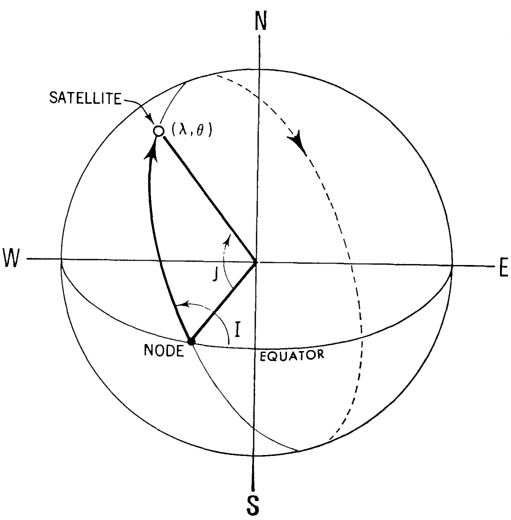

Fig. 1 The orbit of a polar-orbiting sun-synchronous satellite. Twice daily observations are available on the day and night sides. The orbits drift 360° per solar day relative to the rotating earth (after Hayashi, 1980).

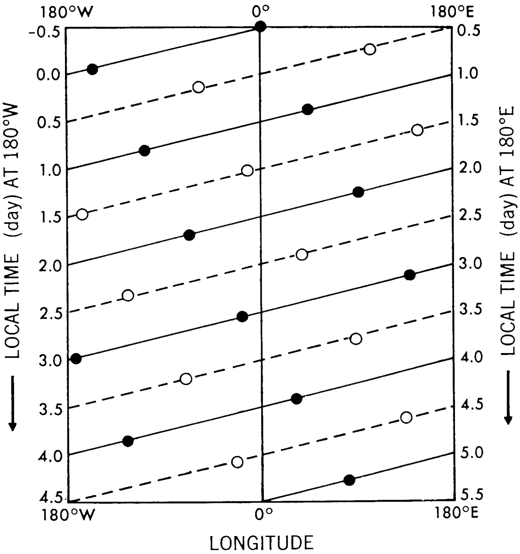

Fig. 2a Longitude-time section of the position of a hypothetical satellite with 2.3 orbits (actually 12∼14 orbits) per day at midnight (full circle) and noon (open circle) at the equator. Solid and dashed lines connect the night and day side points, respectively. These lines are continuous at 180°. The numerals indicate local time at 180°W (left) and 180°E (right) (after Hayashi, 1980).

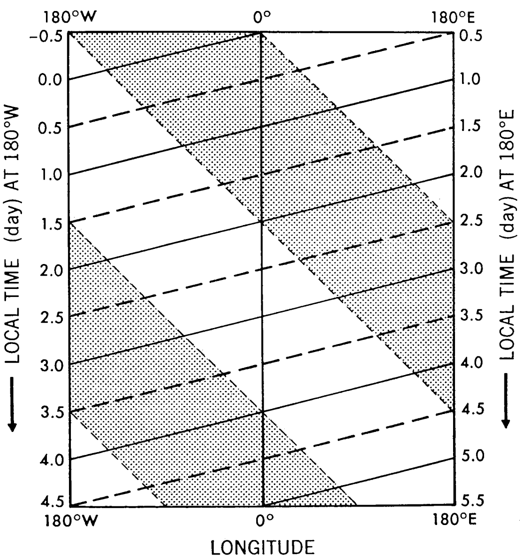

Fig. 2b Longitude-time section of an eastward moving wave (indicated by shading) with wavenumber 1 and a period of 4 days. The wavenumber 1 is measured as 1.25 along the slanted lines connecting the same local time (after Hayashi, 1980).

2-day period planetary waves (Rogers and Prata, 1981; Salby, 1981). Moreover, once-daily data are associated with more aliasing errors than twice-daily data.

On the other hand, Hayashi (1980) modified the conventional method of Hayashi (1971) in order to estimate space-time spectra correctly from asynoptic satellite data which are sampled at the same local time. When these data are regarded as spatial data, the wavenumber m is shifted by the frequency f (see Fig. 2b), as pointed out by Hartmann (1976). Hayashi (1980) showed that the correct spectra can be retrieved by a time spectral analysis of the Fourier transform of the asynoptic field with respect to the frequency-shifted wavenumber $(m-f)$. This method assumes that twice-daily satellite data are of equal time increments. However, this assumption does not hold well in high latitudes due to the orbital tilt of the sun-synchronous satellite. For example, Nimbus III sample data at intervals of 10 and 14 hours at 60°N. This method also assumes that asynoptic data are somehow interpolated to regular longitude-latitude grid points. A simple linear interpolation will greatly distort the high wavenumbers components.

Recently, Salby (1982) discussed a sampling theory for asynoptic satellite observations and proposed an ingeneous method of retrieving space-time spectra and synoptic fields from uneven twice-daily satellite data. This method rotates both space and time axes and transforms both wavenumber and frequency. This transformation is similar to but different from the relativistic Lorentz transformation. The transformed "space"-like data points consist of only two uneven interval points, while the transformed "time"-like data points consist of many equal interval points. This data configuration gives only two transformed "wavenumbers" for each transformed "frequency" in a resolvable wavenumber-frequency range. The non-orthogonal two-point Fourier series are correctly inverted by solving for two unknown space-time Fourier coefficients for each transformed "frequency." Synoptic fields can then be retrieved by a space-time Fourier resynthesis.

In order to make use of twice-daily data, however, there is no need to transform *both* space and time coordinates. In Part I of the present paper, Salby's non-orthogonal Fourier transform method is simplified by transforming frequency only by the use of the Galilean transformation as in Chapman *et al.* (1974). In Part II (Hayashi, 1983), the wavenumber transform method of Hayashi (1980) is also modified by the use of the non-orthogonal Fourier inversion. It is also the purpose of the present paper to describe the computational scheme in detail.

In Section 2 the modified frequency transform method is described. In Section 3, this method is tested with respect to its aliasing characteristics. Summary and remarks are given in Section 4. Appendix A lists symbols, while Appendix B lists the computer code of this method. Appendix C gives space-time spectral formulas.

2. The modified frequency transform method

2.1 Space-time Fourier series

The space-time Fourier series for continuous longitude λ and time t is given by

$$w(\lambda, t)=R_e \sum_{m=0}^{\infty} \sum_{f=-\infty}^{\infty} \delta_m \hat{w}(m, f)\exp(im\lambda+i2\pi ft) \tag{2.1a}$$

$$=\frac{1}{2} \sum_{m=-\infty}^{\infty} \sum_{f=-\infty}^{\infty} \hat{w}(m, f)\exp(im\lambda+i2\pi ft), \tag{2.1b}$$

where $\hat{w}(m, f)$ are the space-time Fourier coefficients and $\delta_m=1$ except for $\delta_0=0.5$. Positive and negative frequencies f indicate westward and eastward phase velocities, respectively, for positive wavenumber m.

2.2 Space coordinate transformation

The zonal coordinate λ' whose origin moves westward relative to the earth with the zonal angular velocity (2π day^{-1}) of a sun-synchronous satellite[2)] is related to λ as

$$\lambda=\lambda'-2\pi t, \tag{2.2}$$

where λ' coincides with λ at $t=0$

Inserting (2.2) into (2.1b) gives

$$w(\lambda'-2\pi t, t)$$
$$=\frac{1}{2} \sum_{m=-\infty}^{\infty} \sum_{F=-\infty}^{\infty} \hat{w}(m, f)\exp(im\lambda'+i2\pi Ft), \tag{2.3}$$

where F is the Doppler-shifted frequency (day^{-1}) measured along the rotated time axis (slanted lines in Fig. 2a) and is given by

$$F=f-m. \tag{2.4}$$

When data are sampled at discrete t, aliasing occurs in (2.3) as

$$\exp[i2\pi(F_0+2lF_N)t]=\exp(i2\pi F_0 t), \tag{2.5}$$

where l is an arbitrary integer.

The Nyquist frequency F_N is given by

$$F_N=N/(2T), \tag{2.6}$$

where N is the number of data in T days.

2.3 Space-Fourier series

By virtue of orthogonality and (2.5), taking a discrete time Fourier transform of (2.3) with respect to $F=F_0$ gives

$$\tilde{w}(\lambda', F_0)=\sum_{m=-\infty}^{\infty} \sum_{f=-\infty}^{\infty} \hat{w}(m, f)\exp(im\lambda'), \tag{2.7}$$

where

$$\tilde{w}(\lambda', F_0)\equiv\frac{2}{N} \sum_{t=0}^{T} w(\lambda'-2\pi t, t)\exp(-i2\pi F_0 t), \tag{2.8}$$

and the summation in (2.7) is taken over such f and m that satisfy

$$f-m=F=F_0+2lF_N, \tag{2.9}$$

for a given F_0 and arbitrary integers l.

In order to determine the coefficients $\hat{w}(m, f)$ in (2.7) uniquely from two sets of ascending and descending branches of satellite data, these coefficients are assumed to be zero except for

$$|f|<f_N, \tag{2.10}$$

and

$$|m|<m_N, \tag{2.11}$$

where f_N is the Nyquist frequency of twice-daily data and m_N is the largest integer satisfying $m_N \leq F_N$. $2F_N$ is defined by (2.6) and is equal to the average number of data in a longitude circle sampled during one day.

As illustrated in Fig. 3, only one or two pairs of integer wavenumber and frequency satisfy (2.10) and (2.11) for given F_0 and arbitrary l in (2.9). If the coefficients of the Nyquist wavenumber were *not* assumed to be zero, the Nyquist region should be parallel to the isoline of $m-f$ as in Salby (1982). On the other hand, there is no reason why the Nyquist region should *not* be parallel to the wavenumber axis, although Salby (1982) oriented this region in the direction of aliasing.

The above wavenumber-frequency pairs (p, f_p) and (q, f_q) can be explicitly determined, following Salby (1982), as

First, integer p, integer Tf_p, and F_0 are chosen

2) For a non-sunsynchronous satellite which drifts around the earth once per δ day ($\delta \neq 1$), time and period must be measured in units of δ days to apply the present method. (See Fig. 1).

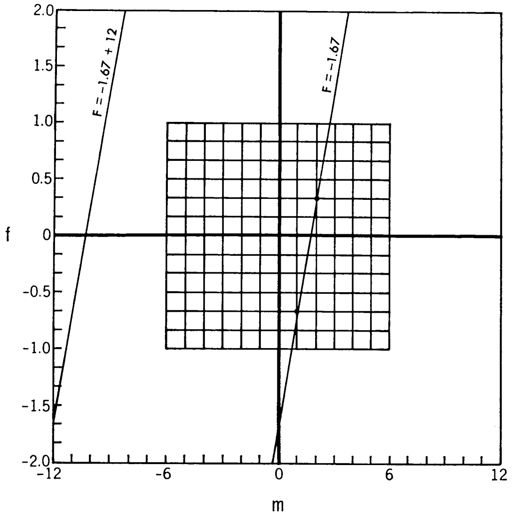

Fig. 3 Wavenumber-frequency points in a wavenumber-frequency domain. The mesh indicates the region bounded by the isolines of the Nyquist wavenumber and frequency (day^{-1}) of discrete data (twice-daily in 6 days and 12 points in longitude). The slanted lines are the isolines of the Doppler-shifted frequencies ($F=-1.67$) and its aliased value ($F=-1.67+12$). The dots indicate two pairs of wavenumber-frequency point which fall at integer wavenumbers inside the Nyquist region.

to satisfy

$$0 \leqq p < m_N, \tag{2.12}$$

$$|f_p| < f_N, \tag{2.13}$$

and

$$F_0 = f_p - p. \tag{2.14}$$

Second, (q, f_q) is determined as

$$(q, f_q) = (p-1, f_p-1) \text{ for } f_p > 0 \tag{2.15a}$$

$$= (p+1, f_p+1) \text{ for } f_p \leqq 0, \tag{2.15b}$$

where the choice of the sign in (2.15) is to satisfy $|f_q| < f_N$.

Since $\hat{w}(m, f)$ is zero except for the above two pairs, (2.7) can be truncated as

$$\tilde{w}(\lambda_a', F_0) = \hat{w}(p, f_p)\exp(ip\lambda_a') + \hat{w}(q, f_q)\exp(iq\lambda_a'), \tag{2.16}$$

$$\tilde{w}(\lambda_d', F_0) = \hat{w}(p, f_p)\exp(ip\lambda_d') + \hat{w}(q, f_q)\exp(iq\lambda_d'), \tag{2.17}$$

where λ'_a and λ'_d are given by the longitude of the ascending and descending branches of the orbits at $t=0$.

2.4 *Nonorthogonal Fourier inversion*

Since the above Fourier series with uneven data increments are non-orthogonal, the coefficients must be determined by explicitly solving the above two equations, following Salby (1982), as

$$\hat{w}(p, f_p)\Delta = \tilde{w}(\lambda_a', F_0)\exp(iq\lambda_d') - \tilde{w}(\lambda_d', F_0)\exp(iq\lambda_a'), \tag{2.18}$$

$$\hat{w}(q, f_q)\Delta = \tilde{w}(\lambda_d', F_0)\exp(ip\lambda_a') - \tilde{w}(\lambda_a', F_0)\exp(ip\lambda_d'), \tag{2.19}$$

where

$$\Delta = \exp(ip\lambda_a' + iq\lambda_d') - \exp(ip\lambda_d' + iq\lambda_a'), \tag{2.20}$$

and

$$q = p-1 \text{ for } f_p > 0 \tag{2.21a}$$

$$= p+1 \text{ for } f_p \leqq 0. \tag{2.21b}$$

Eqs. (2.18) and (2.19) correspond to Eqs. (39.1) and (39.2) of Salby (1982).

If the twice-daily data are of equal interval ($\lambda_a'=0$, $\lambda_d'=\pi$), the above coefficients are reduced to

$$\hat{w}(p, f_p) = [\tilde{w}(\lambda_a', F_0) + \tilde{w}(\lambda_d', F_0)]/2, \tag{2.22}$$

$$\hat{w}(q, f_q) = [\tilde{w}(\lambda_a', F_0) - \tilde{w}(\lambda_d', F_0)]/2, \tag{2.23}$$

where p and q are odd and even numbers, respectively.

These coefficients are just the half sum and difference of the Fourier transforms of ascending and descending data as expected intuitively.

For once-daily data, the coefficients $\hat{w}(p, f_p)$ are given by

$$\hat{w}(p, f_p) = \tilde{w}(\lambda_a', F_0)\exp(-ip\lambda_a'), \tag{2.24}$$

or

$$\hat{w}(p, f_p) = \tilde{w}(\lambda_d', F_0)\exp(-ip\lambda_d'), \tag{2.25}$$

where $\hat{w}(p, f_p)$ have been determined uniquely from (2.7) by assuming that the space-time Fourier components are zero except for $|p| < m_N$ and $|f| < 0.5$.

2.5 *Time-Fourier transform*

Since the initial data points are not exactly at the same synoptic time due to the finite orbital velocity of a satellite, t must be replaced by t', defined by

$$t = t' + \tau_\lambda, \tag{2.26}$$

where τ_λ is a small time correction ($\sim$1 hour) for the initial data.

Inserting (2.26) and $F_0 = f - m$ into (2.8) gives

$$\tilde{w}(\lambda', F_0) = \frac{2}{N}\left\{\sum_{t'=0}^{T} w(\lambda' - 2\pi t, t)\exp(-i2\pi F_0 t')\right\} \times \exp(-i2\pi F_0 \tau_\lambda) \tag{2.27a}$$

$$=\frac{2}{N}\left\{\sum_{t'=0}^{T}[w(\lambda'-2\pi t, t)\exp(i2\pi mt')]\right.$$
$$\left.\times\exp(-i2\pi ft')\right\}\exp(-i2\pi F_0\tau_\lambda). \qquad (2.27b)$$

The Fourier transform (2.27a) is made with respect to the Doppler-shifted frequency F_0 in the t' coordinate. On the other hand, the Fourier transform (2.27b) can be made with respect to the ordinary frequency f with w multiplied by a factor $\exp(i2\pi fmt')$.

2.6 Computational procedure

The computational procedure of the modified frequency transform method is summarized as follows (see Appendix B for the computer code).

1) Compute the Fourier coefficients (2.27) of day and night time data in the range $0\leq m<m_N$ and $|f|<1$ with $F_0=f-m$.

2) Compute the space-time Fourier coefficients (2.18) with $p=m$ and q defined by (2.21).

3) Space-time cross spectra are obtained from the space-time Fourier coefficients by the use of formulas given in Appendix C.

In order to reduce leakage, the original time series should be tapered at each end (see Bendat and Piersol, 1971, p. 323). An improved tapering procedure is suggested by Garcia and Geisler (1981, p. 2196).

3. Test of the method

The input wave is given by

$$w(\lambda, t)=2\cos(m\lambda+2\pi ft), \qquad (3.1)$$

which consists of a pair of complex conjugates as

$$w(\lambda, t)=\exp(im\lambda+i2\pi ft)$$
$$+\exp(-im\lambda-i2\pi ft). \qquad (3.2)$$

The satellite sampling of (3.1) is given by use of (2.2) and (2.26) as

$$w(\lambda', t')=2\cos[m\lambda'+2\pi F(t'+\tau_\lambda)], \qquad (3.3)$$

The response to this input has actually been computed by use of the conventional and the modified method.

Fig. 4 shows aliased wavenumber-frequency points for an ordinary space-time spectral analysis applied to once-daily synoptic data ($T=6$ days and 12 longitudinal points). The dots and crosses which are symmetric with respect to the origin represent complex conjugates. For example, one of the conjugate pairs $(m, f)=(\pm3, \pm1.33)$ is indicated by a large dot and a large cross. The mesh indicates the Nyquist wave-

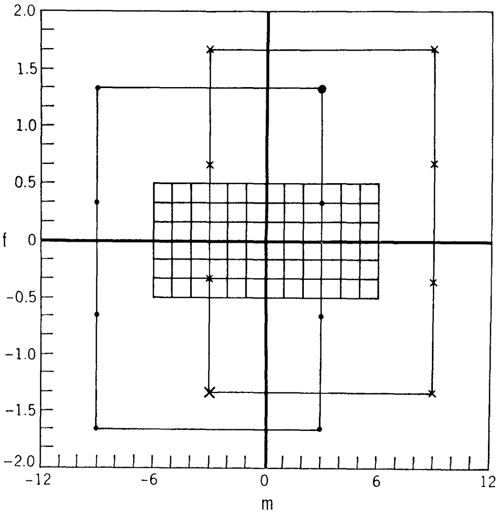

Fig. 4 Aliased wavenumber-frequency points for an ordinary space-time spectral analysis applied to a regular space-time sampling (once daily in 6 days and 12 points in longitude). The mesh represents the Nyquist wavenumber-frequency region.

number-frequency region. When the input is given on any of the points outside the mesh, it is aliased to the points inside the mesh.[3] These input and output points are located at the vector intervals of $(2m_N, 2f_N)=(12, 1)$. In a one-sided domain where the signs of wavenumber and frequency are disregarded, two of the dots and crosses become mirror images with respect to the multiples of m_N and f_N.

Fig. 5 shows the aliased wavenumber-frequency points for the modified frequency transform method (2.24) applied to the once-daily artificial asynoptic data ($T=6$ days and $N=6\times12$). As expected from (2.9), aliasing occurs along the slanted isolines of $f-m=$constant at vector intervals of $(1, 2f_N)=(1, 1)$. Additional aliasing occurs at intervals of $(0, 2F_N)=(0, 12)$. This aliasing occurs in parallel with the f axis and the aliased points fall outside the outer frame of Fig. 5. A linear combination of these two intervals also results in $(2m_N, 2m_N-2F_N)=(12, 0)$ which turns out to be parallel to the m-axis for this particular example ($m_N=F_N=$integer) as illustrated by Fig. 5. The aliasing configuration of the modified method for once-daily data is identical to that of the frequency transform

3) If the input frequency does not coincide with one of the discrete frequencies of the finite length data, the response spreads to the adjacent frequencies.

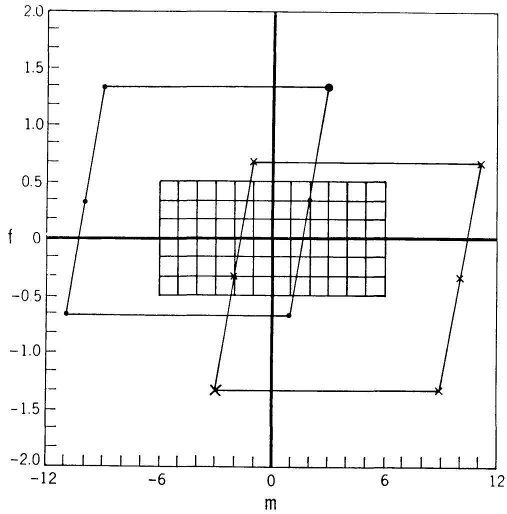

Fig. 5 As in Fig. 4 except for the frequency transform method applied to artificial satellite data (once daily in 6 days and 12 points in longitude). The slanted lines indicate the isolines of the Doppler shifted frequency ($F=F_0+2F_N$).

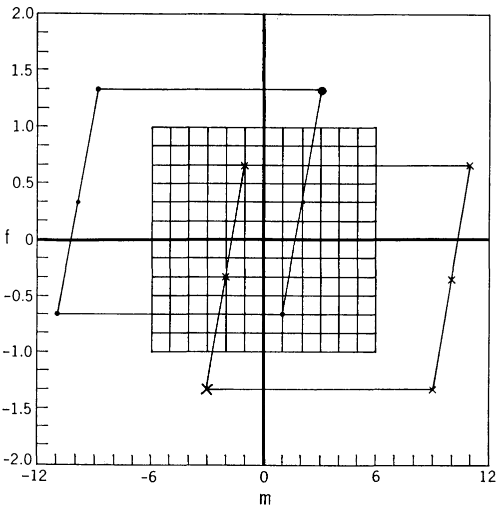

Fig. 6 As in Fig. 4 (frequency transform method) except for uneven twice-daily data. This aliasing configuration is for the input given outside the Nyquist region. (When the input is given inside, no aliasing occurs inside.)

method (Chapman *et al.* 1974) and the wavenumber transform method (Hayashi, 1980, 1982) for once-daily data. When the frequency is transformed, the wavenumber axis is rotated to be parallel to the slanted isolines. When the wavenumber is transformed, the frequency axis is rotated to be parallel to the slanted isolines.

Fig. 6 is the same as Fig. 5 (modified method) except for uneven twice-daily data and the input which is given *outside* the Nyquist region. This input is aliased to the two points inside this region with their magnitude altered. In particular, a diurnal oscillation and its harmonics are aliased to one another and also to the zero frequency along the slanted isoline. When the input is given only inside the Nyquist region, no aliasing occurs inside this region (not illustrated). The two aliasing directions of the frequency transform method are not necessarily perpendicular to each other, while those of the wavenumber-frequency transform method of Salby (1982) are perpendicular.

Fig. 7 shows the space-time power spectra of

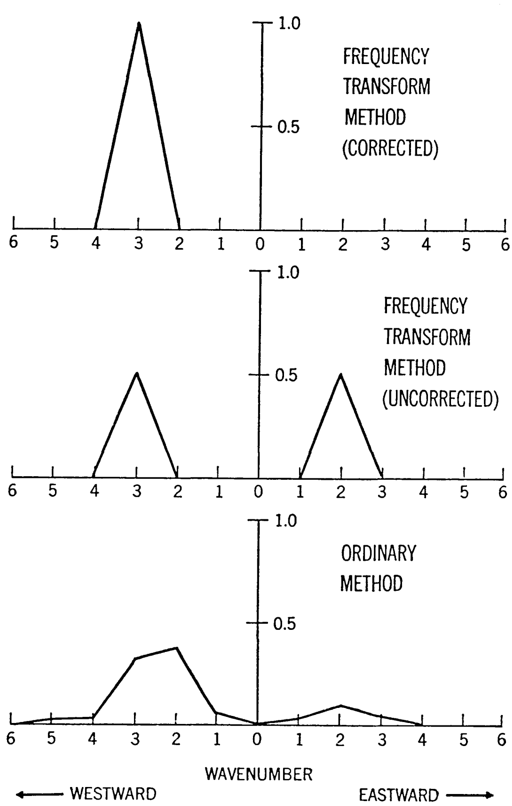

Fig. 7 Wavenumber distributions (period=2 days) of the space-time power spectra of sinusoidal waves (wavenumber=3, period=2 days, westward moving) estimated from asynoptic data with time increments of 6 and 18 hours. The frequency transform method with (top) and without (middle) a correction for uneven time increments. The ordinary method (bottom).

a given sinusoidal wave which are estimated from asynoptic data with uneven time intervals of 6 and 18 hours. These spectra are correctly estimated by the frequency transform method with a correction for the uneven time increments (top), whereas they are aliased without this correction (middle). They are also aliased by the ordinary method (Hayashi, 1971) of space-time spectral analysis (bottom).

4. Summary and remarks

A simplification is made of the non-orthogonal Fourier transform method of Salby (1982) for estimating space-time spectra from uneven twice-daily data sampled by a polar-orbiting satellite. The modified method transforms frequency only by the use of the Galilean transformation, while Salby's method transforms both frequency and wavenumber. The modified method is summarized as follows:

1) The space-time spectra are obtained from a linear combination of the time-Fourier transforms of day and night data which are associated with the Doppler-shifted frequency as viewed from a satellite.

2) The non-orthogonal Fourier series with uneven increments is inverted by explicitly solving for its coefficients. These coefficients are uniquely determined by assuming that the spectra are confined in the resolvable wavenumber-frequency region.

3) Aliasings occur among wavenumber-frequency points when the input wave is imposed outside the resolvable wavenumber-frequency region. When the input wave is imposed inside this region, no aliasing occurs within this region.

The modified frequency transform method is convenient, when the original satellite data are available. This method is useful not only for analyzing short period planetary waves but for correctly isolating long period waves. It gives the correct spectra except for the zero and diurnal frequencies, even if a diurnal oscillation and its harmonics are present.

The modified frequency transform method uniquely determines wavenumber-frequency spectra from *once*-daily data alone by assuming that these spectra are confined in the resolvable wavenumber-frequency range. Without this assumption, Chapman *et al.*'s (1974) frequency transform method does not by itself determine the spectra uniquely from once-daily data.

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Appendix A

Symbols

θ : Latitude,
λ : zonal coordinate,
λ' : moving zonal coordinate ($\lambda=\lambda'-2\pi t$),
$\lambda_a'(\theta)$, $\lambda_d'(\theta)$: longitudes of ascending and descending orbits at $t=0$,
t : time (days, see footnote [2])
t' : time relative to the initial data point ($t=t'+\tau_\lambda$),
$\tau_a(\theta)$, $\tau_d(\theta)$: time corrections for the initial data points of ascending and descending orbits
T : length of time series (days, see footnote[2])
N : number of data points in T days,
f : frequency (day^{-1}, see footnote[2])
f_N : Nyquist frequency of f,
F : Doppler-shifted frequency ($F=f-m$),
F_N : Nyquist frequency of $F(2F_N=N/T)$,
m : wavenumber,
m_N : Nyquist wavenumber of $m(m_N\leq F_N)$.
(p, f_p), (q, f_q) : wavenumber-frequency pairs associated with the same $F(f_p-p=f_q-q=F)$.

Appendix B

Computer Code of the Frequency Transform Method

SUBROUTINE STFC (List 1) computes space-time Fourier coefficients (2.18) from twice-daily sunsynchronous satellite data (see footnote 2 for non-sunsynchronous satellite data) for a given m and f by use of SUBROUTINE TFC (List 2) which computes time-Fourier coefficients (2.27a). When SUBROUTINE STFC is called for the first time, set ITIME$=1$ to compute and store cosine and sine functions in SUBROUTINE TFC.

The input arguments are given by

$N=$closest even number of data points during IT days,

$$XA=\lambda_a', \quad XD=\lambda_d'$$
$$TA=\tau_a, \quad TD=\tau_d$$
$$WA(I)=w(\lambda_a, t'), \quad WD(I)=w(\lambda_d, t')$$
$$M=m, \ (0\leq m<m_N)$$

and

List 1

```
      SUBROUTINE STFC(ITIME,N,IT,XA,XD,
     *TA,TD,WA,WD,M,IF,F,WMF)
      PARAMETER ID=N
      DIMENSION WA(ID),WD(ID)
      COMPLEX CI,C1,C2,WAF,WDF,WMF
      CI=(0.0,1.0)
      PI=3.141592653
      F=FLOAT(IF)/FLOAT(IT)
      KF=IF-M*IT
      CALL TFC(ITIME,PI,CI,N,WA,KF,WAF)
      CALL TFC(ITIME,PI,CI,N,WD,KF,WDF)
      P=M
      Q=M-1
      IF(IF.LE.0) Q=M+1
      C1=CEXP(CI*Q*XD+CI*2.0*PI*(P-F)*TA)
      C2=CEXP(CI*Q*XA+CI*2.0*PI*(P-F)*TD)
      WMF=WAF*C1-WDF*C2
      C1=CEXP(CI*P*XA+CI*Q*XD)
      C2=CEXP(CI*P*XD+CI*Q*XA)
      WMF=WMF/(C1-C2)
      RETURN
      END
```

List 2

```
      SUBROUTINE TFC(ITIME,PI,CI,N,W,KF,WF)
      PARAMETER ID=N
      COMPLEX CI,WF
      DIMENSION  W(ID),C(ID),S(ID)
      NH=N/2
      IF(ITIME.GE.2) GO TO 15
      DO 10 I=1,N
      RAD=FLOAT(I-1)*PI/FLOAT(NH)
      C(I)=COS(RAD)
   10 S(I)=SIN(RAD)
      ITIME=2
   15 CS=0.0
      SN=0.0
      DO 20 I=1,N
      J=IABS(KF)*(I-1)
      MI=MOD(J,N)+1
      CS=CS+W(I)*C(MI)
   20 SN=SN+W(I)*S(MI)
      WF=(CS-CI*SN)/FLOAT(NH)
      IF(KF.LT.0) WF=CONJG(WF)
      RETURN
      END
```

$$IF=fT(-1<f<1)\,.$$

The output arguments are given by

$$F=f(\mathrm{day}^{-1})$$

and

$$WMF=\hat{w}_{m,f}(\mathrm{complex})$$

Appendix C

Space-time Cross Spectra

The space-time power spectra $P_{m,f}$, cospectra $K_{m,f}$, quadrature spectra $Q_{m,f}$, phase difference $Ph_{m,f}$, and coherence $Coh_{m,f}$ between two sets of space-time series (w, w') are given by

$$P_{m,\pm f}(w)=\delta_m\langle|w_{m,\pm f}|^2\rangle\,, \tag{C.1}$$

$$K_{m,\pm f}(w,w')=\delta_m R_e\langle\hat{w}^*_{m,\pm f}\hat{w}'_{m,\pm f}\rangle\,, \tag{C.2}$$

$$Q_{m,\pm f}(w,w')=\delta_m I_m\langle\hat{w}^*_{m,\pm f}\hat{w}'_{m,\pm f}\rangle\,, \tag{C.3}$$

$$Ph_{m,\pm f}(w,w')=\arg[\langle\hat{w}^*_{m,\pm f}\hat{w}'_{m,\pm f}\rangle]\,, \tag{C.4}$$

and

$$Coh^2_{m,\pm f}(w,w')=\frac{|\langle\hat{w}^*_{m,\pm f}\hat{w}'_{m,\pm f}\rangle|^2}{\langle|\hat{w}_{m,\pm f}|^2\rangle\langle|\hat{w}'_{m,\pm f}|^2\rangle}\,, \tag{C.5}$$

where the asterisk denotes the complex conjugate and the angle brace denotes ensemble average which can be replaced by a narrow frequency band average. ($\delta_m=0.5$ except for $\delta_0=0.25$).

In addition, the coherence $Coh_{m,f}(w)$ between eastward and westward moving components (Hayashi, 1977, 1979) is given by

$$Coh^2_{m,f}(w)=\frac{|\langle\hat{w}^*_{m,+f}\hat{w}^*_{m,-f}\rangle|^2}{\langle|\hat{w}_{m,+f}|^2\rangle\langle|\hat{w}^*_{m,-f}|^2\rangle}\,. \tag{C.6}$$

The power spectra of standing (w^s) and traveling (w^t) wave components are given by use of this coherence as

$$P_{m,f}(w^s)=2P^{1/2}_{m,f}(w)P^{1/2}_{m,-f}(w)Coh_{m,f}(w)\,, \tag{C.7}$$

and

$$P_{m,\pm f}(w^t)=P_{m,\pm f}(w)-P_{m,f}(w^s)/2\,. \tag{C.8}$$

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時空間スペクトルを極軌道衛星データから求める修正方法

第1部：振動数変換法

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極軌道衛星により採集された1日2回の不等時間隔データから時空間スペクトルを求める非直交フーリエ変換法 (Salby, 1982) を簡単化した。Salby の方法は振動数と波数の両方を変換するのに対し，この修正方法はガレリー変換により振動数だけを変換すれば良い。波数—振動数 aliasing の性質を調べ，計算機プログラムも例示した。