

Theories/models must be **elegant**
if they are to have lasting value

Elegance \Leftrightarrow capturing key sources of
complexity in as simple a dynamical framework as possible;

Claim: Our understanding of the climate system
in the 21st century
will be embedded in **elegant hierarchies** of climate models

Hope: simulation models will eventually become elegant
by being subsumed within these hierarchies

Importance of Model Hierarchies

Molecular Biology

Climate theory

- Hierarchy provided by nature
 - **Experimental** science
 - Nature of evolution => simpler organisms directly relevant to Man
 - Relatively easy to focus on model organisms (E.coli, fruit fly)
- Must create own hierarchy
 - **Theoretical** science
 - Relevance depends on imperfect ability to design appropriate models
 - Difficult to focus attention of community on specific models

$$\frac{\partial q_i}{\partial t} = -J(\psi_i, q_i) + \text{Heat} + \text{Friction} \quad (i = 1, 2)$$

Conservation of QG PV

$$J(\psi, B) \equiv \frac{\partial \psi}{\partial x} \frac{\partial B}{\partial y} - \frac{\partial \psi}{\partial y} \frac{\partial B}{\partial x}$$

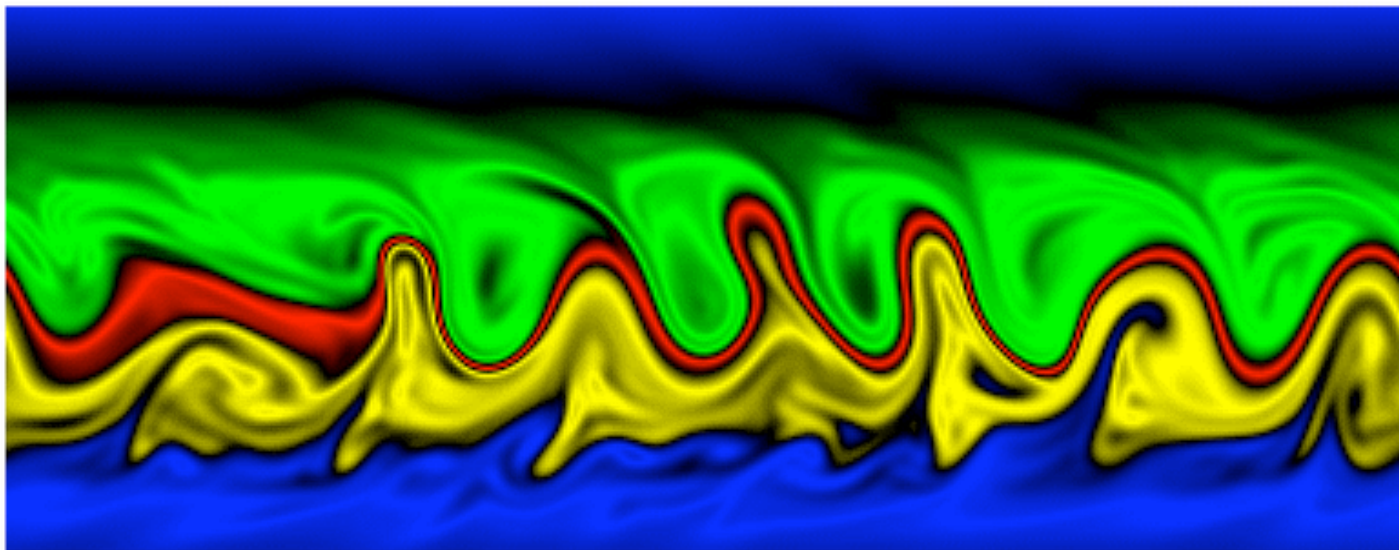
Geostrophic advection

$$q_1 = \beta y + \nabla^2 \psi_1 - \frac{\psi_1 - \psi_2}{\lambda^2}$$

PV in upper layer

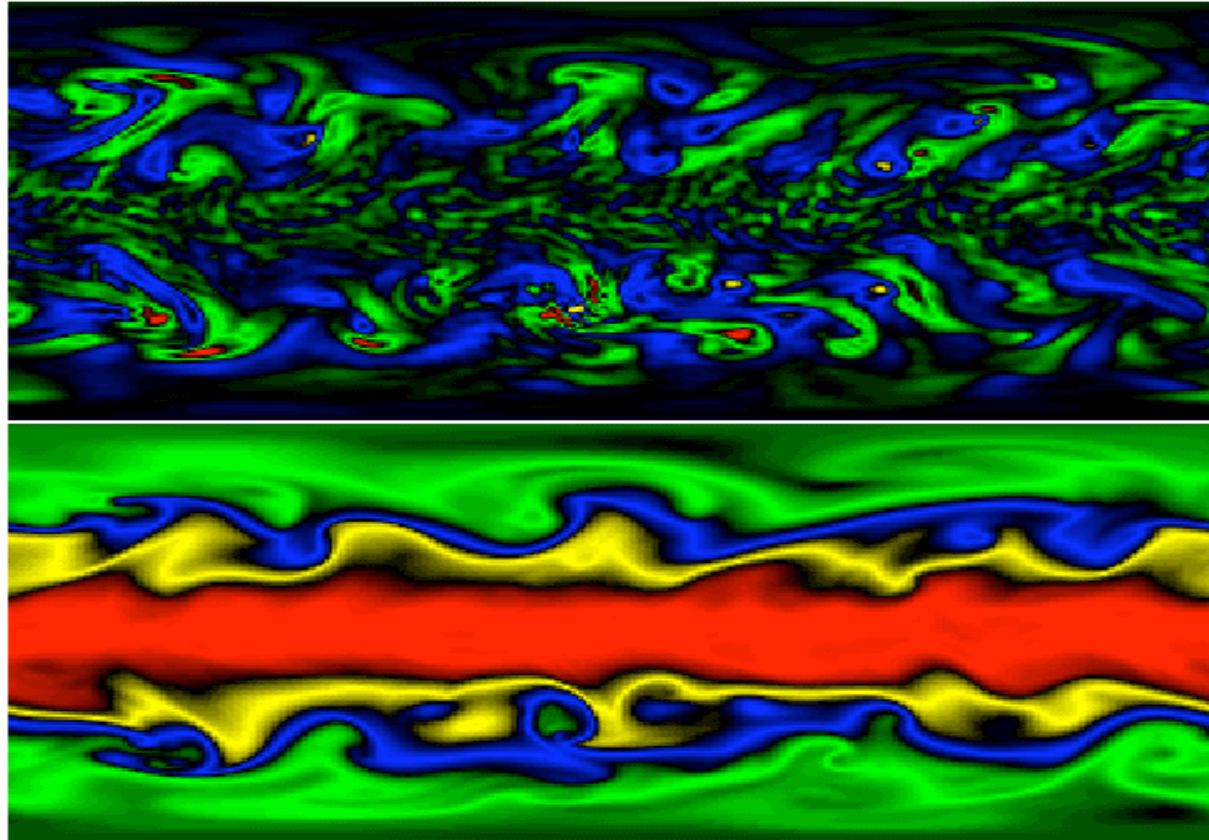
$$q_2 = \beta y + \nabla^2 \psi_2 + \frac{\psi_1 - \psi_2}{\lambda^2}$$

PV in lower layer



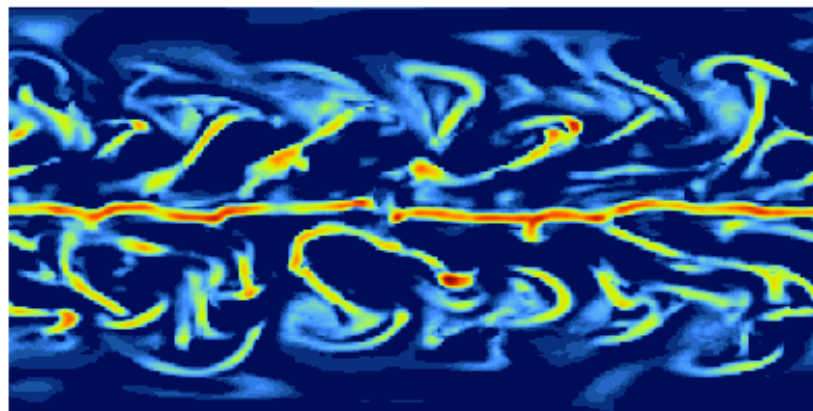
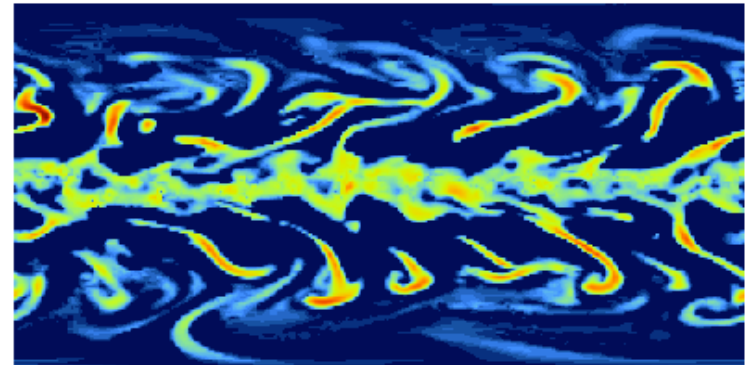
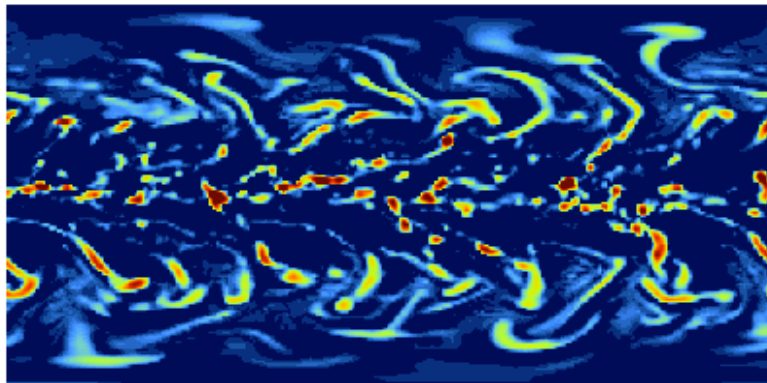
The **E. Coli.** of climate models?

Dry atmosphere, zonally symmetric climate,
no seasons, no diurnal cycle



The fruit fly of climate models?

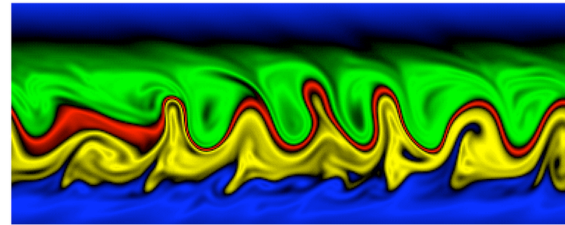
Idealized moist atmospheric model
Zonally symmetric climate,
No seasons, no diurnal cycle, no clouds
3 different idealized convection schemes shown



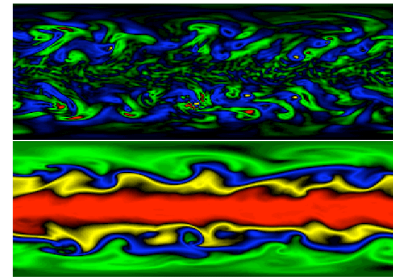
The mouse of climate models

Some open questions in general circulation theory

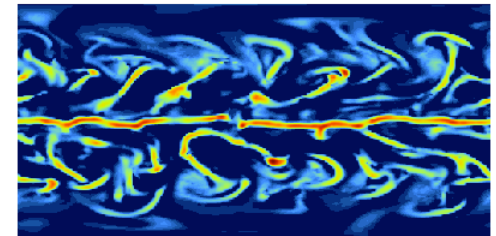
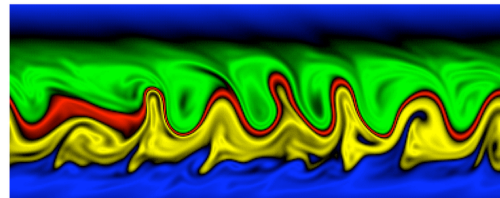
- Poleward eddy heat flux



- Momentum transport



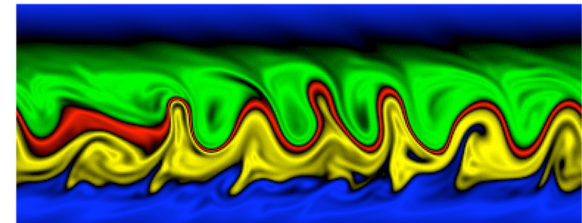
- Latent heat release
- and midlatitude eddies

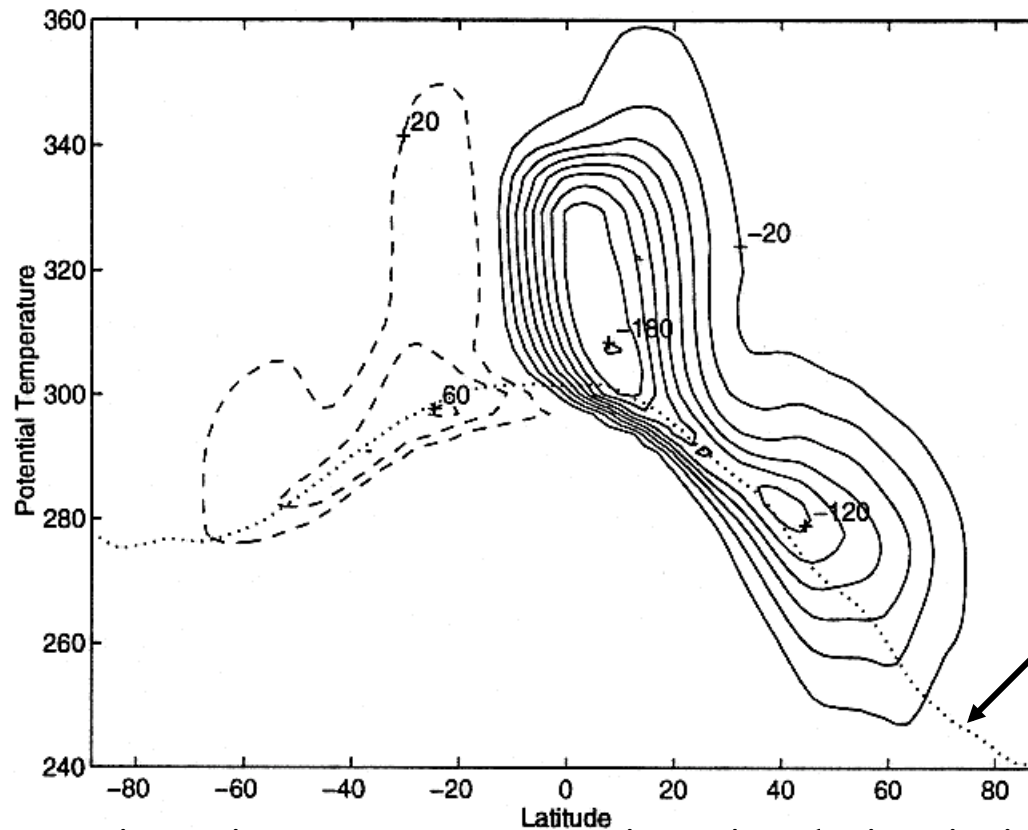


Theory for baroclinic eddy heat fluxes is at the heart of any theory of the general circulation:

I have personally focused on a diffusive picture – why?

- 1) Experience with utility of simple diffusive energy balance models
- 2) QG simulations show that there can be scale separation between scale of eddies and scale of the mean inhomogeneity in direction of transport
- 3) Possibility of creating “laboratory” for measuring (and cleanly testing hypotheses about) diffusivity in homogeneous turbulence model





From Held and Schneider,
1999

Mean surface
Pot. temperature

Wintertime mass overturning circulation in isentropic coordinates;
(most of the return flow occurs in “cold air outbreaks”
at temperatures colder than the mean)

In QG theory the isentropic depth of this layer is $O(\epsilon)$

Actual surface return flow occurs in layer with
isentropic depth comparable to that of poleward flow,
as in 2 layer model!

Homogeneous theory for 2-layer QG model

External parameters:

β = Planetary vorticity gradient

λ = radius of deformation

U = imposed vertical shear ($U_1 - U_2$)

(ignore parameters describing dissipation)

Predicting

ε = rate of (inverse) cascade of energy =
rate of dissipation of kinetic energy

\mathcal{D} = diffusivity (of what?)

Key hypothesis:

Inverse energy cascade stop at the Rhines' scale

$$\Rightarrow \text{Diffusivity} = \mathcal{D} \sim \varepsilon^{3/5} \beta^{-4/5}$$

Homogeneous theory for 2-layer QG model

$$\mathcal{D} \sim \varepsilon^{3/5} \beta^{-4/5}$$

But diffusivity for what?

Held + Larichev: in the limit as $\beta \Rightarrow 0$,
PV gradient in either layer \sim thickness gradient in either layer \Rightarrow
Can think of \mathcal{D} interchangeably as diffusivity for PV or
thickness in either layer

$$\varepsilon = \mathcal{D} U^2 \lambda^{-2} = \mathcal{D} T^{-2} \Rightarrow$$

$$\mathcal{D} \sim \beta^{-2} T^{-3}$$

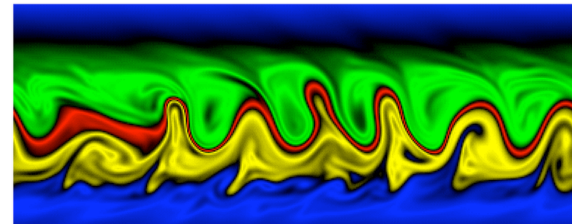
Lapeyre and Held (2003):

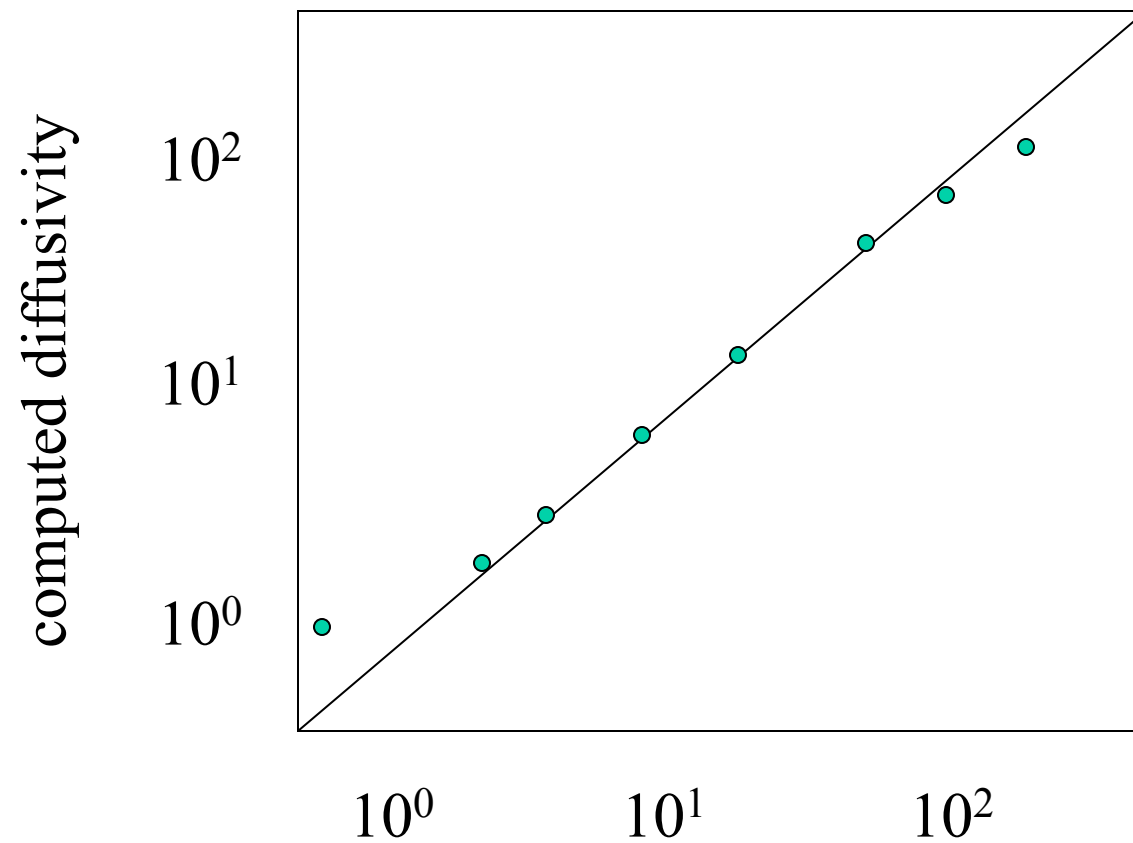
for arbitrary β , diffuse lower layer PV - why?

$\mathcal{D}_1 = \varepsilon^{3/5} \beta^{-4/5}$ then implies

$$\mathcal{D}_1 = (U\lambda) \xi^2 (1 - \xi^{-1})^{3/2}$$

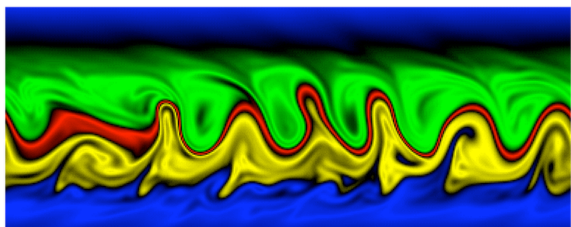
$$\xi = U/(\beta\lambda^2)$$

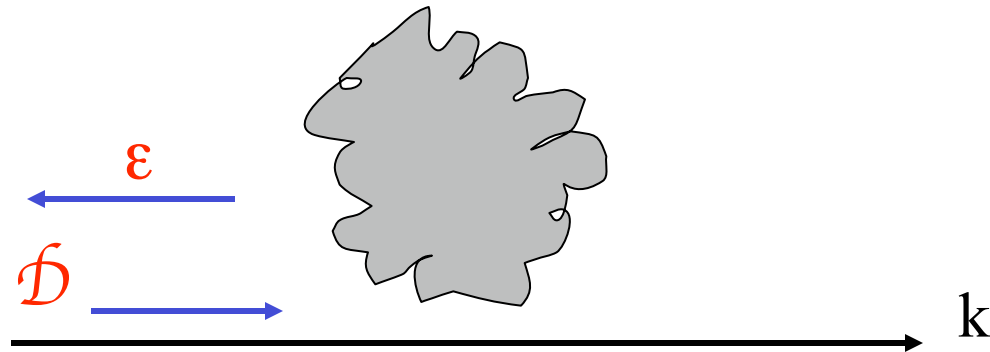




Predicted diffusivity

$$\xi^2(1 - 1/\xi)^{3/2}$$





Barry, Craig, Thuburn: 2002

Dry entropy: $[\varepsilon/T] \sim \varepsilon/T = [Q/T] = [(\mathbf{div} \mathbf{F})/T] = [\mathbf{F} \cdot \mathbf{grad} T]/T^2$

To connect to QG version, suppose \mathbf{F} is oriented along isentropes:

$$\varepsilon \sim \mathbf{grad}_{\Theta} T |\mathbf{F}| S, \quad S = \text{isentropic slope}$$

$$\sim (g/c_p) |c_p v' T'| S \sim D dT/dy S \sim D U^2 \lambda^{-2}$$

Moist entropy:

need to account for entropy generation due to “moist processes”

Moist static energy flux = $D dh/dy$ (moist enthalpy gradient)

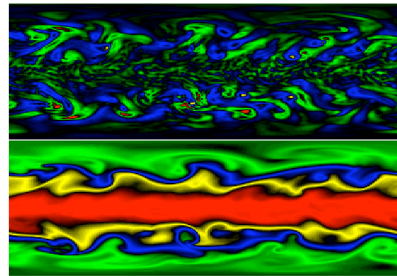
Mixing slope steepened (reduced effective static stability)

$$\varepsilon \sim \gamma D dh/dy S$$

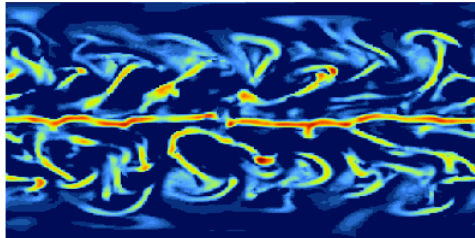
But have we learned anything about the atmosphere?

The D- ϵ relation: $D \sim \epsilon^{3/5} \beta^{-4/5}$ does not appear to hold in

either



or



Essence of the problem:

- static stability maintenance?
- need to think more clearly about moist entropy?

A moist 2-layer QG model
(Lapeyre and Held)

add moisture variable **m** to lower layer
Precipitation does not allow m to exceed m_s

$$\frac{D_2 m}{Dt} + m_0 \nabla \cdot \mathbf{u}_2 = E - P + \dots$$

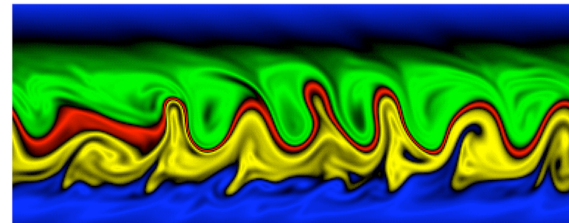
$$\frac{m_s}{m_0} = \mathcal{C} \frac{\eta}{H_2} = \mathcal{C} \frac{f_0}{g^* H_2} (\psi_1 - \psi_2)$$

Moist available potential energy

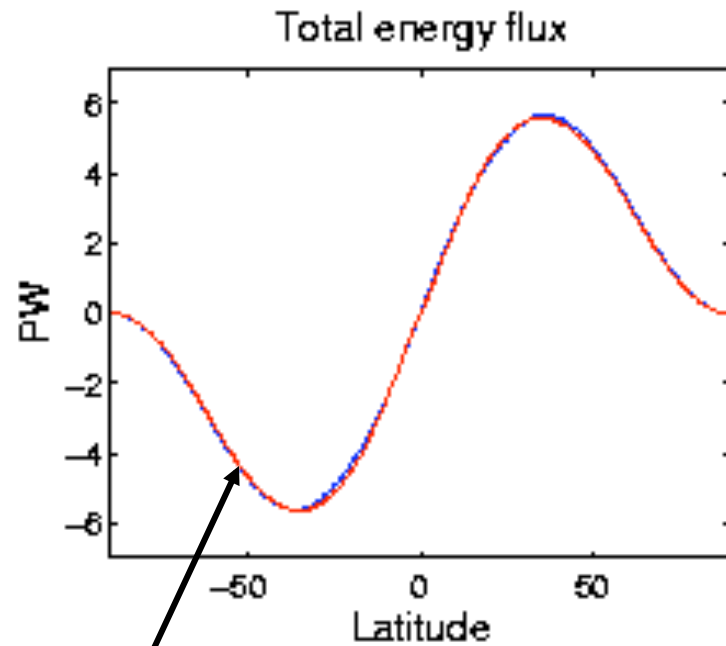
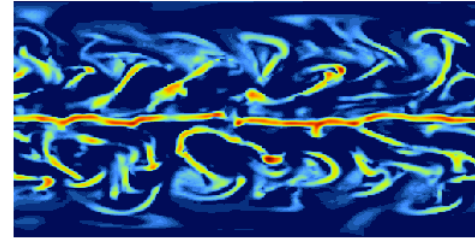
$$\frac{\partial}{\partial t} MAPE = \langle v_2 q_m \rangle - \mathcal{N} - \mathcal{F}_m - \mathcal{F}_T - \epsilon$$

$$\mathcal{N} = \langle E(m_s - m) \rangle$$

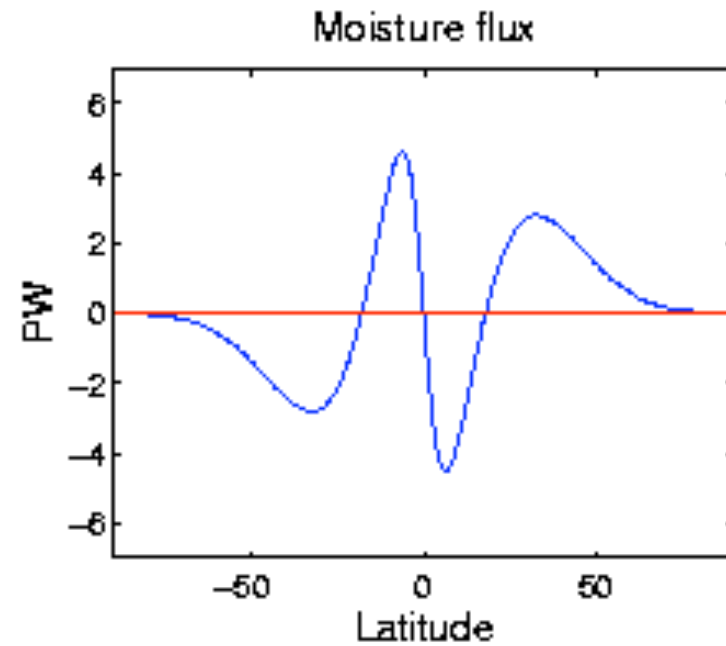
$$MAPE = a \langle (m_s - m)^2 \rangle + b \langle (\eta + \mathcal{L}m)^2 \rangle$$



Idealized moist GCM



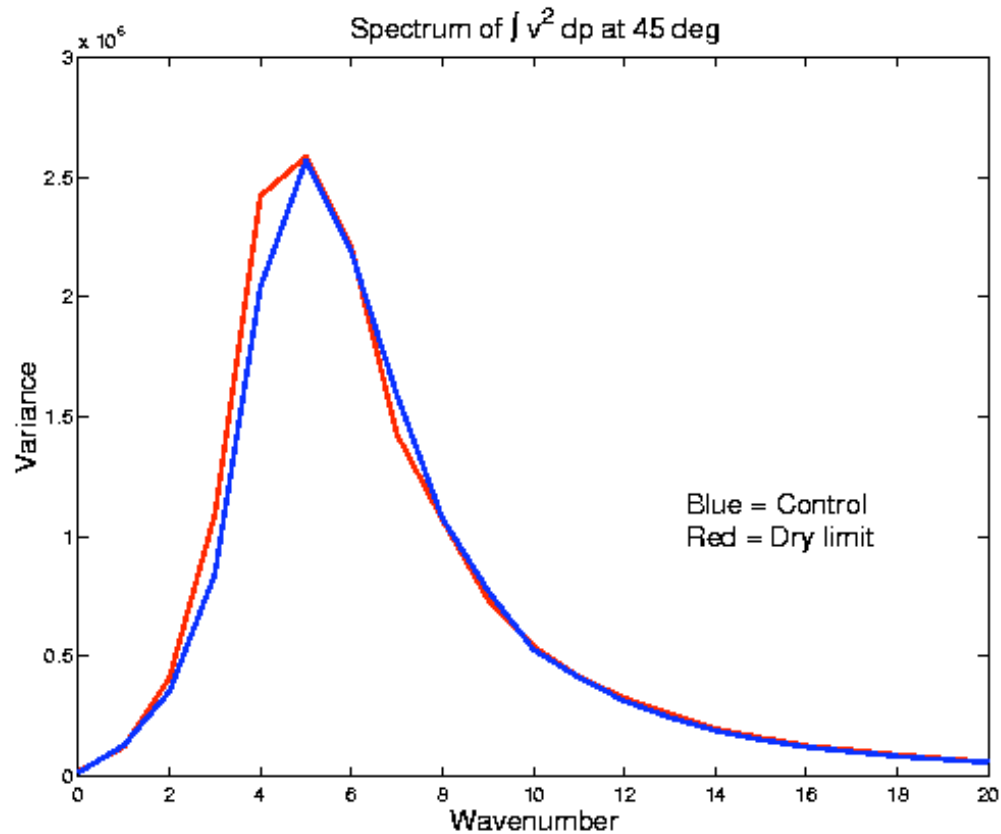
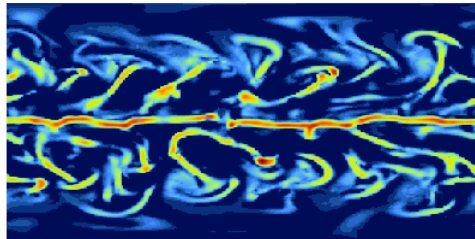
Both control and dry
total poleward energy fluxes



Poleward latent flux in control

Fluxes identical to within ~1% despite very different
(dry) static stabilities and surface temperature gradients

Zonal spectrum of v' in idealized moist model



Eddy scale does not change despite increase in dry stability

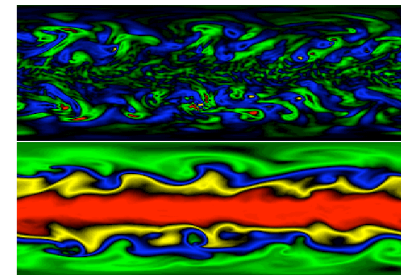
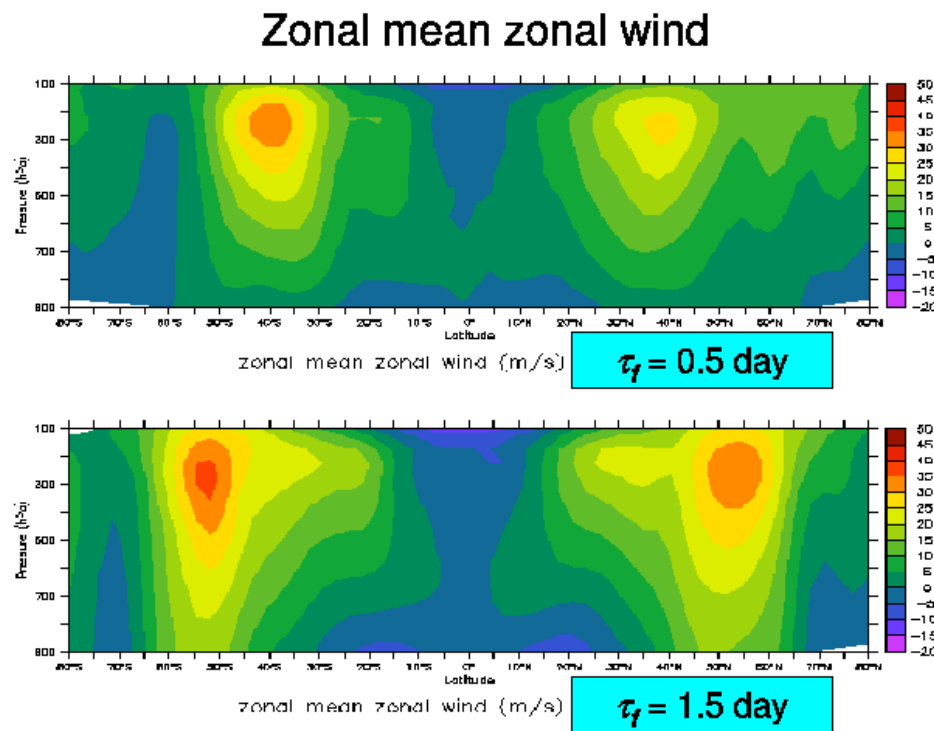
Moral:

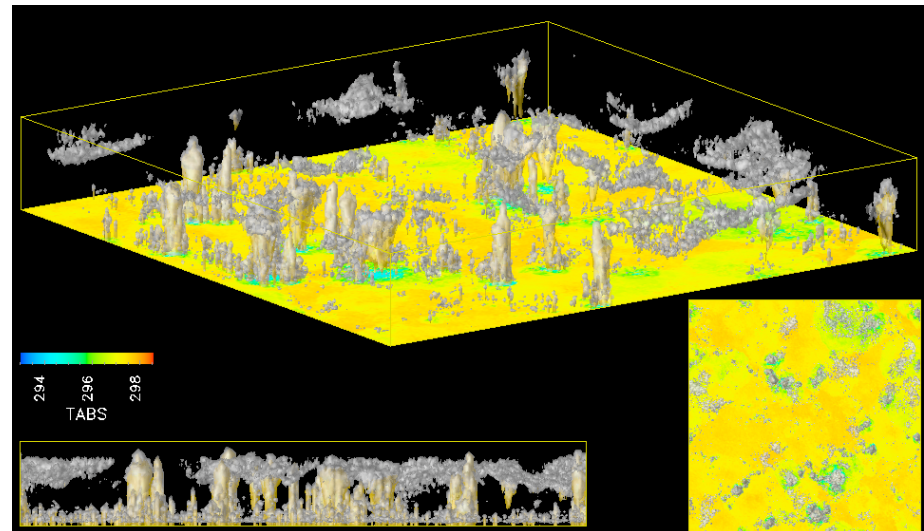
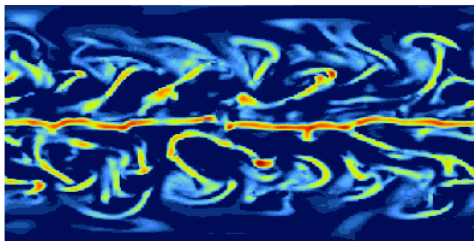
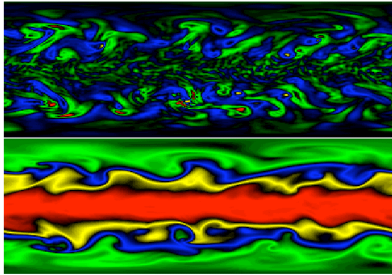
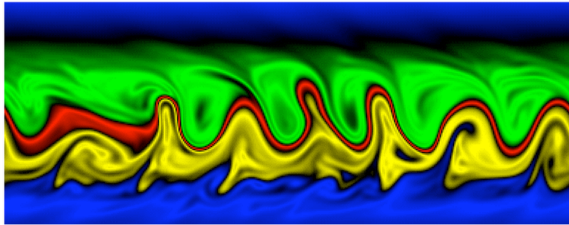
Theory for moist mid-latitude eddies in a moist atmosphere must consider eddies as moist entities, not as dry entities modified by latent heat release

Changing the strength of surface friction

Gang Chen,
AOS/Princeton

following W. Robinson





Radiative-Convective Equilibrium

Courtesy of
P. Blossey, C. Bretherton, U. of Washington