

A Modification of the Atmospheric Energy Cycle

YOSHIKAZU HAYASHI

Geophysical Fluid Dynamics Laboratory/NOAA, Princeton University, Princeton NJ 08542

(Manuscript received 27 October 1986, in final form 24 January 1987)

ABSTRACT

A modification is made of the conventional energy cycle by combining the eddy flux convergence and the mean meridional circulation terms in the mean momentum and heat equations. The combined terms are interpreted as the effective flux convergences in the extratropics where the steady state mean circulation is regarded as essentially being induced by eddies. In the presence of mean heating, the modified energy cycle is simpler and less misleading than the transformed energy cycle based on the transformed Eulerian-mean equations.

This modification suggests that the major energy source of tropospheric planetary waves can be traced to the thermal generation of mean potential energy and that the stratospheric planetary wave is maintained by the total (mean plus eddy) vertical flux of energy from the troposphere. The conventional energy cycle of observed tropospheric planetary waves is, however, not as complicated as that of theoretical planetary waves in the quasi-nonacceleration condition. This is due to the fact that the observed tropospheric eddy heat flux convergence is counterbalanced by the mean heating and does not induce a large mean circulation in the steady state.

1. Introduction

It has been recognized that the zonal mean meridional circulation is not only forced by mean heating but also induced by eddies (Eliassen, 1952; Phillips, 1954; Kuo, 1956) to maintain the mean geostrophic and hydrostatic balance. Under the nonacceleration condition (no dissipation, no wave transience, and no wave singularity), the eddy momentum flux convergence does not result in accelerating the mean flow (Charney and Drazin, 1961; Andrews and McIntyre, 1976; Boyd, 1976; Nakamura, 1979). This convergence is counterbalanced by the Coriolis force due to the mean circulations induced by the eddy heat and momentum flux convergences. The conventional momentum balance in the quasi-nonacceleration condition is complicated by the presence of this induced circulation. Although the induced circulation by itself accelerates easterlies aloft, the combined effect of the eddy flux convergence and the induced circulation depends on their relative magnitudes. If the mean-induced circulation dominates the eddy momentum convergence, the combined effect is to accelerate the mean easterlies. If the adiabatic cooling due to ascending mean motion dominates the eddy heat flux convergence, the combined effect is to cool the mean temperature.

In order to describe the net effect of eddies on the mean flow in a simple manner, Andrews and McIntyre (1976) transformed the conventional Eulerian-mean equations by the use of the Eliassen-Palm flux convergence (EPFC) of Eliassen and Palm (1960) and a residual mean circulation. The EPFC consists of the eddy

momentum flux convergence and the vertical derivative of the eddy heat flux, which effectively acts like a momentum convergence. The residual circulation is formally defined as that part of a mean circulation which is not balanced by the eddy heat flux convergence. The residual circulation is not only forced by the mean heating but is also induced by eddies. Under certain conditions (Andrews and McIntyre, 1976; Hayashi, 1985a), the EPFC dominates the induced residual mean circulation and approximates the mean acceleration due to eddies. In this case, the induced circulation can be interpreted as being mainly due to the eddy heat flux convergence. Although this approximation holds rather well for stratospheric planetary waves, it does not hold well for tropospheric transient waves due to the influence of the lower boundary (Hayashi, 1985a; Pfeffer, 1987). The transformed Eulerian-mean equations are also interpreted as approximating the Lagrangian-mean equations, provided that the Stokes drift can be approximated by the eddy heat flux divergence (Andrews and McIntyre, 1978; Matsuno and Nakamura, 1979). Although this approximation holds for stratospheric planetary waves, it does not hold well for baroclinically unstable waves (Uryu, 1979).

The induced Eulerian-mean and residual mean circulations complicate the energy cycle. Under the nonacceleration condition, energy is transferred among the mean and eddy energies in such a way as to have no deposit of energy (Dickinson, 1969). In order to describe in a simple manner how the mean flow will change due to eddies, Plumb (1983) and Kanzawa (1984) transformed the energy equations based on the

transformed Eulerian-mean equations. The transformed energy equations allow a mean-eddy transfer of kinetic energy only, while the conventional energy equations allow a mean-eddy transfer of both kinetic and potential energies. However, in the presence of mean heating, the energy cycle is complicated by the steady state mean residual circulation which is induced by the EPFC, as will be shown in section 3.

In the presence of mean heating and mean friction, eddies not only induce a mean meridional circulation and modify the zonal flow and mean temperature but also modify mean heating and mean friction, which, in turn, further modifies the mean circulation (Hayashi, 1985b). Eddies also modify mean condensational heating due to the eddy moisture flux and modify the EPFC (Stone and Salustri, 1984).

In view of the ambiguity between the eddy flux and the mean meridional circulation, section 2 modifies the momentum and energy equations by combining the eddy flux convergence and the mean meridional circulation. This combined term will be interpreted as the effective eddy flux convergence in the extratropics where the mean meridional circulation is regarded as essentially being induced by eddies. Section 3 compares the conventional, transformed and modified energy cycles in order to examine the sensitivity of the energy cycle to its different formulations. Conclusions and remarks are given in section 4. Appendix A lists symbols. Appendix B discusses the relation between the form drag and energy flux. Appendix C gives the energy equations in spherical and pressure coordinates.

2. Modification of the momentum and energy equations

In this section, a modification is made of the conventional momentum and energy equations in Cartesian, log-pressure coordinates. The conventional energy equations in spherical and pressure coordinates are given in appendix C. The notations are listed in appendix A.

a. The transformed Eulerian-mean equations

The conventional Eulerian-mean equations are transformed as

$$\bar{u}_t = -(\overline{u'v'})_y - \rho^{-1}(\overline{\rho u'w'})_z + f\bar{v} + \bar{X}, \quad (1.1a)$$

$$= \text{EPFC} + f\bar{v}^* + \bar{X}, \quad (1.1b)$$

$$\bar{T}_t = -(\overline{T'v'})_y - N^2\bar{w} + \bar{Q}, \quad (1.2a)$$

$$= -N^2\bar{w}^* + \bar{Q}, \quad (1.2b)$$

where the mean advections are neglected, and X and Q are friction and heating, respectively. The Coriolis frequency f and Brunt-Väisälä frequency N are allowed to vary with y and z , respectively.

Here, (\bar{v}^*, \bar{w}^*) is the "residual mean circulation," defined as

$$\bar{v}^* = \bar{v} - \rho^{-1}(\overline{\rho N^{-2}T'v'})_z, \quad (1.3)$$

$$\bar{w}^* = \bar{w} - N^{-2}(\overline{T'v'})_y. \quad (1.4)$$

The EPFC in (1.1b) is the "Eliassen-Palm flux convergence," defined as

$$\text{EPFC} = -(\overline{u'v'})_y - \rho^{-1}(\overline{\rho u'w'})_z + f(\bar{v} - \bar{v}^*), \quad (1.5a)$$

$$= -(\overline{u'v'})_y - \rho^{-1}(\overline{\rho u'w'})_z + f\rho^{-1}(\overline{\rho N^{-2}T'v'})_z, \quad (1.5b)$$

$$= -Y_y - \rho^{-1}(\rho Z)_z, \quad (1.5c)$$

and the (Y, Z) in (1.5c) is the "Eliassen-Palm flux" (sign reversed), which is defined as

$$(Y, Z) = (\overline{u'v'}, -fN^{-2}\overline{T'v'} + \overline{u'w'}). \quad (1.6)$$

Under the "tall mean flow condition" (Andrews and McIntyre, 1976; Hayashi, 1985a), the residual mean circulation in the absence of mean heating can be neglected in (1.1b). In this case, the EPFC can be interpreted as approximating the mean flow acceleration due to the combined effects of eddies and induced mean circulations.

b. The modified Eulerian-mean equations

In order to isolate the mean meridional circulation that is induced by eddies in the presence of mean heating, the most reasonable reference state is a hypothetical steady mean state in the absence of eddies.

The eddy-induced mean circulation (\hat{v}, \hat{w}) is isolated as

$$(\hat{v}, \hat{w}) = (\bar{v}, \bar{w}) - (\bar{v}^{**}, \bar{w}^{**}), \quad (2.1)$$

where $(\bar{v}^{**}, \bar{w}^{**})$ is the hypothetical "adiabatic mean circulation," which is forced by the mean heating in the presence of mean friction.

The anomalous eddy-induced mean circulation $(\Delta\hat{v}, \Delta\hat{w})$ is given by

$$(\Delta\hat{v}, \Delta\hat{w}) = (\Delta\bar{v}, \Delta\bar{w}) - (\Delta\bar{v}^{**}, \Delta\bar{w}^{**}), \quad (2.2)$$

where Δ denotes a deviation from the climatological mean. The climatological mean is an appropriate reference state for an anomaly of the meridional circulation induced by eddies whose intensity varies climatologically.

The diabatic mean circulation is determined by the steady state Eulerian-mean equations in the absence of eddies as

$$0 = f\bar{v}^{**} + \bar{X}, \quad (2.3)$$

$$0 = -N^2\bar{w}^{**} + \bar{Q}. \quad (2.4)$$

The diabatic mean circulation vanishes in the inviscid limit in the extratropics where mean advections are neglected by quasi-geostrophic scaling (Schneider, 1977; Held and Hou, 1980). This theorem holds in the presence of surface friction. However, the surface flow and the surface friction vanish in the steady state in the absence of eddies. In the presence of eddies, the surface friction does not vanish but affects the atmosphere above the boundary layer through Ekman pumping.

The modified Eulerian-mean equations assume that the mean meridional circulation in the extratropics is entirely induced by eddies. This implies, in particular, that the *steady state* mean meridional circulation is entirely induced by the eddy momentum flux convergence. This assumption is consistent with the conventional estimation of time-mean meridional circulations from observed eddy momentum flux convergence (see Lorenz, 1967, p. 94). Under this assumption, the Eulerian-mean equations can be modified as

$$\bar{u}_t = \text{EMFC} + \bar{X}, \quad (2.5)$$

$$\bar{T}_t = \text{EHFC} + \bar{Q}, \quad (2.6)$$

Here, EMFC and EHFC are the “effective momentum flux convergence” and “effective heat flux convergence,” respectively, and are defined as

$$\text{EMFC} = -(\bar{u}'v')_y - \rho^{-1}(\rho\bar{u}'w')_z + f\bar{v}, \quad (2.7a)$$

$$= \text{EPFC} + f\bar{v}^*, \quad (2.7b)$$

$$\text{EHFC} = -(\bar{T}'v')_y - N^2\bar{w}, \quad (2.8a)$$

$$= -N^2\bar{w}^*. \quad (2.8b)$$

EHFC is related to the EPFC through (2.7b), (2.8b) and the continuity equation as

$$\rho^{-1}(\rho N^{-2}\text{EHFC})_z = -(f^{-1}\text{EPFC} - f^{-1}\text{EMFC})_y. \quad (2.9)$$

The steady state EMFC is balanced by the mean friction (\bar{X}) in (2.3), and the steady state EHFC is given by

$$\rho^{-1}(\rho N^{-2}\text{EHFC})_z = -(f^{-1}\text{EPFC} + f^{-1}\bar{X})_y. \quad (2.10)$$

Above the boundary layer in the extratropics, the mean friction (\bar{X}) in (2.10) can be neglected against EPFC and the steady state EHFC can be approximated as

$$\text{EHFC} = N^2\rho^{-1} \int_z^\infty \rho(f^{-1}\text{EPFC})_y dz, \quad (2.11a)$$

$$= N^2 \left[\rho^{-1} \int_z^\infty \rho f^{-1}(\bar{u}'v')_y dz - f^{-1}\bar{u}'w' \right]_y - (f^{-1}\bar{T}'v')_y. \quad (2.11b)$$

This approximation is not valid in the boundary layer.

The eddy momentum flux terms in (2.11b) approximate the induced circulation. To this approximation ($\bar{X} = 0$), the eddy heat flux does not induce any mean circulations in the steady state, since it is counterbalanced by the mean heating. This approximation is consistent with the observational estimate of time-mean heating from quasi-geostrophic potential vorticity flux (Wiin-Nielsen and Sela, 1971, p. 450).

c. The conventional energy equations

The conventional energy equations can be written as

$$\bar{K}_t = -(\bar{K} \cdot K') + (\bar{P} \cdot \bar{K}) + C(\bar{K}) + \bar{u}\bar{X}, \quad (3.1)$$

$$\bar{P}_t = -(\bar{P} \cdot P') - (\bar{P} \cdot \bar{K}) + C(\bar{P}) + \bar{T}\bar{Q}, \quad (3.2)$$

$$K'_t = (\bar{K} \cdot K') + (P' \cdot K') + C(K') + \bar{u}\bar{X}', \quad (3.3)$$

$$P'_t = (\bar{P} \cdot P') - (P' \cdot K') + C(P') + \bar{T}'\bar{Q}', \quad (3.4)$$

where the nonlinear terms have been omitted.

In the above equations, \bar{K} and \bar{P} are the mean kinetic and available potential energies, while K' and P' are the eddy kinetic and available potential energies. They are defined as

$$\bar{K} = \rho\bar{u}^2/2, \quad (3.5)$$

$$\bar{P} = \rho N^{-2}\bar{T}^2/2, \quad (3.6)$$

$$K' = \rho(\bar{u}'^2 + \bar{v}'^2)/2, \quad (3.7)$$

$$P' = \rho N^{-2}\bar{T}'^2/2. \quad (3.8)$$

The energy conversion terms are defined as

$$(\bar{K} \cdot K') = -\rho\bar{u}'v'u'_y, \quad (3.9)$$

$$(\bar{P} \cdot P') = -\rho N^{-2}\bar{T}'v'T'_y, \quad (3.10)$$

$$(\bar{P} \cdot \bar{K}) = \rho\bar{T}\bar{w}, \quad (3.11)$$

$$(P' \cdot K') = \rho\bar{T}'w', \quad (3.12)$$

$$C(\bar{K}) = -(\rho\bar{\phi}\bar{v} + \rho\bar{u}\bar{u}'v')_y - (\rho\bar{\phi}\bar{w} + \rho\bar{u}\bar{u}'w')_z, \quad (3.13)$$

$$C(\bar{P}) = -\rho N^{-2}(\bar{T}\bar{T}'v')_y, \quad (3.14)$$

$$C(K') = -(\rho\bar{\phi}'v')_y - (\rho\bar{\phi}'w')_z, \quad (3.15)$$

$$C(P') = 0. \quad (3.16)$$

d. The transformed energy equations

The transformed energy conversion terms $\{ \}$ are defined as

$$\{\bar{K} \cdot K'\} = -\rho\bar{u}\text{EPFC}, \quad (4.1a)$$

$$= (\bar{K} \cdot K') + (\bar{P} \cdot P') + (\rho\bar{u}Y)_y + (\rho\bar{u}Z)_z, \quad (4.1b)$$

$$\{\bar{P} \cdot P'\} = 0, \quad (4.2)$$

$$\{\bar{P} \cdot \bar{K}\} = \rho\bar{T}\bar{w}^*, \quad (4.3a)$$

$$= (\bar{P} \cdot \bar{K}) + (\bar{P} \cdot P') - C(\bar{P}), \quad (4.3b)$$

$$\{P' \cdot K'\} = (P' \cdot K') - (\bar{P} \cdot P'), \quad (4.4)$$

$$C\{\bar{K}\} = -(\rho\bar{\phi}\bar{v}^*)_y - (\rho\bar{\phi}\bar{w}^*)_z, \quad (4.5)$$

$$C\{\bar{P}\} = 0, \quad (4.6)$$

$$C\{K'\} = -\{\rho\bar{\phi}'v'\}_y - \{\rho\bar{\phi}'w'\}_z, \quad (4.7a)$$

$$= C(K') - (\rho\bar{u}Y)_y - (\rho\bar{u}Z)_z, \quad (4.7b)$$

$$C\{P'\} = 0, \quad (4.8)$$

where $\{\bar{\phi}'v'\}$ and $\{\bar{\phi}'w'\}$ are the transformed eddy energy fluxes defined as

$$\{\overline{\phi'v'}\} = \overline{\phi'v'} + \bar{u}Y, \quad (4.9)$$

$$\{\overline{\phi'w'}\} = \overline{\phi'w'} + \bar{u}Z. \quad (4.10)$$

The relation (4.1b) is consistent with the work done by a quasi-geostrophic vorticity flux derived by Pfeffer (1981, p. 1359), Lau and Holopainen (1984, p. 327) and Holopainen (1984, p. 2509).

The transformed energy fluxes are related to the conventional fluxes as

$$C\{K'\} + C\{\bar{K}\} = C(K') + C(\bar{K}) + C(\bar{P}), \quad (4.11)$$

$$\{\overline{\phi'w'}\} + \bar{\phi}\bar{w}^* = \overline{\phi'w'} + (\bar{\phi}\bar{w} + \partial A/\partial y + \bar{u}\bar{u}'w'), \quad (4.12)$$

$$\{\overline{\phi'v'}\} + \bar{\phi}\bar{v}^* = \overline{\phi'v'} + (\bar{\phi}\bar{v} - \rho\partial A/\partial z + \bar{u}\bar{u}'v' + N^{-2}\bar{T}\bar{T}'v'), \quad (4.13)$$

where A is defined by

$$A = N^{-2}\bar{\phi}\bar{T}'v'. \quad (4.14)$$

It should be noted that the terms in (4.12) and (4.13) involving A do not appear in the conventional mean energy fluxes. These are nondivergent fluxes and do not contribute to the energy flux convergence.

The relation (4.1b) is similar to that derived by Pfeffer (1981, p. 1359) for quasi-geostrophic flows.

e. The approximate transformed energy equations

In the absence of dissipation, instability, nonlinearity and diabatic heating in waves, the conventional wave energy fluxes are related (Eliassen–Palm relations) to the Eliassen–Palm fluxes as

$$(\overline{\phi'v'}, \overline{\phi'w'}) = (c - \bar{u})(Y, Z). \quad (5.1)$$

For quasi-stationary waves ($|c| \ll \bar{u}$) above the boundary layer, these relations can be approximated as

$$(\bar{u}Y, \bar{u}Z) = (-\overline{\phi'v'}, -\overline{\phi'w'}). \quad (5.2)$$

With the use of this approximation, the transformed energy conversion terms in section 2d can be approximated as

$$\{\bar{K} \cdot K'\} = (\bar{K} \cdot K') + (\bar{P} \cdot P') + C(K'), \quad (5.3)$$

$$C\{\bar{K}\} = C(\bar{K}) + C(\bar{P}) + C(K'), \quad (5.4)$$

$$C\{K'\} = 0, \quad (5.5)$$

while the transformed energy fluxes are approximated by the use of (4.13), (4.14), (4.16) and (4.17) as

$$\{\overline{\phi'w'}\} = 0, \quad (5.6)$$

$$\{\overline{\phi'v'}\} = 0, \quad (5.7)$$

$$\bar{\phi}\bar{w}^* = \overline{\phi'w'} + (\bar{\phi}\bar{w} + \partial A/\partial y + \bar{u}\bar{u}'w'), \quad (5.8)$$

$$\bar{\phi}\bar{v}^* = \overline{\phi'v'} + (\bar{\phi}\bar{v} - \rho^{-1}\partial\rho A/\partial z + \bar{u}\bar{u}'v' + N^{-2}\bar{T}\bar{T}'v'). \quad (5.9)$$

Under this approximation, the transformed eddy energy fluxes vanish. These approximate terms exactly

satisfy the energy balance. The advantage of this approximation is that the conventional energy cycle above the boundary layer can be transformed by rearranging and recombining its energy conversion terms without explicitly calculating the Eliassen–Palm flux, although the transformed energy cycle is somewhat distorted by this approximation.

f. The modified energy equations

The modified energy equations can be derived from the modified Eulerian-mean equations and perturbation equations by defining the modified energy conversion terms $\langle \quad \rangle$ as

$$\langle \bar{K} \cdot K' \rangle = -\rho\bar{u}\text{EMFC}, \quad (6.1a)$$

$$= (\bar{K} \cdot K') - (\bar{P} \cdot \bar{K}) - C(\bar{K}), \quad (6.1b)$$

$$\langle \bar{P} \cdot P' \rangle = -\rho\bar{T}\text{EHFC}, \quad (6.2a)$$

$$= (\bar{P} \cdot P') + (\bar{P} \cdot \bar{K}) - C(\bar{P}), \quad (6.2b)$$

$$\langle \bar{P} \cdot \bar{K} \rangle = 0, \quad (6.3)$$

$$\langle P' \cdot K' \rangle = (P' \cdot K') + (\bar{P} \cdot \bar{K}), \quad (6.4)$$

$$C\langle \bar{K} \rangle = 0, \quad (6.5)$$

$$C\langle \bar{P} \rangle = 0, \quad (6.6)$$

$$C\langle K' \rangle = C(K') + C(\bar{K}), \quad (6.7)$$

$$C\langle P' \rangle = C(\bar{P}). \quad (6.8)$$

The modified energy fluxes are defined as

$$\langle \bar{\phi}\bar{w} \rangle = 0, \quad (6.9)$$

$$\langle \bar{\phi}\bar{v} \rangle = 0, \quad (6.10)$$

$$\langle \overline{\phi'w'} \rangle = \overline{\phi'w'} + (\bar{\phi}\bar{w} + \bar{u}\bar{u}'w'), \quad (6.11)$$

$$\langle \overline{\phi'v'} \rangle = \overline{\phi'v'} + (\bar{\phi}\bar{v} + \bar{u}\bar{u}'v'). \quad (6.12)$$

Equations (6.1a) and (6.2a) indicate that the modified mean-eddy transfers $\langle \bar{K} \cdot K' \rangle$ and $\langle \bar{P} \cdot P' \rangle$ of kinetic and potential energies are equal to the work of the EMFC and EHFC, respectively. The sum of these transfers are related to the transformed mean-eddy transfer term as

$$\langle \bar{K} \cdot K' \rangle + \langle \bar{P} \cdot P' \rangle = \{\bar{K} \cdot K'\} - C\{\bar{K}\}. \quad (6.13)$$

In the stratosphere, the transformed mean-eddy transfer term $\{\bar{K} \cdot K'\}$ is dominated by $C\{\bar{K}\}$ and is not by itself an appropriate measure of wave–mean flow interactions, as will be shown in section 3.

Equation (6.4) indicates that the modified conversion $\langle P' \cdot K' \rangle$ between the eddy kinetic and potential energies is equal to the sum of the conventional eddy and mean conversions. Also, (6.7) indicates that the modified eddy energy flux is equal to the sum of the conventional eddy and mean energy fluxes. Appendix B shows that the modified and transformed eddy energy fluxes vanish at the surface when integrated latitudi-

nally, being consistent with the zero work done by stationary topography.

Figure 1 illustrates the procedures for obtaining the approximate transformed energy cycle and the modified energy cycle from the conventional energy cycle by rearranging and recombining the energy terms. The meridional energy flux convergences vanish when integrated latitudinally. The exact transformed energy cycle is associated with some eddy energy fluxes.

3. Applications to wave-mean flow interactions

In this section, the conventional, transformed and modified energy equations are applied to both theoretical and observational waves.

a. The baroclinically unstable waves

Figure 2 shows the conventional (CEC), transformed (TEC) and modified (MEC) energy cycles of theoretical baroclinically unstable waves (Charney, 1947) in the vertical shear. The CEC indicates that baroclinic waves grow by extracting energy from \bar{P} , which in turn extracts energy from \bar{K} , resulting in the decay of both \bar{P} and \bar{K} . The TEC indicates that these waves grow by extracting energy from \bar{K} , which in turn extracts energy

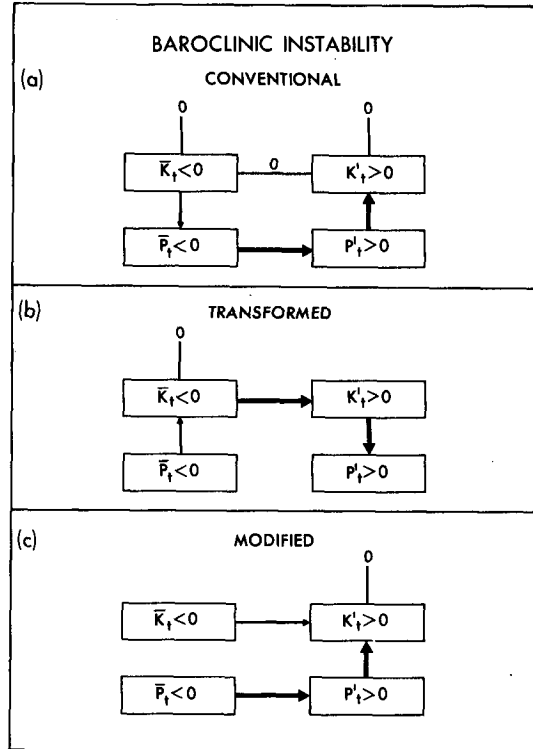


FIG. 2. Energy cycles of theoretical baroclinic instability in the vertical shear. The boxes represent a time-derivative of energy.

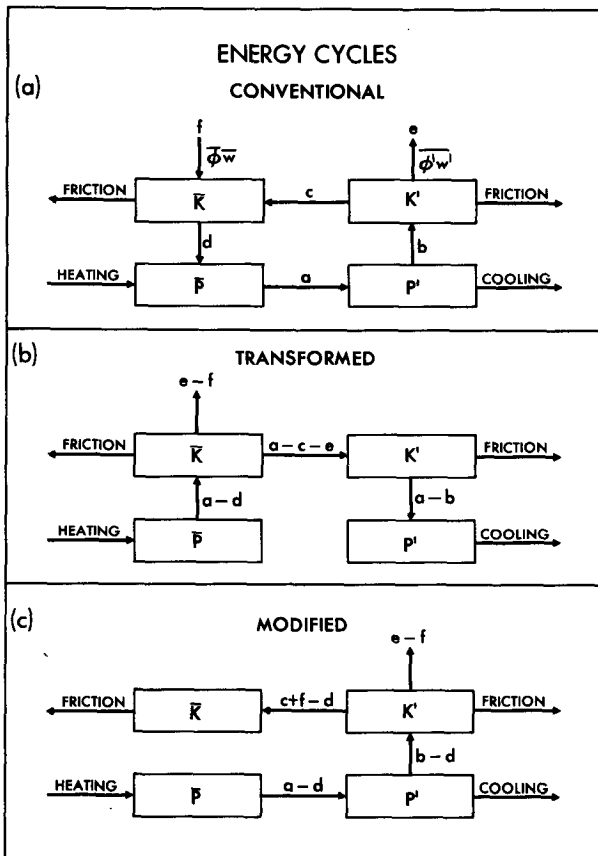


FIG. 1. Schematic diagram for the (a) conventional, (b) transformed and (c) modified energy cycles. The transformed energy cycle is approximated by the use of an approximate Eliassen-Palm relation.

from \bar{P} . Since the exchange of energy between \bar{K} and \bar{P} is entirely due to the eddy-induced circulation, neither the CEC nor the TEC gives a simple description of wave-mean flow interactions. On the other hand, the MEC is simpler in that the baroclinic waves extract energy from both \bar{P} and \bar{K} and there is no exchange of energy between \bar{P} and \bar{K} due to the induced circulation. When kinetic energy and potential energy are combined, the three energy cycles become identical, since the boundary flux terms vanish upon integration. It should be mentioned that the transformed eddy and mean vertical energy fluxes vanish individually at the surface when integrated latitudinally, provided that the surface mean flow vanishes. On the other hand, the transformed vertical energy fluxes of the Eady baroclinic unstable wave (Eady, 1949), shown by Plumb (1983, Fig. 7b) and Kanzawa (1984, Fig. 4), occur at the upper boundary. Thus, the approximate transformed energy equations are not applicable to the Eady wave. The transformed vertical energy fluxes of Charney baroclinic waves vanish at an infinite height.

b. The topographical planetary waves

Figure 3 shows the energy cycles of theoretical stationary planetary waves forced by topography in the presence of vertical shear and Newtonian eddy cooling as studied by Plumb (1983). The radiation condition is imposed at 67.5 km. This condition requires that

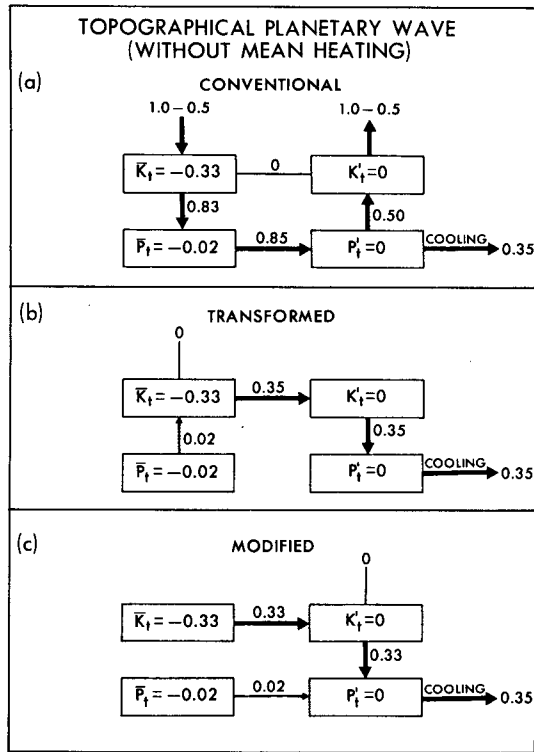


FIG. 3. Energy cycles of topographically forced planetary waves in the vertical shear in the absence of mean heating. The boxes represent a time-derivative of energy.

there be no downward energy flux from above. All three cycles indicate that the energy loss due to eddy cooling is 0.35 and is equal to the time change of \bar{K} plus \bar{P} .

The CEC indicates that the mean-to-eddy transfer $(\bar{P} \cdot P') = 0.85$ of potential energy is much larger than the eddy cooling (0.35). This is due to a large loss of eddy energy by the net boundary flux (1.0-0.5), which consists of upward fluxes of 1.0 and 0.5 at the top and bottom, respectively, and a large gain of mean energy by the net boundary flux (1.0-0.5), which consists of downward fluxes of 1.0 and 0.5 at the top and bottom, respectively. Thus, the CEC of theoretical planetary waves in the quasi-nonacceleration condition is misleading in that the mean-eddy transfer $(\bar{P} \cdot P')$ of potential energy is largely counterbalanced by the conversion $(\bar{K} \cdot \bar{P})$ from mean kinetic energy due to the induced mean circulation.

The TEC is simpler than the CEC in that there are no boundary fluxes at the upper and lower boundaries and the $\{\bar{K} \cdot K'\}$ is equal to the eddy cooling. The MEC is almost identical to the TEC in that the $\langle \bar{K} \cdot K' \rangle$ primarily represents wave-mean flow interaction. The good agreement between the TEC and the MEC indicates that the eddy-induced mean circulation is well approximated by the eddy heat flux convergence in the absence of mean heating.

Figure 4 is the same as Fig. 3 except that mean heating is added to maintain the mean energies. These CEC

and TEC are somewhat different from those of Plumb (1983, Fig. 16, Fig. 18), since Plumb did not add mean heating in such a way as to maintain the mean energies. The CEC indicates that $(\bar{P} \cdot P')$ is still largely counterbalanced by $(\bar{K} \cdot \bar{P})$, although $(\bar{K} \cdot \bar{P}) = 0.5$ is somewhat smaller than that (0.85) in the absence of mean heating. The TEC indicates that $\{\bar{K} \cdot K'\}$ is counterbalanced by $\{\bar{P} \cdot \bar{K}\}$, which is due to the eddy-induced residual mean circulation. Thus, the TEC does not describe wave-mean flow interactions in terms of $\{\bar{K} \cdot K'\}$ only. The MEC indicates that energy is transferred from \bar{P} to P' without a loop.

It is not realistic to use the radiation condition that the wave energy should propagate to infinite height without dissipation. When it is assumed that all the wave energy is dissipated in the upper atmosphere, the eddy heat flux does not induce a steady state circulation, and $\bar{\phi}\bar{w}$ and $(\bar{K} \cdot \bar{P})$ vanish in the absence of eddy momentum flux and mean friction. In this case the CEC (Fig. 4a) becomes identical to the MEC (Fig. 4c) without a loop.

c. Observed troposphere

Figure 5 shows the energy cycles of observed eddies (stationary plus transient waves) in the troposphere (100-925 mb, 25°-75°N, December 1964-February 1965) based on the conventional energy diagnostics of Tomatsu (1979). The unknown downward conven-

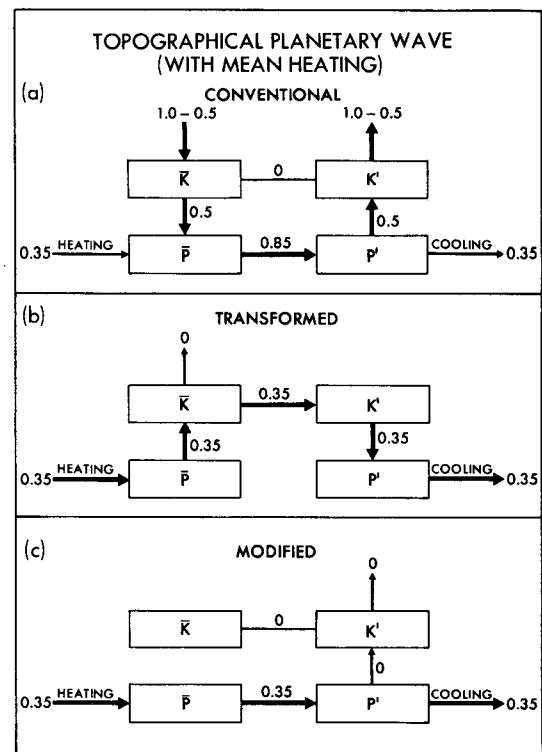


FIG. 4. As in Fig. 3 except for the presence of mean heating.

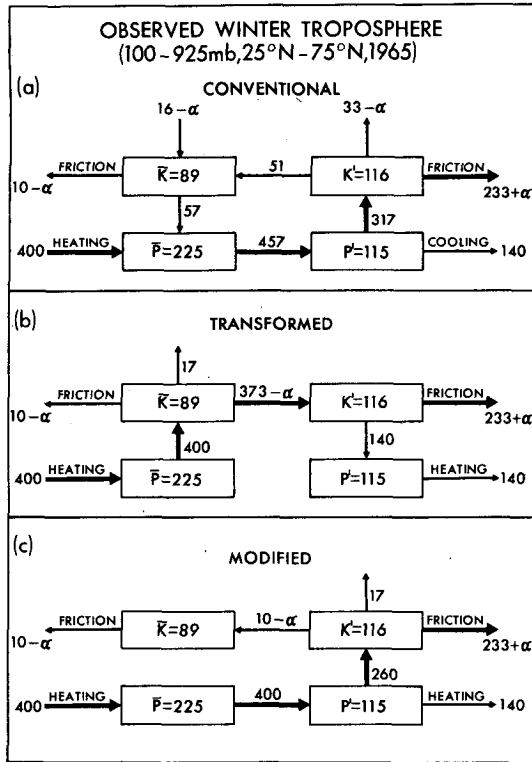


FIG. 5. Energy cycles of observed winter troposphere (100–925 mb, 25–75°N, December 1964–February 1965) based on Tomatsu (1979). The boxes represent energy. The α denotes unknown topographical vertical energy flux. Energy is given in (10^4 J m^{-2}); energy conversion is given in (10^{-2} W m^{-2}).

tional eddy energy flux at 925 mb due to Ekman pumping is not estimated but is combined with the frictional term, which is given by the residual of the energy balance. The unknown topographical boundary energy flux is denoted as α . This α is theoretically positive and can be diagnostically estimated by the use of the Eliassen–Palm relation (5.2) from the poleward heat flux of stationary eddies. Also, this α must be less than 10 in order that the mean frictional dissipation ($10 - \alpha$) remain positive. The friction and heating (or cooling) terms are given by the residuals. These residuals also contain a lateral energy flux convergence which is not available from Tomatsu’s diagnostics. In particular, the “thermal generation” of \bar{P} in the mid-latitudes can, to some extent, be due to the meridional flux convergence $C(\bar{P})$ of \bar{P} , which is associated with the eddy heat flux. The transformed energy flux at 925 mb is not estimated but is combined with the friction term, since the approximated Eliassen–Palm relation (5.2) does not hold well at this level.

The CEC (Fig. 5a) indicates that the thermal generation (400) of \bar{P} due to mean heating is mainly transferred to P' and K' , which are almost completely dissipated by eddy cooling (140) and eddy friction ($233 + \alpha$) in the troposphere, while only a small portion of

TABLE 1. Tropospheric energy conversion (10^{-2} W m^{-2}) of stationary planetary waves (December 1964–February 1965, 25°–75°N, 100–925 mb) and their wavenumber components ($n = 1, 2, 3$) based on Tomatsu (1979).

Level (mb)	Energy terms	n			
		Stationary	1	2	3
100	$\rho \overline{\phi'w'}$	6.1	5.7	0.6	−0.8
100–925	$(\bar{P} \cdot P')$	164.3	52.5	38.5	63.9
100–925	$(P' \cdot K')$	36.5	11.2	7.8	9.5
100–925	$(K' \cdot \bar{K})$	19.6	5.2	3.5	8.5

eddy energy ($33 - \alpha$) leaks into the stratosphere. This CEC is not as misleading as that of the theoretical stationary waves which transport a large amount of eddy energy upward into the stratosphere in the absence of large dissipation in the troposphere. On the other hand, the TEC (Fig. 5b) indicates that \bar{P} is converted to \bar{K} and then transferred to K' and P' . However, this convergence is mainly due to the eddy-induced residual mean circulation. The MEC (Fig. 5c) indicates that \bar{P} is transferred to P' and then converted to K' as in the CEC. According to this MEC, \bar{K} is maintained by $\langle \bar{K} \cdot K' \rangle$ alone. However, the $\langle \bar{P} \cdot \bar{K} \rangle$ due to the diabatic mean circulation may not be negligible in the mean

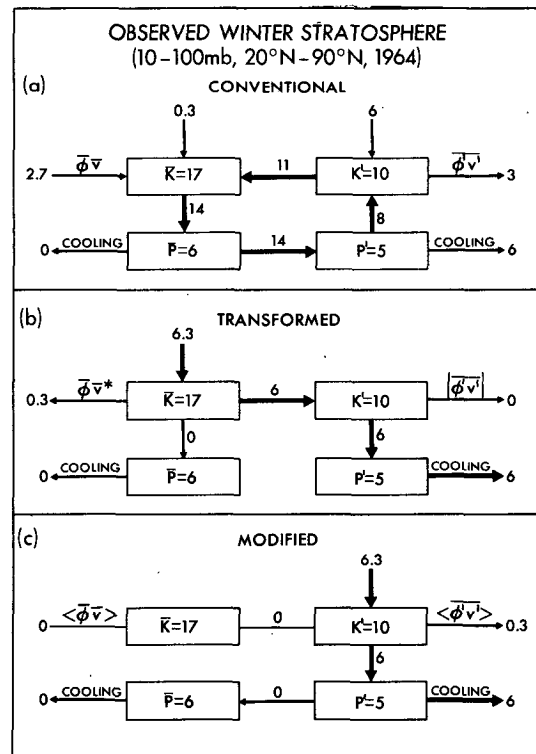


FIG. 6. Energy cycles of observed winter stratosphere (10–100 mb, 20–90°N, January–February 1964 and December 1964) based on Dopplack (1971). Energy is given in (10^4 J m^{-2}); energy conversion is given in (10^{-2} W m^{-2}).

kinetic energy balance, although it may be negligible in the mean potential energy balance in the extratropics.

The large discrepancy between the energy cycles of the observed tropospheric eddies and the theoretical stationary waves is due to the fact that these eddies include transient baroclinic waves that do not propagate into the stratosphere. However, there is also the possibility that the theoretical stationary eddy flux of energy is much larger than that of the observed eddy flux. In order to clarify this possibility, Table 1 lists the stratospheric and tropospheric stationary (3 month mean) eddy energy conversions and the eddy vertical flux at 100 mb and their wavenumber 1, 2, 3 components based on Tomatsu (1979). This table indicates that the stationary eddy flux at 100 mb is mostly due to the wavenumber 1 component and is about 50% of the $(P' \cdot K')$ of the same component in the troposphere. This means that 50% of this kinetic energy conversion is dissipated in the troposphere. Moreover, this flux is only about 10% of the $(\bar{P} \cdot P')$ for the same component, which is about five times as large as $(P' \cdot K')$. This means that 80% of this potential energy conversion $(\bar{P} \cdot P')$ is dissipated in the troposphere. This loss is due not only to the thermal diffusion and radiative cooling but also to interactions between stationary and transient eddies, as demonstrated by the energy diagnostics of a general circulation model (Hayashi and Golder, 1985). The large energy flux of the theoretical stationary wave is due to the absence of large eddy damping in the theoretical model of Plumb (1983).

d. Observed stratosphere

Figure 6 shows the energy cycles of the observed winter stratosphere (10–100 mb, 20°–90°N) based on Dopplack (1971). The residual terms in the mean and eddy kinetic energy balance are regarded as lateral boundary fluxes $(\phi\bar{v}, \phi'v')$, since these terms agree fairly well with their direct estimates. The residual terms in the mean and eddy potential energy balance are regarded as due to radiative heating, since these terms agree fairly well with their direct estimates.

The CEC (Fig. 6a) is similar to a theoretical energy cycle in the nonacceleration condition in that the energy is transferred in a loop. The CEC is complicated in that the mean-eddy transfer $(\bar{P} \cdot P')$ of potential energy is almost exactly counterbalanced by the conversion $(\bar{K} \cdot \bar{P})$ from the mean kinetic energy. This conversion is due to the thermodynamically indirect circulation, which is mainly induced by the eddy momentum flux convergence. It should be noted that \bar{P} is not generated by radiative heating.

The TEC (Fig. 6b) is simpler than the CEC in that there is no loop. However, this TEC is also complicated in that the mean-eddy transfer $\{\bar{K} \cdot K'\}$ of kinetic energy due to the EPFC is replenished by the residual mean vertical energy flux, which is induced by the EPFC itself. Andrews et al. (1983) also found a large

residual circulation in the stratosphere of a general circulation model. It is rather disturbing that the transformed eddy energy flux almost vanishes for quasi-stationary waves in spite of the fact that these waves are forced from below and are maintained against radiative damping.

The MEC (Fig. 6c) is simpler than the TEC in that the major energy source is the modified eddy flux of kinetic energy from the troposphere, which is converted to P' . This modified eddy flux is equal to the sum of the conventional eddy and mean fluxes.

Figure 7 is the same as Fig. 6 except for the summer season. The CEC (Fig. 7a) indicates that the eddy kinetic energy is maintained primarily by the eddy energy flux from the troposphere, while the mean kinetic energy is maintained primarily by the mean energy flux from the troposphere. It should be noted that the radiative heating destroys the mean potential energy.

The TEC (Fig. 7b) indicates that eddy kinetic energy is maintained by the conversion from mean kinetic energy, while the mean kinetic energy is maintained by the mean energy flux from the troposphere. The mean potential energy is maintained against radiative cooling by $\{\bar{K} \cdot \bar{P}\}$, which is associated with the residual mean circulation.

The MEC (Fig. 7c) indicates that eddy kinetic energy is maintained by the modified eddy energy flux from the troposphere, while the mean potential energy is maintained against radiative cooling by the EHFC. This result suggests that the summer lower stratosphere is associated with a larger reverse temperature gradient than expected from radiative equilibrium due to the planetary heat flux convergence. If there were no eddies, the reverse temperature gradient would be maintained by radiative heating occurring at the warm summer pole. In the presence of eddies, radiative cooling occurs at the excessively warm summer pole.

e. Observed stratospheric warming

Figure 8 shows the energy cycles of observed stratospheric warming (4–9 February 1957) based on Reed et al. (1963). The CEC (Fig. 8a) indicates that the eddy energy entering the stratosphere from the troposphere is transferred to \bar{P} and \bar{K} and then returns to the troposphere, resulting in an increase of \bar{P} and a decrease of \bar{K} . It is disturbing that \bar{K} decreases in spite of the eddy-to-mean transfer of kinetic energy. The eddy-to-mean transfer of potential energy is associated with a reverse temperature gradient at the time of the warming and is consistent with the increase of \bar{P} . This energy cycle is consistent with the conventional energy cycle (Kanzawa, 1984, Fig. 6) of a theoretical model cycle (Matsuno and Nakamura, 1979) of stratospheric warming.

The TEC (Fig. 8b) indicates that the residual mean energy flux entering the stratosphere is transferred to \bar{K} due to the residual mean circulation. It should be noted that this circulation can be interpreted as the Lagrangian-mean circulation. Matsuno and Nakamura

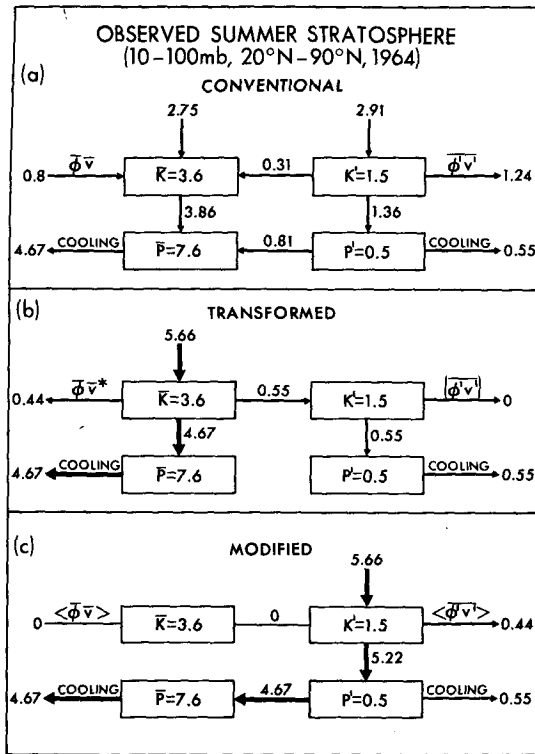


FIG. 7. As in Fig. 6 except for summer.

(1979) interpreted the stratospheric warming as being caused by the adiabatic warming due to the Lagrangian-mean motion. However, this motion is induced by planetary waves.

The MEC (Fig. 8c) indicates in a simple manner that the eddy energy entering the stratosphere is transferred to P' and \bar{P} , being consistent with the increase of \bar{P} , while \bar{K} is transferred to K' , being consistent with the decrease of \bar{K} . It should also be noted that the stratospheric warming is associated with energy cycles that are similar to those of the summer stratosphere. In the summer stratosphere, the reverse temperature gradient exceeds that in radiative equilibrium due to planetary-wave heat flux convergence. In the winter stratosphere, however, the reverse temperature gradient is essentially caused by planetary waves.

4. Conclusions and remarks

The present paper has modified the momentum and energy equations and compared the conventional, transformed and modified energy cycles of theoretical and observational atmospheric waves in order to examine the sensitivity of the energy cycle to its different formulations. The conclusions are summarized as follows:

1) The modified energy equations describe wave-mean flow interaction in terms of the effective momentum flux convergence (EMFC) and the effective

heat flux convergence (EHFC), which are combinations of eddy momentum and heat flux convergences and mean meridional circulations.

2) In the extratropics, where mean friction is negligible against mean Coriolis force, the time-mean circulation is primarily induced by the eddy momentum flux. In this case, the EHFC is given by vertically integrating the meridional gradient of the Eliassen-Palm flux convergence (EPFC). This EHFC consists of the eddy meridional heat flux convergence and the meridional derivative of the meridional and vertical eddy momentum fluxes, which act like effective eddy heat flux convergences.

3) The modified mean energy flux is zero by definition, while the modified eddy energy flux is equal to the sum of the conventional eddy and mean energy fluxes. The transformed eddy energy flux nearly vanishes for quasi-stationary waves, while the transformed mean energy flux is due to the residual mean circulation.

4) The latitudinally integrated vertical component of the modified and transformed eddy energy fluxes due to topographically forced waves vanishes at the surface, being consistent with the zero work done by stationary topography.

5) The modified energy cycle (MEC) is simpler and less misleading than the conventional (CEC) and transformed (TEC) energy cycles in describing the

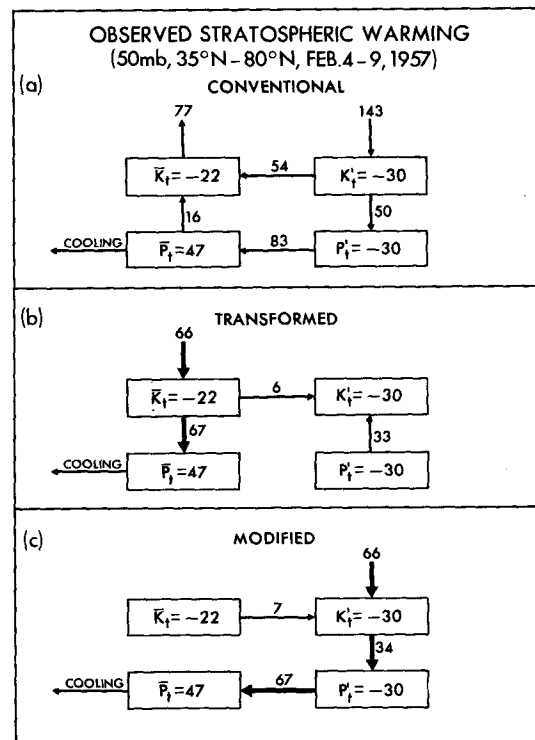


FIG. 8. Energy cycles of stratospheric warming (50 mb, 35°-80°N, 4-9 February 1957) based on Reed et al. (1963). Energy is given in ($10^2 \text{ J m}^{-2} \text{ mb}^{-1}$); energy conversion is given in ($10^{-4} \text{ W m}^{-2} \text{ mb}^{-1}$).

wave-mean flow interactions of baroclinic waves, planetary waves and stratospheric warming.

6) The CEC of the observed troposphere is not as complicated as that of theoretical planetary waves in the quasi-nonacceleration condition, since the observed eddy heat flux convergence is balanced by mean heating and does not induce a large mean circulation.

7) The CEC of the observed stratosphere is complicated, since the observed eddy momentum flux convergence induces a steady state mean circulation. The TEC of the troposphere and stratosphere is complicated, since the EPFC induces the steady state residual mean circulation. The MEC is simpler than the TEC, since there is no induced mean circulation by definition.

8) All the energy cycles suggest that the major energy source of the tropospheric planetary waves can be traced to the thermal generation of the mean potential energy, while the stratospheric planetary waves are maintained primarily by the total (mean plus eddy) vertical flux of energy from the troposphere.

It should be noted that the time-mean meridional circulation induced by the observed eddy momentum flux convergence is not large enough to complicate the tropospheric energy cycle. In the stratosphere, however, an elimination of the time-mean circulation induced by the observed eddy momentum flux convergence results in a simplification of the time-mean energy cycle, since the eddy heat flux convergence is counterbalanced by this mean circulation.

The EPFC is not an appropriate measure of steady state eddy-mean interaction in the presence of mean heating. However, the vertically integrated EPFC is an appropriate measure for the individual effects of stationary and transient planetary waves and gravity waves in the extratropics. In particular, the vertical eddy momentum flux convergence due to gravity waves induces a steady state meridional circulation, and the resulting adiabatic mean vertical motion acts like an effective eddy meridional heat flux convergence. The reverse mean temperature gradient in the upper mesosphere can be interpreted as being maintained by this effective heat flux convergence, while the tropospheric mean temperature gradient can be interpreted as being reduced by this convergence.

In addition to the "conventional," "transformed" and "modified" energy equations, there are probably many other meaningful or meaningless ways of combining the energy conversion terms. The energy cycle can also be simplified by the use of "barotropic" and "baroclinic" energies instead of kinetic and potential energies (e.g., Mak, 1985), although the energy conversion terms of any energy cycle are not unique and can be transformed arbitrarily.

The transformed energy equations assume that the eddy-induced mean circulation is estimated by the eddy heat flux convergence, while the modified energy

equations assume that this circulation can be replaced by the Eulerian-mean circulation. The correct circulation probably lies somewhere between these two extremes. The modified equations do not always properly describe how mean kinetic energy is maintained by wave-mean flow interactions, since the mean momentum balance is sensitive to the diabatic mean circulation, which may be negligible in the mean heat balance in the extratropics. Nevertheless, the modified and transformed energy cycles are helpful for interpreting the conventional energy cycle with caution. The present results should be reexamined with the use of current observed and simulated data.

Acknowledgments. The author is grateful to H. Kida, J. D. Mahlman, I. M. Held and D. G. Golder for their appropriate comments on the original manuscript.

APPENDIX A

List of Symbols

$(\bar{\quad})$	zonal mean
$(\delta\quad)$	deviation from the zonal mean
$(\bar{\quad})^*$	residual mean circulation
$(\bar{\quad})^{**}$	diabatic mean circulation
$\{\quad\}$	transformed quantity
$\langle\quad\rangle$	modified quantity
EPFC	Eliassen-Palm flux convergence
EMFC	effective momentum flux convergence
EHFC	effective heat flux convergence
\bar{K}, K'	mean and eddy kinetic energy
\bar{P}, P'	mean and eddy potential energy
$(\bar{K} \cdot K')$	conversion of \bar{K} to K'
$(\bar{P} \cdot P')$	conversion of \bar{P} to P'
$(\bar{P} \cdot \bar{K})$	conversion of \bar{P} to \bar{K}
$(P' \cdot K')$	conversion of P' to K'
$C(\bar{K})$	flux convergence of \bar{K}
$C(K')$	flux convergence of K'
$C(\bar{P})$	flux convergence of \bar{P}
$C(P')$	flux convergence of P'
t	time
x, y	zonal and meridional Cartesian coordinate
z	vertical "log pressure" coordinates
u, v	zonal and meridional velocities
w	"vertical velocity" in log-pressure coordinate
p	pressure
ρ	basic state density
ϕ	departure from basic state geopotential
T	a measure (ϕ_z) of departure from basic state temperature
X	zonal friction
Q	heating rate per unit mass
τ	zonal stress
h	topographical height
f	Coriolis parameter
N	Brunt-Väisälä frequency

APPENDIX B

Relation between Form Drag and Energy Flux

The mean and perturbation momentum equations are given by

$$\bar{u}_t = f\bar{v} - (\overline{u'v'})_y - \rho^{-1}(\overline{\rho u'w'})_z - \rho^{-1}\bar{\tau}_z, \quad (\text{B1})$$

$$u'_t = fv' - \bar{u}u'_x - \phi'_x - \rho^{-1}\tau'_z, \quad (\text{B2})$$

with the lower boundary conditions (see Charney and Drazin, 1961; Andrews, 1980) at $z = \bar{h}$ given by

$$\bar{w} = (\overline{v'h'})_y, \quad (\text{B3})$$

$$w' = (\bar{u} - c)h_x, \quad (\text{B4})$$

where h is the topographical height and c the zonal speed of the moving wavy surface.

Vertically integrating (B1) gives

$$\int_{\bar{h}}^{\infty} \rho \bar{u}_t dz = F + \bar{\tau}(\bar{h}) - \int_{\bar{h}}^{\infty} (\overline{u'v'})_y dz, \quad (\text{B5})$$

where F is defined as

$$F = \rho(f\overline{v'h'} + \overline{u'w'}) \quad \text{at } z = \bar{h}, \quad (\text{B6a})$$

$$= -\overline{p'h'_x} + \overline{\tau'_z h'} \quad \text{at } z = \bar{h}. \quad (\text{B6b})$$

The first term in (B6b) represents the form drag (see Bretherton, 1969, p. 219), while the second term represents the increase of mean frictional surface stress due to the vertically varying surface flow.

In the absence of friction, this F is reduced to

$$F = \frac{\rho \overline{\phi'w'}}{c - \bar{u}}. \quad (\text{B7})$$

In the inviscid and adiabatic limit, the Eliassen-Palm relation (5.1) in Section 2e reduces (B7) to

$$F = \rho Z \quad \text{at } z = \bar{h}, \quad (\text{B8})$$

where Z is the vertical component of the Eliassen-Palm flux.

The work (cF) done by the moving wavy surface in the absence of friction is related to energy fluxes through the energy equations as

$$\int cF dy = \int (\bar{\phi}\bar{w} + \overline{\phi'w'} + \overline{uu'w'}) dy \quad \text{at } z = \bar{h}, \quad (\text{B9a})$$

$$= \int \langle \overline{\phi'w'} \rangle dy \quad z = \bar{h}, \quad (\text{B9b})$$

where $\langle \overline{\phi'w'} \rangle$ is the modified eddy vertical energy flux.

In the inviscid and adiabatic limit, the cF is related to the energy flux without a latitudinal integration as

$$cF = \frac{c}{c - \bar{u}} \rho \overline{\phi'w'}, \quad (\text{B10a})$$

$$= \rho \overline{\phi'w'} + \bar{u} \frac{\rho \overline{\phi'w'}}{c - \bar{u}}, \quad (\text{B10b})$$

$$= \rho \overline{\phi'w'} + \rho \bar{u} Z, \quad (\text{B10c})$$

$$= \{ \rho \overline{\phi'w'} \}. \quad (\text{B10d})$$

Thus, the transformed vertical eddy energy flux $\{ \rho \overline{\phi'w'} \}$ due to topographically forced waves coincides with cF in the inviscid and adiabatic limit.

APPENDIX C

Energy Equations in Spherical Pressure Coordinates

The conventional energy equations (Lorenz, 1955; Hayashi, 1980) in spherical and pressure coordinates are given by

$$\bar{K}_t = -(\bar{K} \cdot K') + (\bar{P} \cdot \bar{K}) + C(\bar{K}) + \bar{u}\bar{X}, \quad (\text{C1})$$

$$\bar{P}_t = -(\bar{P} \cdot P') - (\bar{P} \cdot \bar{K}) + C(\bar{P}) + a\bar{T}\bar{Q}, \quad (\text{C2})$$

$$K'_t = (\bar{K} \cdot K') + (P' \cdot K') + C(K') + \bar{u}'\bar{X}', \quad (\text{C3})$$

$$P'_t = (\bar{P} \cdot P') - (P' \cdot K') + C(P') + a\bar{T}'\bar{Q}'. \quad (\text{C4})$$

The kinetic and available potential energies of the mean and eddies are defined by

$$\bar{K} = (\bar{u}^2 + \bar{v}^2)/2, \quad (\text{C5})$$

$$\bar{P} = a(\bar{T} - \bar{T}')^2/2, \quad (\text{C6})$$

$$K' = (\bar{u}'^2 + \bar{v}'^2)/2, \quad (\text{C7})$$

$$P' = a\bar{T}'^2/2, \quad (\text{C8})$$

where a is defined as

$$a = \frac{R}{p(\kappa \bar{T}/p - \partial \bar{T}/\partial p)}, \quad (\text{C9})$$

with $\kappa = 2/\gamma$ and \bar{T} denoting the temperature averaged over the globe and time.

The energy conversion terms are approximately given by

$$(\bar{K} \cdot K') = -\left(\frac{\partial \bar{u}}{r \partial \theta} + \frac{\tan \theta}{r} u \right) \overline{u'v'} - \frac{\partial \bar{u}}{\partial p} \overline{u'w'}, \quad (\text{C10})$$

$$(P \cdot P') = -a \frac{\partial \bar{T}}{r \partial \theta} \overline{v'T'}, \quad (\text{C11})$$

$$(\bar{P} \cdot \bar{K}) = \alpha \bar{\omega}, \quad (\text{C12})$$

$$(P' \cdot K') = \alpha' \overline{w'}, \quad (\text{C13})$$

$$C(\bar{K}) = -\frac{\partial}{\partial y} (\bar{\phi}\bar{v} + \overline{uu'v'}) - \frac{\partial}{\partial p} (\bar{\phi}\bar{\omega} + \overline{uu'w'}), \quad (\text{C14})$$

$$C(\bar{P}) = -a \frac{\partial}{\partial y} (\bar{T}\bar{T}'v'), \quad (\text{C15})$$

$$C(K') = -\frac{\partial}{\partial y} (\phi'v') - \frac{\partial}{\partial p} (\phi'w'), \quad (\text{C16})$$

$$C(P') = 0, \quad (\text{C17})$$

where α is the specific volume, ω the vertical pressure

velocity, r the radius of the earth, θ the latitude, and the y -derivative is replaced by

$$\frac{\partial(\quad)}{\partial y} = \frac{\partial \cos\theta(\quad)}{r \cos\theta \partial \theta}. \quad (\text{C18})$$

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