

The GFDL Finite-Volume Cubed-Sphere Dynamical Core Structure and Usage

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for the GFDL FV3 Team
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## FV3 Reference Info

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FV3: Finite-Volume Cubed-Sphere Dynamical Core

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\text { FV3 Home } \quad \text { Key Components } \quad \text { Grids } \quad \text { Performance }
$$

The GFDL Finite-Volume Cubed-Sphere Dynamical Core (FV3) is a scalable and flexible dynamical core capable of both hydrostatic and non-hydrostatic atmospheric simulations. The design of FV3 was guided by these tenets:

1. The discretization should be guided by physical principles as much as possible.
2. A fast model can be a good model, but a good model must be a fast model! Computational efficiency is crucial. FV3 was "reverse engineered" to incorporate properties which have been used in engineering for decades, but only first adopted in atmospheric science by FV3.
FV3 is the dynamical core for all GFDL weather and climate models, including the world-leading AM4 and the powerful SHiE unified weather-to-subseasonal prediction model and SPEAR seamless coupled prediction system. FV3 has been chosen as the dynamical core for the Unified Forecast System (UFS), formerly the Next Generation Global Prediction System project NGGPS). The UFS is designed to unify the National Weather Service's suite of prediction models, including the operational slobal Forecast System (GF5) and regional models to run as a unified, fully-coupled system in the NOAA Environmental Modeling System infrastructure. FV3 was successfully implemented within the GFS, and the FV3-based GFSV15 became perational on 12 June 2019 . Since then, the GFS and GFS Ensemble have both received further updates to FV3 and oth Ensemble Forecast ARREF) and form a key role in the upcoming implementations of the Hurricane Analysis and Forecast System (HAFS) and Rapidy-Refreshing Forecast System (RRFS).
FS is also the dynamical core for NASA's GEOS global model, NASA's next-generation Mars Climate Model, and for other systems worldwide.
his website describes FV3, including the evolution of its development, basic algorithm, and its global variable resolution


FV3 Portal: www.gfdl.noaa.gov/fv3

A Scientific Description of the GFDL
Finite-Volume Cubed-Sphere
Dynamical Core

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\text { Revision v1.0a } 16 \text { June } 2021
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GFDL Weather and Climate Dynamics Division
Technical Memorandum GFDL202100


Harris et al. (2021)
Comprehensive FV3 Scientific Documentation on GitHub and NOAA Institutional Repository

## FV3 Community GitHub


github.com/NOAA-GFDL/GFDL_atmos_cubed_sphere
Official site for FV3 releases, examples, issue tracking, documentation, and more


Examples directory: Jupyter notebooks demonstrating FV3 capabilities. Updates released regularly.

## SHiELD and solo FV3 Container

- Convenient, portable, and reproducible SHiELD and FV3 demonstration
- 35-km regional and nested domains
- Includes idealized "solo_core"
 FV3 tests
- Docker and Singularity containers run on supercomputers, workstations, and laptops


## Literature

Cheng et al. 2022a
Jeevanjee and Zhou (2022)


FV3: The GFDL Finite-Volume Cubed-Sphere Dynamical Core

Lin \& Rood 1996
Efficient 2D high-order conservative FV transport

## FV3 for the 2020s

Rigorous Thermodynamics Flexible dynamics
Adaptable physics interface Variable-resolution techniques Regional \& periodic domains Powerful initialization, DA, and nudging functions

Harris \& Lin 2013, 2016
Variable resolution with two-way nesting and Schmidt grid stretching

Lin \& Rood 1997
FV horizontal solver focusing on nonlinear vorticity dynamics

## The FV3 Way

$>$ Physical consistency
$>$ Fully-FV numerics
$>$ Component coupling
$>$ Computational efficiency


Lin 1998-2004 FV core with "floating" Lagrangian vertical coordinate


Putman \& Lin 2007
Scalable cubed-sphere grid, doubly-periodic domain Consistent Lagrangian nonhydrostatic dynamics

The Global FV3 Community

Past, present, future earth and beyond



## Finite-Volume Dynamical Cores

- All variables are 3D cell- or face-means...not gridpoint values
- We solve not the differential Euler equations but their cell-integrated forms using integral theorems
- Everything is a flux, including the momentum equation. Fully FV!
- Mass conservation ensured to rounding error
- C-D grid: Vorticity computed exactly; accurate divergence computation
- Mimetic: Physical properties recovered by discretization, particularly Newton's $3^{\text {rd }}$ law
- Fully compressible: calculation is horizontally local
- Flow-following Lagrangian vertical coordinate
- FV3 is a fully forward-in-time solver with backwards PGF and acoustic terms


## FV3 time integration sequence

- FV3 is a forward-in-time solver with multiple levels of timeintegration
- Flux-divergence terms and physics tendencies evaluated forward-in-time
- Pressure-gradient and sound-wave terms evaluated backward-in-time for stability
- HEVI: Everything is explicit in the horizontal but implicit in the vertical
- Lagrangian vertical coordinate: flow constrained along time-evolving Lagrangian surfaces. This greatly simplifies the inner "acoustic" or "Lagrangian dynamics" timestep.



## The Cubed-Sphere Grid

 The 3 in FV3- Gnomonic cubed-sphere grid: coordinates are great circles but nonorthogonal
- Solution winds are covariant, advection is by contravariant winds
- Winds $u$ and $v$ are defined internally in the local coordinate; output is always rotated to earth-relative coords
- Special handling at edges and corners


## FV3 Documentation

Chapter 3


## The Cubed-Sphere Grid and Arbitrary Grid Domains

FV3 uses a global cubed-sphere grid or any arbitrary regular non-orthogonal quadrilateral grid This permits Schmidt-stretched grids, two-way nested grids, and uniform regional domains



Regional-Domain Grid-cell Width
Courtesy Jim Purser and
Chan-Hoo Jeon (NCEP/EMC)

## FV Advection

- "Reverse-engineered" forward-in-time 2D scheme constructed from 1D Piecewise-Parabolic Method (PPM) operators
- Mass-conservative
- Correlation-preserving for monotonic limiter
- Cancels splitting error
- Separate Courant number limit in x and y
- Upwinding preserves hyperbolicity and causality
- Tracers are advected with a longer, adaptive timestep using the accumulated mass fluxes
- All quasi-horizontal processes, except PGF, can be represented as advection
- Highly adaptable: Positive-definite tracer advection greatly improves hurricane structure


## FV3 Documentation

## Literature

Chapter 4


## The Piecewise-Parabolic Method: The cornerstone of FV numerics

- Extension to higher order of the Van Leer piecewise-linear method, itself an extension of Godunov's firstorder finite-volume scheme
- The internal variation of each grid cell is approximated by a parabola, from which the fluxes through each cell interface can be integrated


## Literature

Collela \& Woodward 1984


FV3 Documentation

## The Piecewise-Parabolic Method: The cornerstone of FV numerics

- "Vanilla" PPM reconstruction is formally $4^{\text {th }}$ order if $\Delta x$ is constant.
- But you are free to do much more with your degrees of freedom. You can flatten or steepen or ...?
- This is useful for shape-preservation (monotonicity, positive-definite) or for simply eliminating undesirable $2 \Delta x$ noise


## Which solution is the best?

"Accuracy" analyses assume continuous sinusoid modes. They cannot incorporate discontinuities.

Centered-differencing schemes produce a lot of noise at discontinuities!! And staggered grids preserve the junk!

Monotonic schemes are more diffusive-but PPM gives you the freedom to balance shape-
preservation with accuracy
High-order Methods

## Literature

Lin and Rood 1996
X Chen et al. 2018

Notebooks
RHwave, tp_core


## Effect of advection options: 200-mb KE spectra

## WARNING

Variance spectra depend on many
factors, show case-to-case variability, and may not depict scientifically-credible features. Parental discretion is advised.



## T-SHiELD Positive-Definite Advection: Rapid Intensification and Storm Structure

Positive-definite (PD) tracer advection $\rightarrow$ successful rapid intensification (RI) predictions compared to monotonic (MONO)
PD advection enables more WV into eyewall, permitting better updraft and TC structure: may contribute to RI processes

## Literature

Gao et al. 2021



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## Lin-Rood FV Advection

$$
q^{n+1}=\frac{1}{\pi^{n+1}}\left\{\pi^{n} q^{n}+F\left[q^{n}+\frac{1}{2} g\left(q^{n}\right)\right]+G\left[q^{n}+\frac{1}{2} f\left(q^{n}\right)\right]\right\} .
$$

- F, G are flux-form PPM operators, ensuring mass conservation.
- $\mathrm{f}, \mathrm{g}$ are advective form PPM operators.
- This "reverse-engineered" form cancels the leading-order deformation error. The Courant number restriction is then independent in both directions-a truly two-dimensional scheme
- $\max \left(\mathrm{C}_{\mathrm{x}}, \mathrm{C}_{\mathrm{y}}\right) \leq 1$ instead of $\mathrm{C}_{\mathrm{x}}+\mathrm{C}_{\mathrm{y}} \leq 1$


## FV3 Documentation

Chapter 4

## Literature

Lin and Rood 1996
Putman and Lin 2007

## Tracer advection and sub-cycling

- Tracers are advected with a longer timestep than the dynamics
- $\mathrm{U}_{\max } \approx 200 \mathrm{~m} / \mathrm{s}$ but $\mathrm{U}_{\max }+\mathrm{C}_{\mathrm{s}} \approx 540 \mathrm{~m} / \mathrm{s}$
- Split-explicit methods that assume $U \ll c_{s}$ struggle in the stratosphere
- Free-stream preservation: FV3 accumulates mass fluxes during the acoustic timesteps. These fluxes are then used to advect the tracers.
- One or two sub-cycled timesteps is usually enough for stability.
- Adaptively determined timestep from domain-maximum wind speed
- Tracer advection is always monotone or positive definite to avoid new extrema. Explicit diffusion is not used.


## Lagrangian Dynamics in FV3

- FV3 transforms the Euler equations of motion into a Lagrangian vertical coordinate, constraining the flow along quasi-horizontal surfaces
- Lagrangian surfaces deform during the integration. Vertical motion and advection is "free"
- Requires layer thickness $\delta p$ (and $\delta z$ for nonhydro) to be a prognostic variable


## Prognostic Variables

| $\delta p$ | Total air mass (including vapor and condensates) <br> Equal to hydrostatic pressure depth of layer |
| :--- | :--- |
| $\theta_{\mathrm{v}}$ | Virtual potential temperature |
| $\mathrm{u}, \mathrm{v}$ | Horizontal D-grid winds in local coordinate <br> (defined on cell faces) |
| w | Vertical winds (nonhydrostatic) |
| $\delta z$ | Geometric layer depth (nonhydrostatic) |
| $\mathrm{q}_{\mathrm{i}}$ | Passive tracers |

Cell-mean pressure, density, divergence, and specific heat are all diagnostic quantities All variables are layer-means in the vertical: No vertical staggering

## Vorticity Dynamics

- Fluids are strongly vortical at all scales. Vortical motions are especially critical in geophysical flows
- FV3's discretization emphasizes vorticity dynamics:
- Vector-invariant equations: vorticity computed exactly
- C-D Grid Discretization
- Consistent advection of derived vorticial quantities



## Momentum equation

- FV3 solves nonlinear flux-form vector invariant equations using the absolute vorticity fluxes $\Omega \mathrm{v}$, $-\Omega \mathrm{u}$
- D-grid gives exact absolute vorticity $\Omega$ using Stokes' theorem-no averaging!

$$
\frac{\partial \mathbf{V}}{\partial t}=-\Omega \hat{k} \times \mathbf{V}-\nabla\left(\kappa+\nu \nabla^{2} D\right)-\left.\frac{1}{\rho} \nabla p\right|_{z}
$$



- Cell-mean vorticity is advected as a scalar, using the same fluxes as other variables. Products are also advected as scalars!
- ex: Updraft helicity w $\Omega$


## Literature

Lin \& Rood 1996
Harris et al. 2019

## Notebooks

RHwave, HSzuritasuperrotation


## The C-D grid solver

- Flux evaluation requires face-normal and time-mean fluxes.
- The C-grid winds are interpolated and then advanced a half-timestep. These are used to compute the fluxes.

$$
\frac{\partial \mathbf{V}}{\partial t}=-\Omega \hat{k} \times \mathbf{V}-\nabla\left(\kappa+\nu \nabla^{2} D\right)-\left.\frac{1}{\rho} \nabla p\right|_{z}
$$



- Upstream flux also allows consistent computation of the KE gradient term, avoiding the Hollingsworth-Kallberg instability
- Two-grid discretization and time-centered upwind fluxes avoid computational modes, giving FV3 high accuracy and low noise


## Literature

Lin \& Rood 1997

## Backward horizontal pressure gradient force

- Computed from Newton's second and third laws, and Green's Theorem

- Errors lower, with much less noise, compared to traditional evaluations
- Purely horizontal: no along-coordinate projection
- PGF equal and opposite-3 $3^{\text {rd }}$ law! Momentum is conserved
- Curl-free in the absence of density gradients


## FV3 Documentation

Section 6.6

## Literature

Lin 1997
Notebooks
mtn_rest_100km mtn_wave_tests

## The Lagrangian Vertical Coordinate

- Vertical motion and advection is implicit through the deformation of quasihorizontal layers.
- No Courant number restriction or timesplitting
- Computing $\delta p$ and $\delta z$ is sufficient for vertical advection.
- Periodically, a high-order conservative remapping back to the reference "Eulerian" coordinate is done to avoid $\delta p \rightarrow 0$



## Semi-implicit nonhydrostatic solver

- Semi-implicit solver cleanly extends FV Lagrangian dynamics into nonhydrostatic regime
- Start with advected $\mathrm{w}^{*}, \mathrm{z}^{*}$
- Consistent with other variables
- Vertical pressure gradient and non-advective changes to layer depth $\delta z$ are solved by semi-implicit solver
- Simultaneous solution for $w$ and $\delta z$ through diagnosed $p^{\prime}$
- $p^{\prime}$ accurately interpolated to interfaces using cubic spline

$$
\begin{gathered}
\frac{\partial}{\partial \mathrm{t}} \delta z^{*}=\delta w^{*} . \\
\frac{\partial}{\partial \mathrm{t}}\left(w^{*} \delta m\right)=\delta p^{\prime}, \\
\frac{\partial p^{\prime}}{\partial \mathrm{t}}=\gamma \mathrm{p} \frac{\delta w^{*}}{\delta z^{*}} . \\
p=\left(\frac{\delta m}{\delta z} R_{\mathrm{d}} \Theta_{v}\right)^{\gamma} \\
p=p^{*}+p^{\prime}
\end{gathered}
$$

- Vertically-propagating sound waves weakly damped. That's OK.


## FV3 Documentation

Section 7.1

## Lagrangian nonhydrostatic dynamics, how do they work?

- Recall that FV3 uses a hybrid-pressure coordinate. Cell mass $\delta$ p is constant during sound wave processes.
- Nonhydrostaticity creates pressure perturbation
- $p$ computed by ideal gas law, incorporating heating
- $p^{*}$ computed through $\delta p$ above
- Vertical gradients in $p^{\prime}$ create vertical accelerations, deforming the Lagrangian interfaces
- Elastic straining (expansion/compression) of the Lagrangian layers alters $\delta z$
- Adiabatic changes to $\delta z$ changes $p^{\prime}$...

$$
\begin{gathered}
\frac{\partial}{\partial t} \delta z^{*}=\delta w^{*} . \\
\frac{\partial}{\partial t}\left(w^{*} \delta m\right)=\delta p^{\prime}, \\
\frac{\partial p^{\prime}}{\partial t}=\gamma p \frac{\delta w^{*}}{\delta z^{*}} . \\
p=\left(\frac{\delta m}{\delta z} R_{d} \Theta_{v}\right)^{\gamma} \\
p=p^{*}+p^{\prime}
\end{gathered}
$$

## FV3 Documentation

Section 7.2

- MYTH: Numerical diffusion is evil, only used to cover for discretization deficiencies, and should be avoided at all costs.
-TRUTH: Numerical diffusion is a necessary part of any model used for environmental simulation.


## Numerical Diffusion and Physical Dissipation

- All useful atmospheric models have grid-scale motions removed by numerical diffusion (whether they know it or not).
- Energy cascades to grid scales and must be removed since dissipative scales ( $0(1 \mathrm{~cm}$ )) are not explicitly resolved
- Models aren't perfect, noise and errors must be removed
- C-grids produce particularly prodigious noise at discontinuities
- Diffusion is also a powerful tool to improve simulations
- Tompkins and Semie 2017; Pressel et al. 2017; see also Implicit LES


## FV3 Documentation

Sections 8.1, 8.2

## Literature

Zhao et al. 2012
X Chen et al. 2018

## The Turbulent Energy Cascade

"Big whirls have little whirls that feed on their velocity, And little whirls have lesser whirls and so on to viscosity"
-Lewis F. Richardson, 1922

Kinetic energy cascades from the large energy-containing scale to increasingly small-wavelength modes.
In a continuous fluid, the cascade continues until molecular diffusion can dissipate kinetic energy to heat.

In large-scale flows this is complicated by a second upscale turbulent cascade.


Ecke: "The Turbulence Problem" (2005)

## Damping in FV3

## FV3 Documentation

Sections 8.3, 8.4, 8.5

- FV3's physical consistency produces very few computational modes and thus can be minimally-diffusive.
But well-configured diffusion can give improved results
- FV3 applies no direct implicit diffusion to divergent modes which cascade to grid scale unimpeded. Scale-selective divergence damping represents their physical dissipation.
- Rotational modes can be damped implicitly by monotonic advection or explicitly by vorticity damping.
- For consistency also damps $\delta p, \delta z, \theta_{\mathrm{v}}, \mathrm{w}$.

No explicit damping for tracers.

- Note that all implicit (except vertical remapping) and explicit diffusion is along Lagrangian surfaces.


## The Upper Boundary

- FV3 has a flexible constant-pressure upper boundary, greatly reducing reflection of vertically-propagating gravity waves. So the sponge layer can be much shallower.
- Much less problematic than constant-height rigid lid upper boundaries
- In FV3 the top two layers are reserved as sponge layers.
- These layers are very deep ( $\Delta z$ ), already implicitly dissipating verticallypropagating wave.
- In these layers a much stronger, less scale-selective second-order damping is applied on the acoustic timestep.
- Tunable Rayleigh damping and a $2 \Delta z$ filter are also available. Both convert damped KE to heat-energy conserving


## Variable-resolution techniques

## FV3 Documentation

- Variable-resolution is the future of convectivescale modeling (C-SHiELD, HAFS-B)
- Stretched global grid is the easy, simple way to grid refinement.
- Two-way nesting is flexible and highly configurable.
- Inflow BCs "baked-in" to numerics
- Nesting methodology designed to be consistent with numerics
- Concurrent nesting is extremely efficient: Run as many grids as you want at the same time!



## Telescoping Nesting

As many levels as you want


C768_1n3


Triple-nests onto Hurricane Laura

## Literature

Mouallem et al. 2022


Tele-SHiELD: 1.4 km onto Northeast Corridor and Taiwan

## Rigorous Thermodynamics and Physics-Dynamics Coupling

- Mass $\delta p$ in FV3 includes water vapor and all condensates. Condensate loading and moist-mass effects are baked-in.


## FV3 Documentation

Sec. 7.2, Chapter 9

- FV3 incorporates the heat content of water vapor and condensates in adiabatic processes and diabatic heating
- Diabatic heating is applied consistently with the dynamics
- $c_{p}$ in hydrostatic: $\delta p$ constant, $\delta z$ dependent on $T$ (hypsometric equation)
- $\mathrm{c}_{\mathrm{vm}}$ in nonhydrostatic: $\delta z$ constant, p dependent on T (ideal gas law)


## LMARS horizontal solver and Duo-Grid

- Efficient Riemann solver allows accurate unstaggered solution to improve physics coupling

- Shows way to true total energy conservation
- Duo-grid eliminates grid imprinting and provides unified approach to cubed-sphere grid design



## Al2 Pace

Accelerating to km-scale


- The CPU-MPI era is (?) ending
- GT4py Domain Specific Language (DSL) Model re-written in Python and compiled to optimized code for any processor
- GridTools in operations at MeteoSwiss
- Evaluation underway by ECMWF and MPI
- Pace: GT4py implementation of FV3GFS Performance + Python Flexibility


## Literature

Ben-Nun et al. 2022
Gibbon et al. 2022

Python user code


## GT4py FV3



Lines of Code vs. FORTRAN: 0.42x Speedup: 3.92x (P100), 8.48x (A100)

## Literature

Ben-Nun et al. 2022
Gibbon et al. 2022

## 2.6 km FV3 Dycore



Socket-for-socket comparison on Piz Daint (CSCS Switzerland, Intel Haswell + NVIDIA P100)


