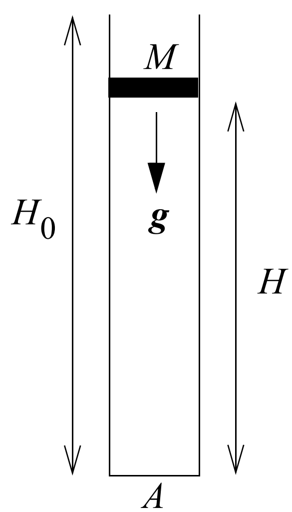
AOS 571 Midterm Test

October 2009

1. Oscillations in a sealed tube.

A disk of mass M just fits into an upright tube of height H_0 and cross-section A that is open at the top. The ambient air density and pressure are approximately constant, say ρ_0 and p_0, respectively. The acceleration due to gravity is g and the air is an ideal gas.

a) The disk is placed in the tube and allowed to come to rest. Find an expression for the equilibrium height H of the disk, assuming that the air in the tube is in *thermal* equilibrium with the surroundings, that the disk forms an air-tight seal with the sides of the tube, and that the effects of friction can be neglected. Hint: start by relating H to the change in density in the sealed part of the tube assuming H is much smaller than the density scale height.



b) Now suppose the tube is perfectly insulated so that the potential temperature θ_0 is preserved? Is the equilibrium level higher or lower, and by how much? Hint: the pressure now varies as $(\rho/\rho_0)^\gamma$ and $\gamma \neq 1$.

c) Assume that the disk is now perturbed *slightly* from its equilibrium position. What is the frequency of its oscillation in each of the scenarios (a) and (b) above?

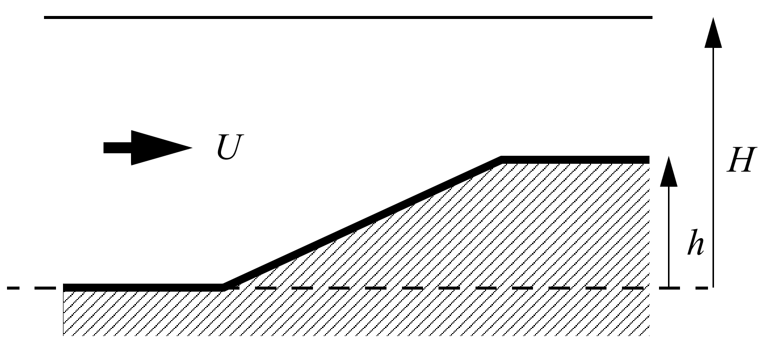
d) Finally, suppose $M = \rho_M hA$ where ρ_M is the density of the disk. Discuss the special case where $\rho_M = \rho_0$ and $h = H$. In particular, how is h related to the properties of a linear sound wave?

2. Flow in a shallowing channel.

Consider a straight, shallow channel with constant width L but bottom elevation rising from $z = 0$ at the far left to $z = h$ at the far right. There is a rigid lid at $z = H > h$. A homogeneous, inviscid fluid flows from left to right, entering with depth H and uniform speed U.

a) Determine the profile $u(y)$ at the far right when: *i*) the channel is not rotating; *ii*) the channel is on an f-plane.

b) In terms of h/H, what is the maximum nondimensional width $L(f/U)$ that allows a steady flow without any changes in the upstream conditions?



3. Scale analysis in a rotating atmosphere.

Consider a rapidly rotating Boussinesq fluid on an *f*-plane. "Rapidly rotating" means the rotation period is short compared to the advective time scale.

a) Restricting attention to horizontal variations, show that the pressure divided by the density scales like $\phi \sim fUL$.

b) Split the horizontal velocity into a "geostrophic" part $\boldsymbol{V_g} = f^{-1}\nabla^{\perp}\phi$ and an "ageostrophic" residual $\boldsymbol{V_a} = \boldsymbol{V} - \boldsymbol{V_g}$. Show that the ratio of magnitudes of $\boldsymbol{V_a}$ to $\boldsymbol{V_g}$ is estimated by $U/(fL)$.

c) Show that the scaling $W \sim UH/L$ is an overestimate for the magnitude of the vertical velocity by deriving the correct estimate. What is the weakest condition on H/L to ensure hydrostatic balance?

d) For the buoyancy, write $b = N^2 z + b'$, with N^2 constant. The corresponding pressure decomposition is $\phi = N^2 z^2/2 + \phi'$ and you already have a scale for ϕ'. Find two scales for b' -- one based on hydrostatic balance and one based on buoyancy conservation. Use these to show that $L \sim NH/f$.

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